6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

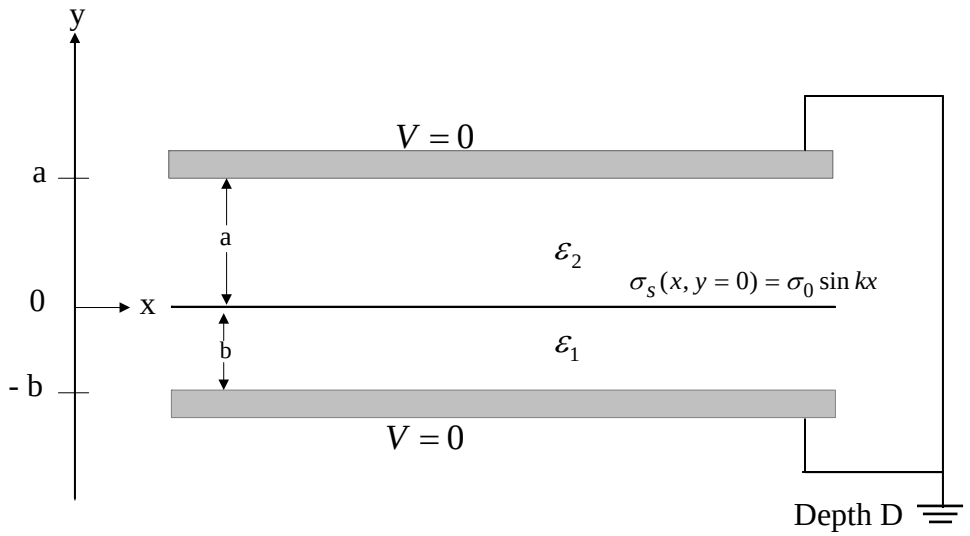
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Final Exam

5/20/2009

NOTE: 6.641 Formula Sheets at the end of exam. You are also allowed both sides of two 8½"x11" pages for 6.641 course material that you have prepared yourself.

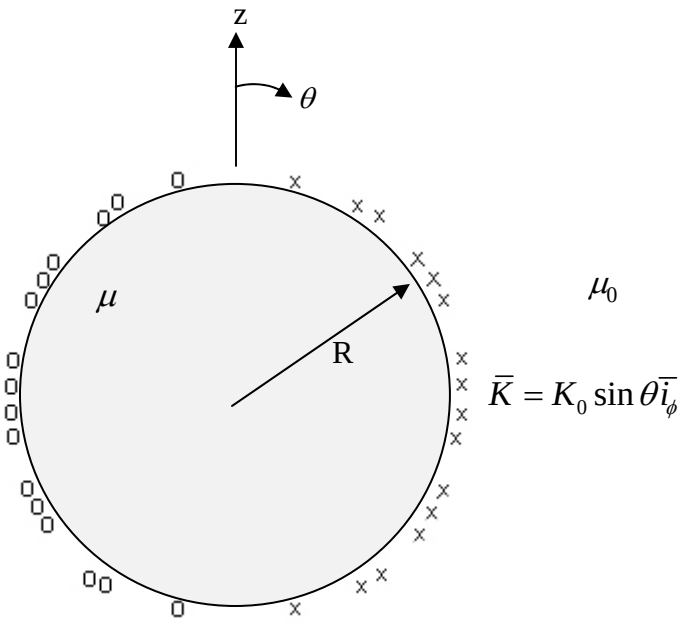
1. 25 points



A sheet of surface charge with surface charge distribution $\sigma_s(x, y = 0) = \sigma_0 \sin kx$ is placed at $y = 0$, parallel and between two parallel grounded perfect conductors at zero potential at $y = -b$ and $y = a$. The regions above and below the potential sheet have dielectric permittivities of ε_2 and ε_1. Neglect fringing field effects.

a) What are the electric potential solutions in the regions $0 \le y \le a$ and $-b \le y \le 0$?
b) What are the electric field distributions in the regions $0 < y < a$ and $-b < y < 0$?
c) What are the free surface charge distrbutions at $y = -b$ and $y = a$?
d) What is the potential distribution at $y = 0$?

2. 25 points



A surface current sheet $\bar{K} = K_0 \sin\theta \bar{i}_\phi$ is placed on the surface of a sphere of radius R. The inside of the sphere ($r < R$) has magnetic permeability μ and the outside region ($r > R$) is free space with magnetic permeability μ_0. The magnetic field at $r = \infty$ is zero.

