Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion
Quiz 2
April 20, 2005

You are allowed to use a one page (both sides) formula sheet that you have prepared for Quiz 2. In addition, the 6.641 Formula Sheet is attached in the study materials section.

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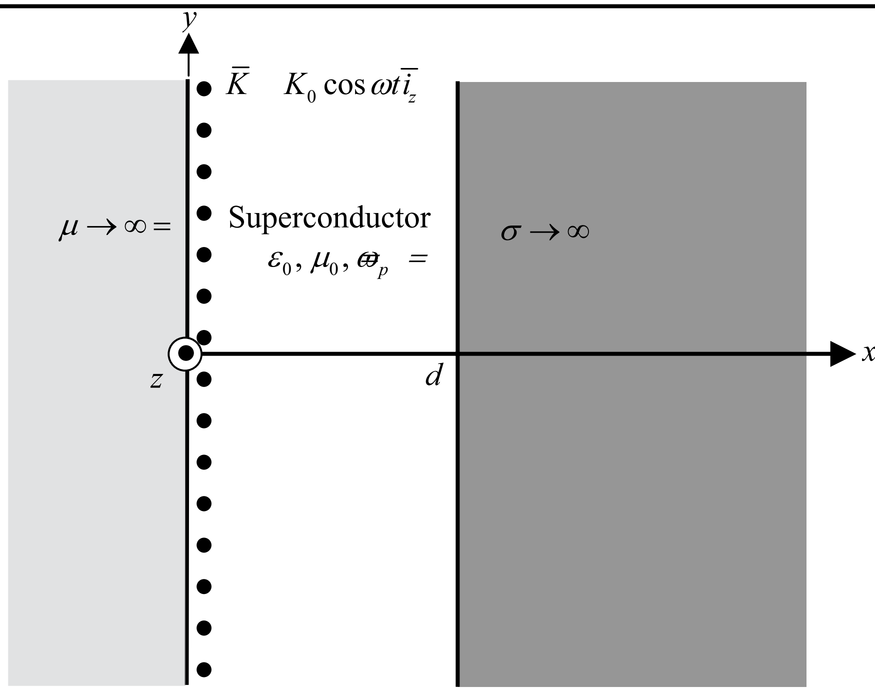
1. (35 points)



A surface current of infinite extent in the *y* and *z* directions, $\bar{K}(x = 0, t) = K_0 \cos \omega t \bar{i}_z$, is located in the *x* = 0 plane between a region with infinite magnetic permeability, $\mu \to \infty$ for *x*<0, and a superconductor in the region $0 < x < d$ described by plasma radian frequency ω_p, dielectric permittivity ε_0, and magnetic permeability μ_0. The region for x > d is a perfect conductor with $\sigma \to \infty$.

In the superconducting region, $0 < x < d$, the volume free current density $\bar{J}$ is related to the electric field $\bar{E}$ as

$$\frac{\partial \bar{J}}{\partial t} = \omega_p^2 \varepsilon_0 \bar{E}$$

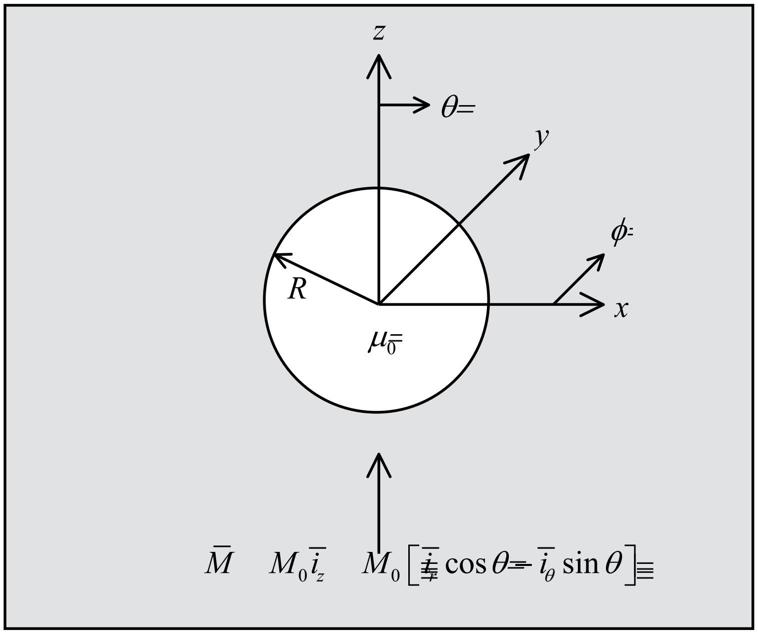
and the magnetic field $\bar{H}$ obeys the equation

$$\nabla^2 \bar{H} - \omega_p^2 \varepsilon_0 \mu_0 \bar{H} = 0$$

Because the system is uniform and of infinite extent in the *y* and *z* directions, all fields only depend on the *x* coordinate.

