Wave-Based Surveillance

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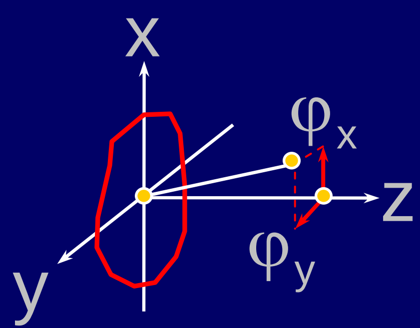
Antenna Aperture Transform Relations and Resolution

Define angular spectrum $\underline{\overline{E}}(\varphi_x, \varphi_y)$ for incoming monochromatic signals



Not to be confused with the radially expanding and diminishing waves characterized by $\overline{E}(\theta, \varphi, R)$

$$\underset{[\text{aperture}]}{\underline{\overline{E}}(x,y)} \cong \int_{4\pi} \underbrace{\underline{\overline{E}}(\varphi_x,\ \varphi_y)}_{\left[\text{vm}^{-1}\text{ster}^{-1}\right]} e^{+j\frac{2\pi}{\lambda}(x\varphi_x + y\varphi_y)} d\Omega$$

$$\underline{\overline{E}}(\varphi_x, \varphi_y) \cong \frac{1}{\lambda^2} \int_A \underline{\overline{E}}(x,y) e^{-j\frac{2\pi}{\lambda}(x\varphi_x + y\varphi_y)} dxdy \left(\text{For } \varphi_x,\ \varphi_y << \frac{\pi}{2} \right)$$

Equivalently we let $x/\lambda \triangleq x_\lambda;\ y/\lambda \triangleq y_\lambda$

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Equivalently we let $x/\lambda \triangleq x_\lambda$; $y/\lambda \triangleq y_\lambda$

$$\underline{\overline{E}}(x_\lambda, y_\lambda) \cong \int_{4\pi} \underline{\overline{E}}(\varphi_x, \varphi_y)\, e^{+j2\pi(x_\lambda \varphi_x + y_\lambda \varphi_y)} d\Omega$$

$$\underline{\overline{E}}(\varphi_x, \varphi_y) \cong \iint_A \underline{\overline{E}}(x, y)\, e^{-j2\pi(x_\lambda \varphi_x + y_\lambda \varphi_y)} dx_\lambda dy_\lambda$$

Thus: $\underline{\overline{E}}(x_\lambda, y_\lambda) \overset{\sim}{\leftrightarrow} \underline{\overline{E}}(\varphi_x, \varphi_y)$

$\downarrow \qquad \downarrow$

$$\underline{R}_{\underline{E}}(\overline{\tau}_\lambda) \overset{\sim}{\leftrightarrow} \left|\underline{\overline{E}}(\varphi_x, \varphi_y)\right|^2 \propto G(\overline{\varphi})\,(\text{transmitting})$$

where $\underline{R}_{\underline{E}}(\overline{\tau}_\lambda) \triangleq \int \int_{-\infty}^{\infty} \underline{\overline{E}}(\overline{r}_\lambda)\underline{\overline{E}}^*(\overline{r}_\lambda - \overline{\tau}_\lambda)\, dx_\lambda dy_\lambda$

(Note : $\underline{\overline{E}}$ is not stochastic)

Single Aperture Resolution Limits

Source image $= T_A(\overline{\varphi}) = G(\overline{\varphi}) * T_B(\overline{\varphi})$

$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$

$f_\varphi \triangleq$ cycles per radian (angle)

$$\underbrace{T_A(\overline{f}_\varphi) = G(\overline{f}_\varphi) \bullet T_B(\overline{f}_\varphi)}$$

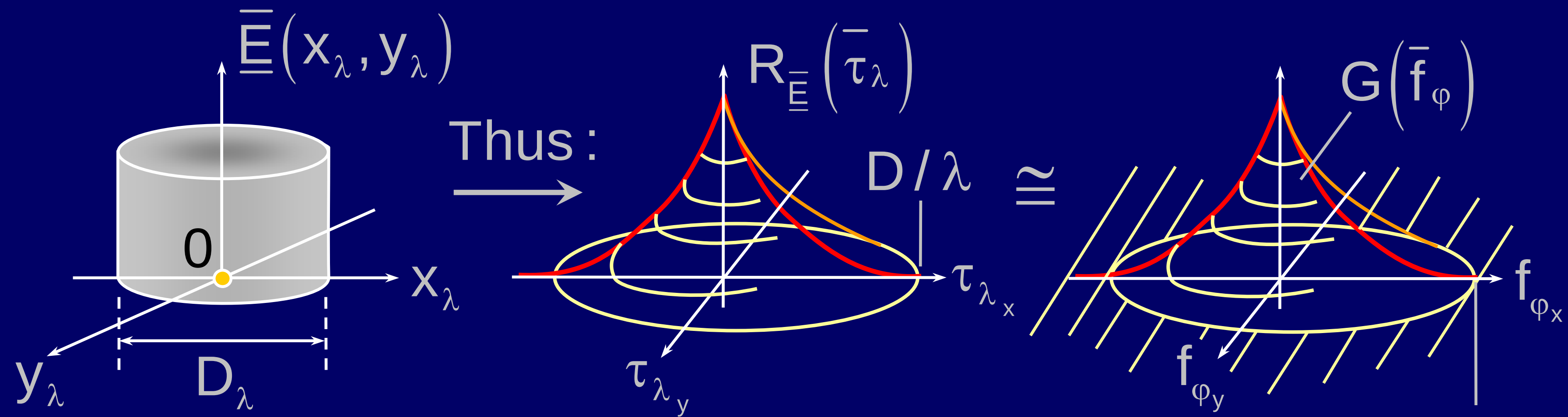
If $G(\overline{f}_\varphi) = 0$, there is <u>no</u> response in the image spectrum $T_A(\overline{f}_\varphi)$