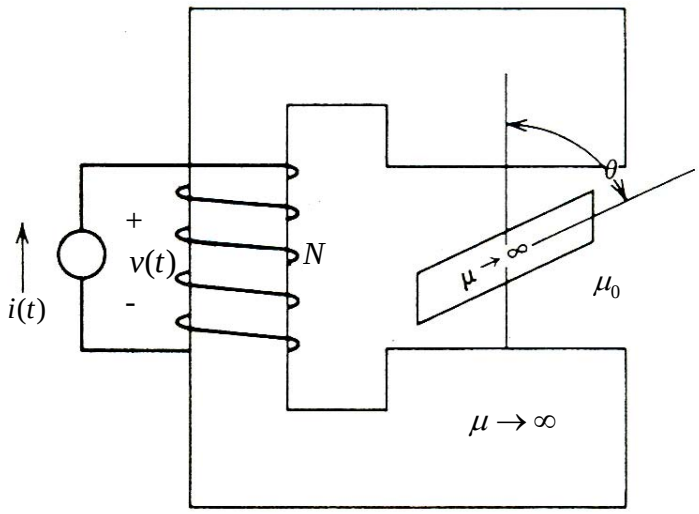
a) What are the boundary conditions on the magnetic field at $r = 0$ and $r = R$?
b) What are the general form of the solutions for the magnetic scalar potential inside and outside the sphere?
c) Use the boundary conditions of part (a) and solve for the magnetic scalar potential and the magnetic field $\bar{H}$ inside and outside the sphere.
d) The scalar magnetic potential for a point magnetic dipole of moment $m\bar{i}_z$ at the origin is:

$\bar{H} = -\nabla\chi$, $\chi = \frac{m\cos\theta}{4\pi r^2}$

What is the effective magnetic moment of the sphere and surface current sheet for $r > R$?
e) What is the equation for the magnetic field line that passes through the point ($r = R_0, \theta = \pi/2$) where $R_0 > R$.
f) For the field line in (e), if $R_0 = 2R$, at what angles of θ does the field line contact the sphere?

3. 25 points

A reluctance motor is made by placing a high permeability material, which is free to rotate, in the air gap of a magnetic circuit excited by a current $i(t)$.



The inductance of the magnetic circuit varies with rotor angle θ as

$$L(\theta) = L_0 + L_1 \cos 2\theta,\ L_0 > 0,\ 0 < L_1 < L_0$$

where the maximum inductance $L_0 + L_1$ occurs when $\theta = 0$ or $\theta = \pi$ and the minimum inductance $L_0 - L_1$ occurs when $\theta = \pm \pi/2$.

a) What is the magnetic torque, T_{mag}, on the rotor as a function of the angle θ and current $i(t)$?

b) With $i(t)$ a DC current I, a constant positive mechanical stress $T_{mech} > 0$ is applied. What is the largest value of $T_{mech} = T_{max}$ for which the rotor can be in static equilibrium?

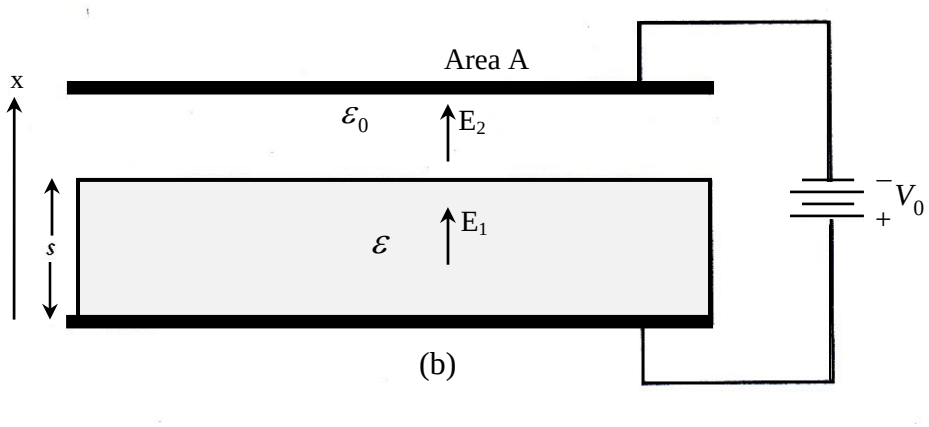
c) If $T_{mech} = \frac{1}{2} T_{max}$, plot the total torque $T_{mag} + T_{mech}$. Use a graphical method to determine the equilibrium values of θ and label which are stable and which are unstable.

d) If the rotor has moment of inertia J and is slightly perturbed from a stable equilibrium position θ_{eq} at $t = 0$ by an angle position $\theta'(t)$, what is the general frequency of oscillation? What is the oscillation frequency for θ_{eq} found for stable equilibrium in part (c)?

e) If the initial conditions of the perturbation are $\left.\frac{d\theta'}{dt}\right|_{t=0} = 0$ and $\theta'(t=0) = \Delta\theta$ what is $\theta'(t)$ for $t > 0$. Neglect any damping.

f) If $i(t)$ is a DC current I and a motor drives the rotor angle θ at constant angular speed Ω so that $\theta = \Omega t$, what is the voltage $v(t)$ across the coil?

4. 25 points



A parallel plate capacitor with electrodes of area A has its upper electrode in a free space region in series with a solid dielectric of thickness s and dielectric permittivity ε. The $x = s$ interface has no free surface charge.

a) What are the electric fields E_1 and E_2 in the dielectric and free space regions?
b) What is the free surface charge density on the lower electrode?
c) What is the capacitance $C(x)$ of the capacitor?
d) What is the electric force on the upper electrode?