(a) What is the direction of $\bar{H}$ for $0 < x < d$ and what is $\bar{H}$ for $x < 0$ and $x > d$?
(b) What boundary conditions must $\bar{H}$ satisfy at $x = 0$ and $x = d$?
(c) What are the directions of $\bar{J}$ and $\bar{E}$ in the region $0 < x < d$? What boundary condition must $\bar{E}$ satisfy at $x = d$?
(d) What is the general form of solution of $\bar{H}$, $\bar{J}$, and $\bar{E}$ for $0 < x < d$?
(e) Apply the boundary conditions of (b) and (c) and solve for $\bar{H}$, $\bar{J}$, and $\bar{E}$.
(f) What is the surface current $\bar{K}(x = d, t)$ on the $x = d$ plane?
(g) What is the force per unit area on the $x = d$ plane?

2. (30 points)



A permanently magnetized medium of infinite extent in all directions has magnetization:
$\bar{M} = M_0 \bar{i}_z = M_0 \left[\bar{i}_r \cos\theta - \bar{i}_\theta \sin\theta\right]$

The medium has zero conductivity so that the free volume current density, $\bar{J}$, is zero everywhere. The medium has a spherical hole of radius *R* filled with free space with magnetic permeability μ_0. The magnetic field $\bar{H}$ as radius *r* from the center of the spherical hole goes to infinity is zero, $\bar{H}(r \to \infty) = 0$. The field solutions are axisymmetric so that there is no variation with the azimuthal angle ϕ.

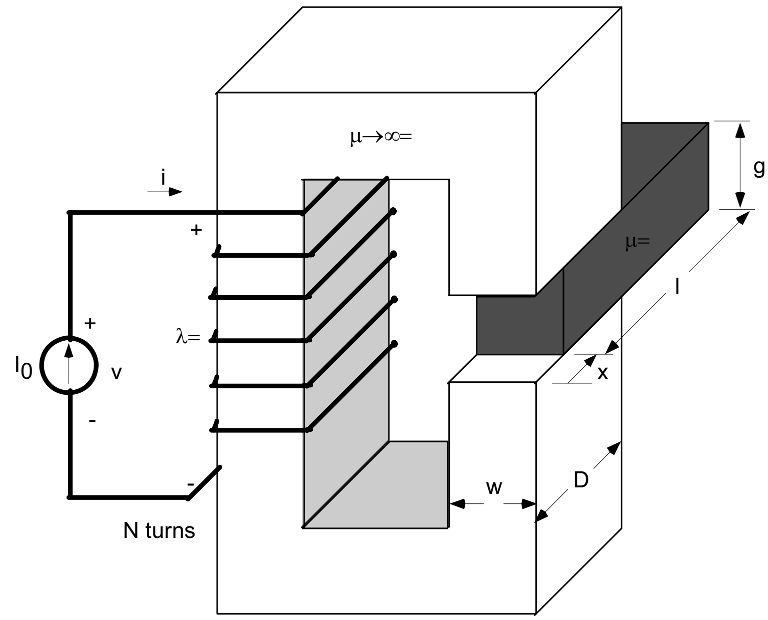
(a) Because $\bar{J} = 0$, the magnetic field $\bar{H}$ has zero curl. This allows the definition of a magnetic scalar potential $\chi(r,\theta)$ where $\bar{H} = -\nabla\chi$. What are the governing equations for χ in the regions $r < R$ and $r > R$?

(b) What boundary conditions must be satisfied at *r* = *R*?

(c) Solve for the magnetic scalar potential $\chi(r,\theta)$ and magnetic field $\bar{H}(r,\theta)$ in regions $r < R$ and $r > R$.

(d) For $r > R$, what is the effective magnetic dipole moment of the spherical hole?
Hint: The magnetic scalar potential for a point magnetic dipole with moment $\bar{m}$ directed in the *z* direction is $\chi(r,\theta) = \dfrac{|\bar{m}|\cos\theta}{4\pi r^2}$.

3. (35 points)



The figure illustrates a magnetic yoke with infinite permeability; and an air gap with magnetic permeability μ_0, length *g*, and cross-sectional area Dw. An incompressible block with length $l >> D$, magnetic permeability μ and cross-sectional area wg can move in the *x* direction as shown. Both the magnetic block and the yoke can be assumed to have negligible electrical conductivity. There is an *N* turn winding around the magnetic yoke driven by a DC current source I_0. Assume throughout that the position *x* of the moveable block is bounded as $0 \le x \le D$.

(a) Find the flux linkage (λ) – current (i) relationship for the winding in terms of x, μ, μ_0, N and the dimensions of the magnetic circuit.
(b) Find the force in the *x* direction on the incompressible block due to the current source I_0.
(c) If the magnetic block is forced to move with a displacement given by $x = x_0 + x'\cos(\omega t)$ find the voltage developed across the current source.
(d) The current source and the block position are controlled to traverse the $i - x$ plane in a cycle as shown. Find the work done by the current source for each cycle.