Note : $R_{\underline{\underline{E}}}(\overline{\tau}_\lambda) \overset{\sim}{\leftrightarrow} G(\overline{\varphi}) \leftrightarrow G(\overline{f}_\varphi)$ so $R_{\underline{\underline{E}}}(\tau) \cong G(f_\varphi)$

Single Aperture Resolution Limits

Note: $R_{\underline{\underline{E}}}(\bar{\tau}_\lambda) \overset{\sim}{\leftrightarrow} G(\bar{\varphi}) \leftrightarrow G(\bar{f}_\varphi)$ so $R_{\underline{\underline{E}}}(\tau) \cong G(f_\varphi)$

Example:



$$f_{max} = \frac{D}{\lambda}\,\frac{\text{cycles}}{\text{radian}}$$

Note: Zero response to source angular spectral components with spatial frequencies beyond $f_m = D/\lambda$ cycles/radian

Antenna Responses for Stochastic Signals

Let $\overline{E}(x_\lambda,\ y_\lambda,\ t)(vm^{-1}\ ster^{-1}) = \text{Re}\{\underline{\overline{E}}(t,\ x_\lambda, y_\lambda)e^{j\omega t}\}$

$\uparrow$ Slowly varying, narrowband random signal

Assume stochastic signals from different directions are uncorrelated (so no systematic intensity variations in aperture).

$$\text{Then: } \underline{\overline{E}}(x_\lambda,\ y_\lambda,\ t) \leftrightarrow \underline{\overline{E}}(\varphi_x,\ \varphi_y,\ t)$$

$$\downarrow\downarrow \qquad\qquad \downarrow\downarrow$$

$$E\left[R_{\underline{E}}\left(\overline{\tau}_\lambda\right)\right]$$

$$\triangleq$$

$$\varphi_{\underline{\overline{E}}}\left(\tau_{x_\lambda},\ \tau_{y_\lambda}\right) \leftrightarrow E\left[\left|\underline{\overline{E}}(\varphi_x,\ \varphi_y,\ t)\right|^2\right]$$

$$\uparrow$$

$$\left[vm^{-1}\right]^2$$

(Double arrow implies irreversibility for two reasons: expectation and magnitude operators used)

Antenna Responses for Stochastic Signals

Then: $\underline{\bar{E}}(x_\lambda, y_\lambda, t) \leftrightarrow \underline{\bar{E}}(\varphi_x, \varphi_y, t)$

$\downarrow\downarrow \qquad\qquad \downarrow\downarrow$

$E\left[R_{\underline{E}}\left(\bar{\tau}_\lambda\right)\right]$

$\underline{\underline{\Delta}}$

$\varphi_{\underline{\bar{E}}}\left(\tau_{x_\lambda}, \tau_{y_\lambda}\right) \leftrightarrow E\left[\left|\underline{\bar{E}}(\varphi_x, \varphi_y, t)\right|^2\right]$

$\uparrow$

$\left[vm^{-1}\right]^2$

(Double arrow implies irreversibility for two reasons: expectation and magnitude operators used)

Can we deduce $I(\varphi_x, \varphi_y)\left[Wm^{-2}ster^{-1}Hz^{-1}\right]$ from $\underline{\bar{E}}(x_\lambda, y_\lambda, t)$? (Yes)

$$\frac{\phi_{\underline{\bar{E}}}\left(\tau_{x_\lambda}, \tau_{y_\lambda}, f\right)}{2\eta_o B} \leftrightarrow \frac{E\left[\left|\underline{\bar{E}}(\varphi_x, \varphi_y, f)\right|^2\right]}{2\eta_o B} = I(\varphi_x, \varphi_y, f)$$

