MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.630 Electromagnetics — Issued: Week 7
Quiz No. 1 — Time: 3:00pm-5:00pm

Problem 1 (30%)

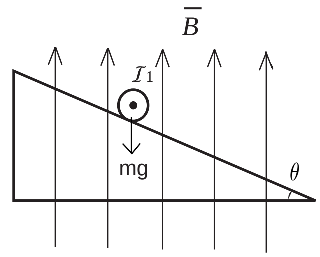

Fig.1(a)

(a) A conducting bar with a current I_1 (with direction as indicated in Fig. 1(a)) is on a smooth slope.If there is an external static magnetic field $\overline{B}$ pointing upwards, as shown in Figure 1(a), write down the magnitude of $\overline{B}$ at which the bar can rest on the slope. (Assume the bar's length is infinite and its gravity per unit length is mg.)

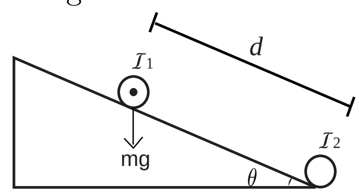

Fig.1(b)

(b) Consider the setup as shown in Fig. 1(b). One conducting bar on the smooth slope is carrying a current I_1. Another conducting bar is carrying current I_2, sitting at the bottom of the slope, parallel to the former bar and keeping it at rest on the slope. What is I_2 and is I_2 flowing into the paper or out of the paper?

Problem 2 (30%)

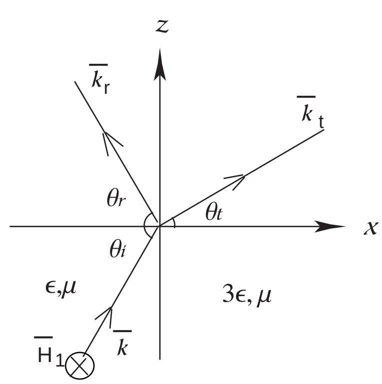

Fig.2

Consider the diagram as shown in Fig. 2. An incident plane wave is propagating in a homogeneous medium with permittivity ϵ and permeability μ for $x < 0$. The magnetic field of the incident wave is

$$\overline{H}_1 = \hat{y} H_0 \cos(\frac{\sqrt{3}}{2}kz + \frac{1}{2}kx - \omega t)$$

(a) Write out the $\overline{k}$ vector and the electric field $\overline{E}_1$ of the incident wave.
(b) Is the incident wave a TE or TM wave?
(c) The half space $x > 0$ is filled with another material with permittivity 3ϵ and permeability μ. What's the reflection coefficient?
(d) Suppose the incident electric field is now $\overline{E} = \overline{E}_1 + \overline{E}_2$. Find $\overline{E}_2$ such that $\overline{E}$ is a right-hand circularly polarized wave.
(e) If the wave is incident from the half space $x > 0$ instead of incident from the half space $x < 0$, what is the range of incident angles for which total reflection occurs?

Problem 3 (40%)

Two Hertzian dipole antennas are located at $(0, 0, 0)$ and $(0, 0, d)$ with dipole moments $p_1 = q_1 l$ and $p_2 = q_2 l$ as shown in Fig. 3. The two in-phase dipoles are oriented in z and x direction respectively.

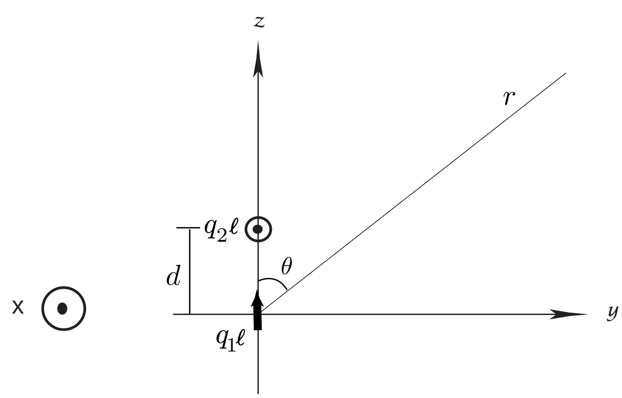

Fig.3

(a) For the x-oriented dipole, the far field ($r \gg 1$) expression of electric field on the yz-plane is:

$$\overline{E}_2 = \hat{x}\frac{k^2 q_2 \ell}{4\pi r \epsilon_0}\cos(k\sqrt{y^2 + (z-d)^2} - \omega t)$$

Show that as $d \ll \sqrt{y^2 + z^2} = r$,

$$\overline{E}_2 = \hat{x}\frac{k^2 q_2 \ell}{4\pi r \epsilon_0}\cos(kr - kd\cos\theta - \omega t)$$

(b) Find the total far field expression of electric field $\overline{E}$ on the yz-plane generated by both dipoles.
(c) Let q_1 and q_2 be real and positive. On the yz-plane, if the far field $\overline{E}$ for $\theta = 60°$ is circularly polarized,

(i) Find the minimum d in terms of λ.
(ii) What is the ratio of q_1/q_2?
(iii) Specify the handness of the circularly polarized wave.