$\uparrow (377\Omega)$

$\left(v\ m^{-1}\right)^2 ohm^{-1}\ Hz^{-1} = W\ m^{-2}\ Hz^{-1}$

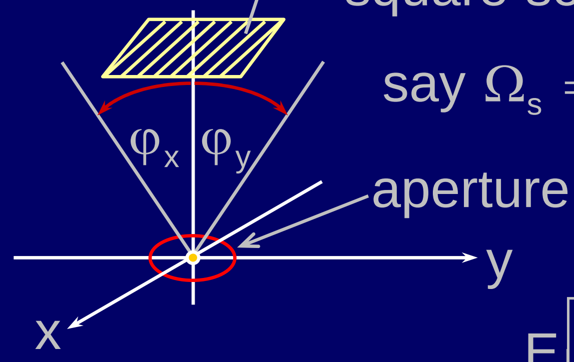
$\dfrac{\left(v\ m^{-1}\ rad^{-1}\right)^2}{ohm\ Hz}$

$W\ m^{-2}\ Hz^{-1}\ ster^{-1}$
[ster not a physical unit]

Aperture Field Correlations for a Thermal Source

square source @ $T_B\left(K^\circ\right)$

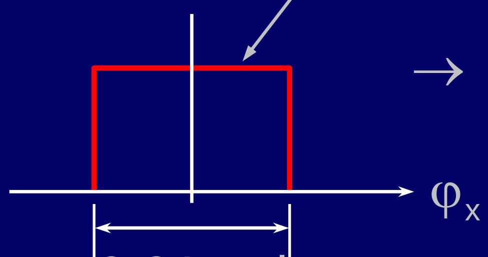
say $\Omega_s = (0.01\ \text{rad})^2 \cong 10^{-4}$ ster



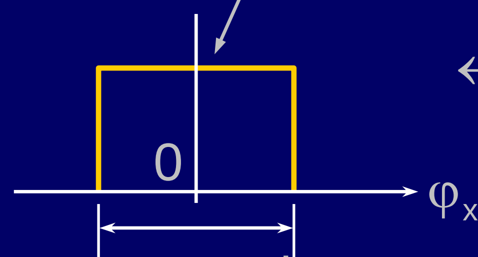
$I_x(\varphi_x, \varphi_y, f) = kT_B/\lambda^2$

$E\left[\left|\bar{E}(\varphi_x, \varphi_y)\right|^2\right] = \frac{kT_B}{\lambda^2} \bullet 2\eta_o B$

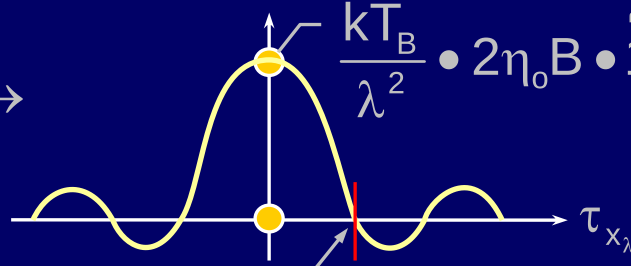
$\phi_{\bar{E}_\lambda}\left(\tau_{x_\lambda}, \tau_{y_\lambda}\right)$



$\rightarrow$



$\leftrightarrow$



$\frac{kT_B}{\lambda^2} \bullet 2\eta_o B \bullet \overbrace{10^{-4}}^{\Omega_s}$

single polarization

B is the signal bandwidth of interest

$\frac{1}{0.01} = \frac{\tau_x}{\lambda}$

Note: $\phi_{\bar{E}_x}(0,\ 0) = E\left\{\left|\bar{E}(x,\ y,\ t)\right|^2\right\} = \underbrace{\left(\frac{kT_B}{\lambda^2}\Omega_s B\right)}_{S_o(\text{watts})} 2\eta_o$ as expected.

Time and Space Field Correlations (3D)

time / frequency

$$\phi_{\underline{\underline{E}}_x}\left(\tau_{\lambda_x}, \tau_{\lambda_y}, \tau\right) / 2\eta_o \left(Wm^{-2}\right) \leftrightarrow \phi_{\underline{\underline{E}}_x}\left(\tau_{\lambda_x}, \tau_{\lambda_y}, f\right) / 2\eta_o \left(Wm^{-2}Hz^{-1}\right)$$

$$\updownarrow \left(\overline{\tau}_\lambda \leftrightarrow \overline{\varphi}\right)$$

$$I_x\left(\overline{\varphi}, f\right) = \frac{kT_B\left(\overline{\varphi}, f\right)}{\lambda^2} Wm^{-2}\ Hz^{-1}\ ster^{-1}$$

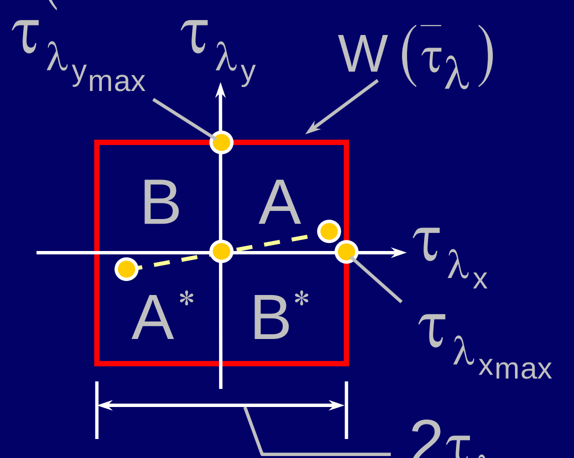
(one polarization)

Aperture Synthesis

Assume field of size $\tau_{\lambda_{x\max}}$ by $\tau_{\lambda_{y\max}}$ within which two small antennas can be moved independently

Note $\phi_{\underline{\bar{E}}}(\bar{\tau}) = \phi^*_{\underline{\bar{E}}}(-\bar{\tau})$

$\left(\text{if stationary w.r.t. } \bar{r}\text{; i.e., true angular decorrelation}\right)$



$$2\tau_{\lambda_{y\max}} = \frac{2L_x}{\lambda}$$

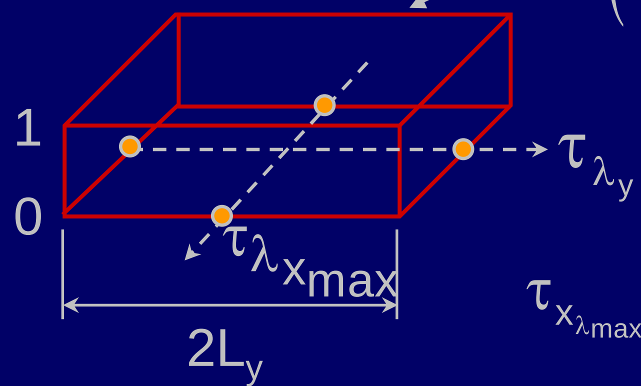
Therefore, in $\bar{\tau}_\lambda$ space, we need to measure combinations in only two quadrants, e.g., A, B because the conjugates A^*, B^* follow.

We observe $W(\bar{\tau}_\lambda) \bullet \phi_{\underline{\bar{E}}}(\bar{\tau}_\lambda)$

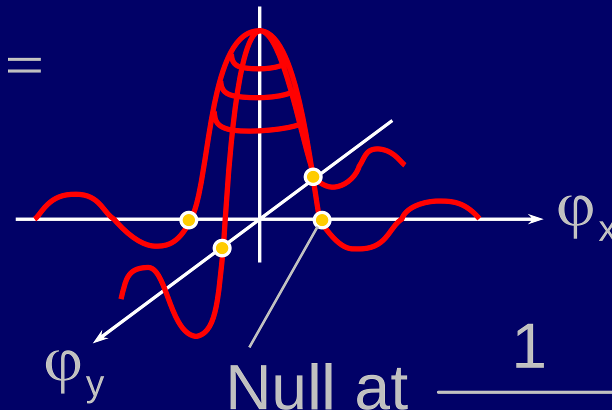
$$\updownarrow$$

and retrieve $W(\bar{\varphi}) * \left|\underline{\bar{E}}(\bar{\varphi})\right|^2$ where $\left|\underline{\bar{E}}(\bar{\varphi})\right|^2$ is the desired image

Maximum Antenna Separation Limits Resolution



$$W\left(\bar{\tau}_\lambda\right) \leftrightarrow W\left(\bar{\varphi}\right) =$$



$$\tau_{x_{\lambda max}} = L_x / \lambda$$

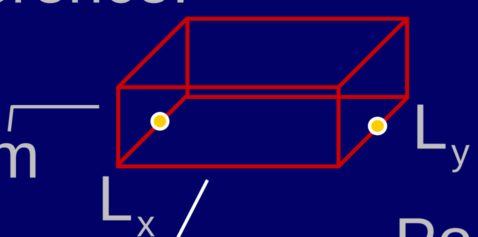
Null at $\dfrac{1}{2\tau_{\lambda_{x_{max}}}} = \dfrac{\lambda}{2L_x}$

Aperture synthesis: $\theta_{3dB} \cong \dfrac{\lambda}{2L_x}, \dfrac{\lambda}{2L_y}$

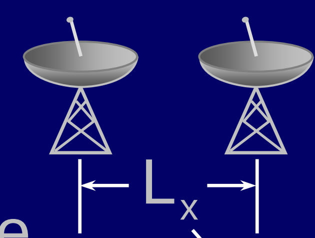
Recall, filled aperture: $\theta_{3DB} \cong \dfrac{\lambda}{L_x}, \dfrac{\lambda}{L_y} \left(\eta_A = 1\right)$

Origin of difference:

Consider:



vs

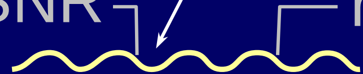


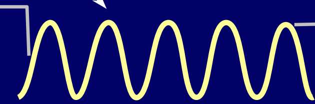
Single uniform aperture

Vanishing SNR

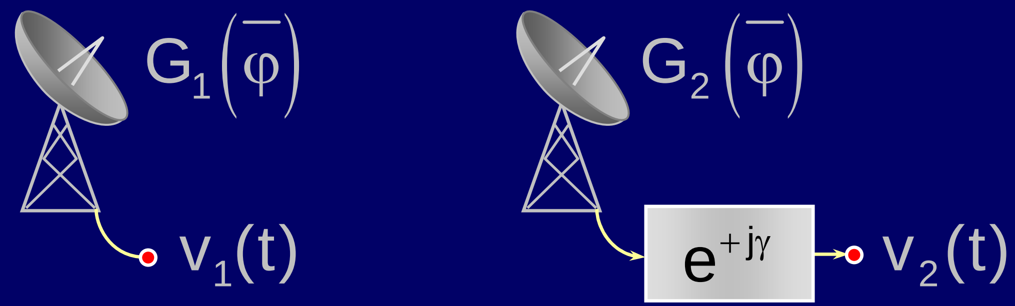
Point-source response functions

Good SNR





Circuits for Interferometers



$$v_1(t) \cong k_1 R_e\left\{\sqrt{G_1(\overline{\varphi})} \bullet \underline{E}(x,y,t)\, e^{j\omega t}\right\}$$

$$v_2(t) \cong k_2 R_e\left\{\sqrt{G_2} \bullet \underline{E}(x-\tau_x, y-\tau_y, t)\, e^{j\gamma} e^{j\omega t}\right\}$$

↑ slowly varying

Narrowband Case

$$\text{Output} = E\left[v_1(t)\, v_2(t)\right] =$$

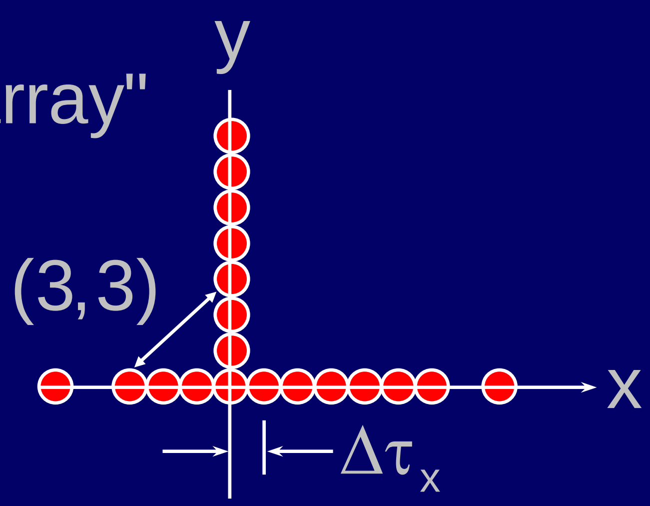
$$\frac{k_1 k_2}{2} \text{Re}\Big\{\underbrace{\sqrt{G_1 G_2}}_{\triangleq G_{12}} \bullet \underbrace{E\left[\underline{\overline{E}}(x,y,t) \bullet \underline{\overline{E}}^*(x-\tau_x, y-\tau_y, t)\right]}_{\phi_{\underline{E}}(\overline{\tau}_\lambda, f)} e^{-j\gamma}\Big\}$$

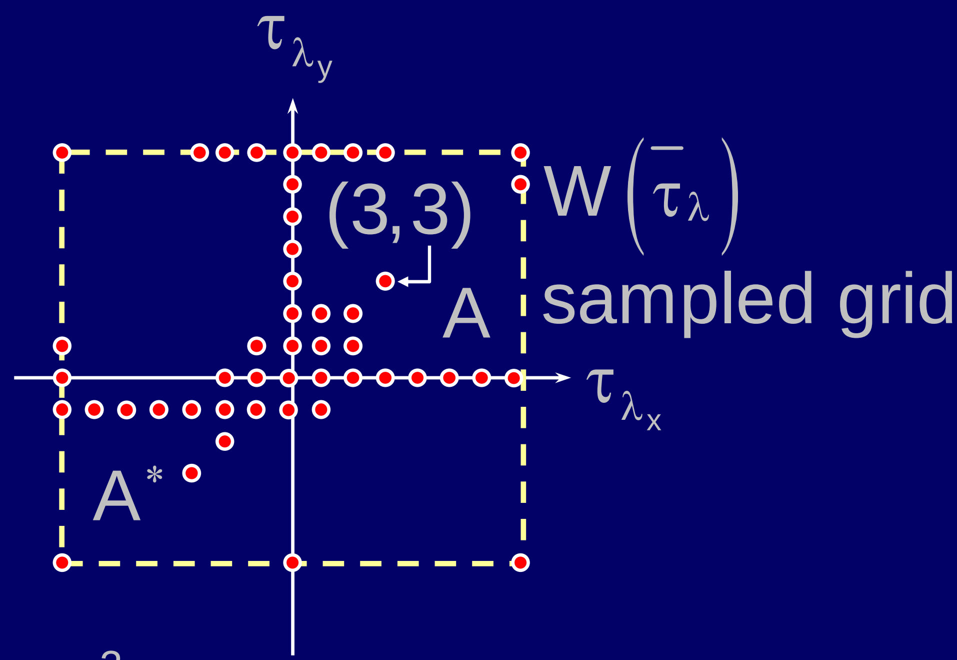
$$\left(\text{Recall } E\left[a(t)b(t)\right] = \frac{1}{2} R_e\left\{\underline{A}\underline{B}^*\right\},\ \text{Re}\left\{\underline{\phi}\, e^{-j\pi/2}\right\} = \text{Im}\left\{\underline{\phi}\right\}\right)$$

Image from Discrete Antenna Array

E.G.

"T-array"



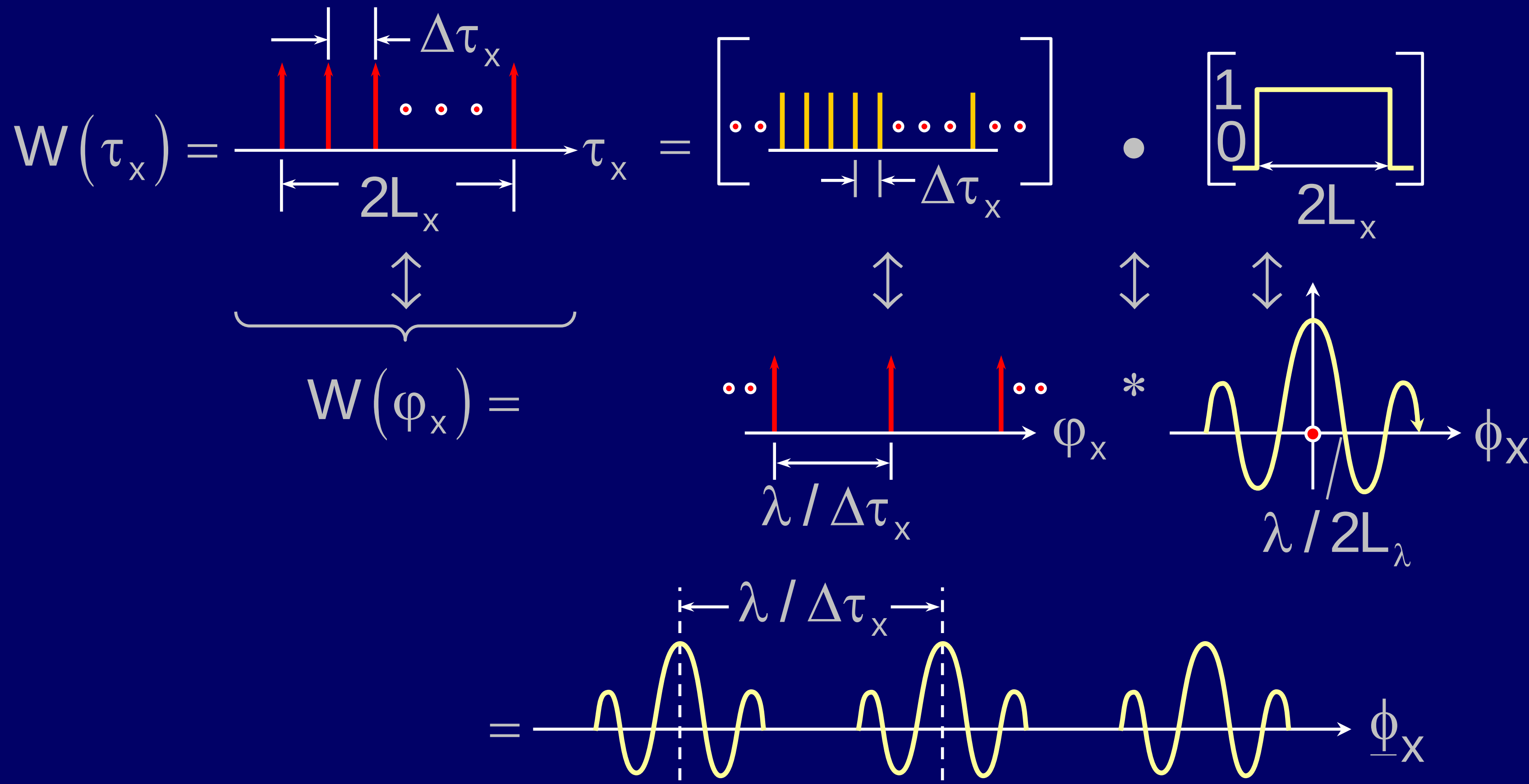


Recall: $$\frac{\underline{\phi}(\tau_{x_\lambda}, \tau_{y_\lambda}, f)}{2\eta_o B} \leftrightarrow \frac{\left|\underline{\overline{E}}(\overline{\varphi}, f)\right|^2}{2\eta_o B} = I(\overline{\varphi}, f)\left(Wm^{-2}ster^{-1}Hz^{-1}\right)$$

Therefore: $$\left[\phi_{\underline{\overline{E}}}(\overline{\tau}_\lambda, f)/2\eta_o B\right] \bullet W(\overline{\tau}_\lambda) \leftrightarrow I(\overline{\varphi}, f) * W(\overline{\varphi})$$

Image from Discrete Antenna Array

$$\left[\phi_{\underline{\underline{E}}}\left(\bar{\tau}_\lambda, f\right)/2\eta_o B\right] \bullet W\left(\bar{\tau}_\lambda\right) \leftrightarrow I\left(\bar{\varphi}, f\right) * W\left(\bar{\varphi}\right)$$

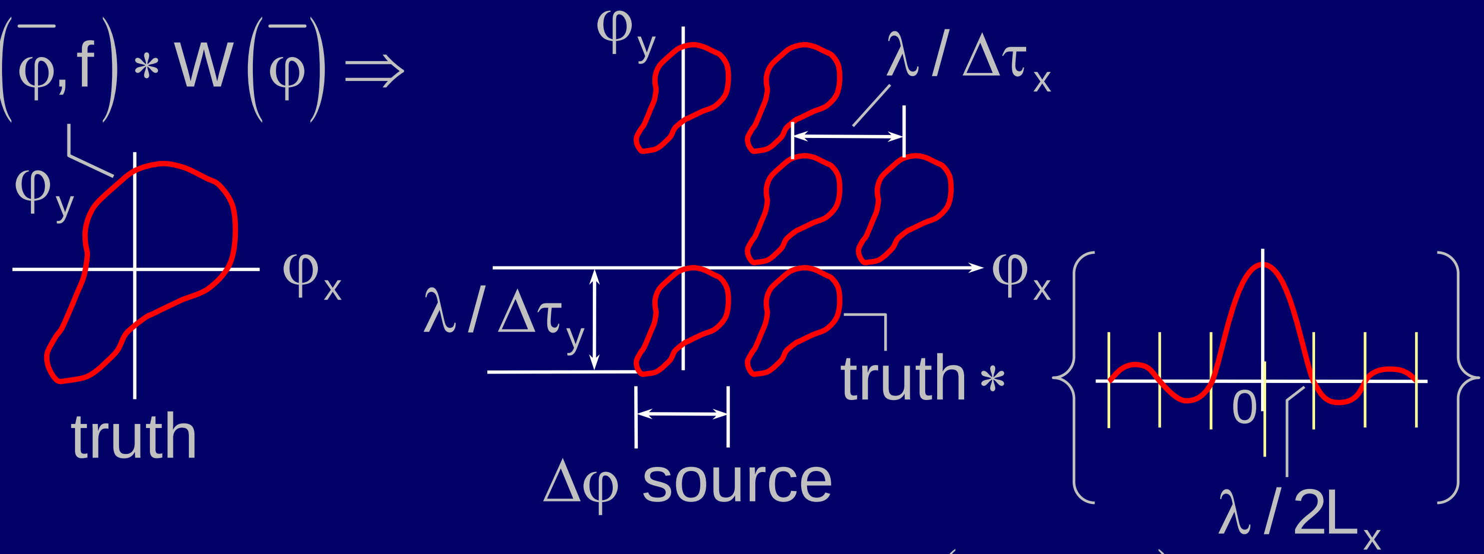


Aliased images are confused if they overlap.

Image Aliasing in Synthesized Images

$$\left(\phi_{\underline{\underline{E}}}\left(\bar{\tau}_\lambda, f\right) / 2\eta_0 B\right) \bullet W\left(\bar{\tau}_\lambda\right) \leftrightarrow \Phi\left(\bar{\varphi}, f\right) * W\left(\bar{\varphi}\right)$$

$$I\left(\bar{\varphi}, f\right) * W\left(\bar{\varphi}\right) \Rightarrow$$



To avoid image aliasing, let $\Delta\tau_x \tilde{<} \lambda / \Delta\phi_x$ (source)

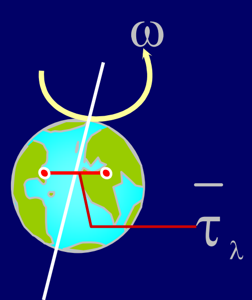
Note that objects in space often are isolated in empty fields, so aliasing is not a problem. Objects imaged on the ground have major aliasing problems, requiring Nyquist-sampled $\bar{\tau}_\lambda$ plane that puts all aliases in the weak sidelobes of $G_{AB}\left(\bar{\varphi}\right)$.

$$\text{Note: } \hat{I}\left(\bar{\varphi}, f\right) = \left[I\left(\bar{\varphi}, f\right) * W\left(\bar{\varphi}\right)\right] \bullet G_{12}\left(\bar{\varphi}\right)$$

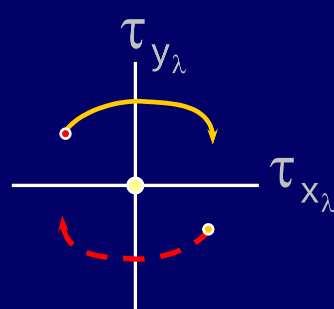
Aperture Synthesis Using Earth Rotation

Moveable on a track

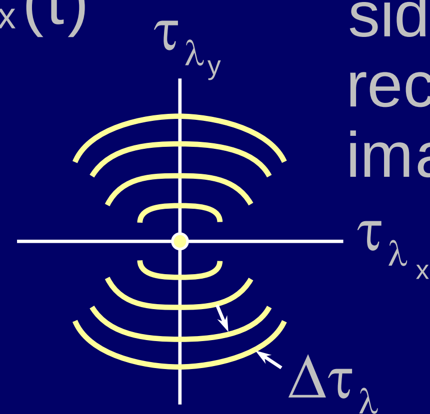
Earth rotation moves effective $\bar{\tau}_x(t)$



Earth spins



1 day per curve



Gaps yield sidelobes in reconstructed image

Radio astronomers call τ_{λ_x}, τ_{λ_y} "u, v"

Choose $\Delta\tau_\lambda$ sufficiently small that no aliasing occurs.

Note: Simple targets that are characterized by the positions or sizes of only a few key features or elements can be deciphered using heavily aliased or burred images.

Interferometer Circuits

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Basic Aperture Synthesis Equation

Recall:

$$\underline{\overline{E}}\left(\overline{r}, t\right) \quad \leftrightarrow \quad \underline{\overline{E}}\left(\overline{\psi}, t\right)$$

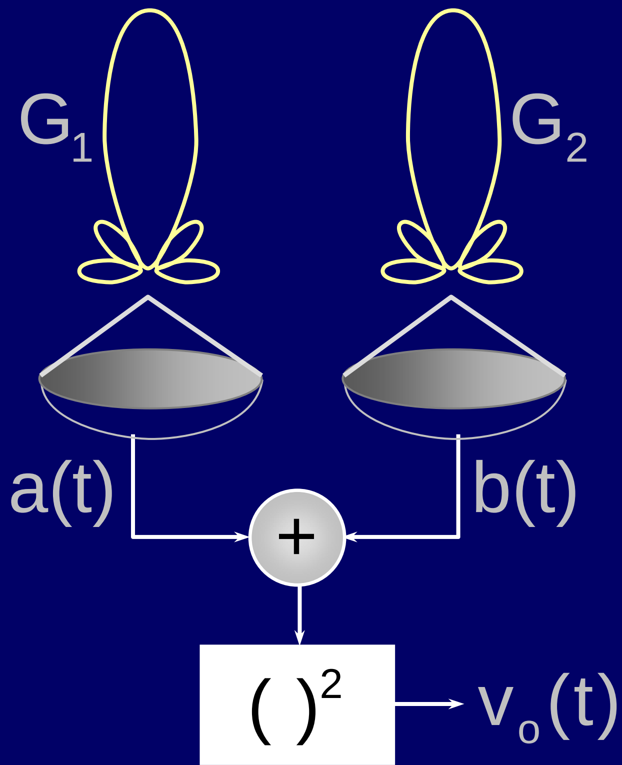
$$\downarrow\downarrow \qquad\qquad \downarrow\downarrow$$

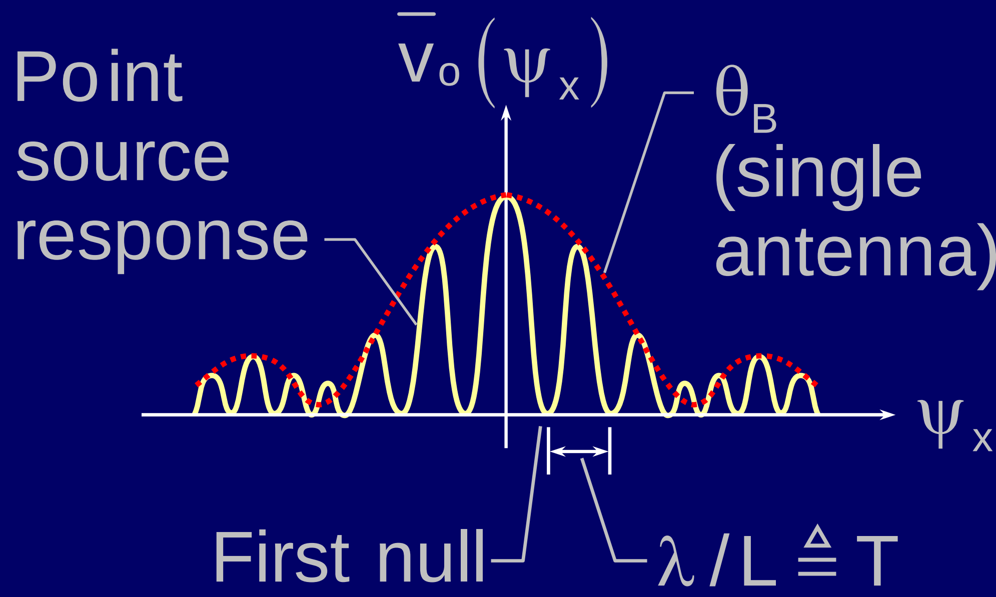
$$E\left[R_{\underline{\overline{E}}}\left(\overline{\tau}_{\lambda}\right)\right] = \phi_{\underline{\overline{E}}}\left(\overline{\tau}_{\lambda}\right) \quad \leftrightarrow \quad E\left\{\left|\underline{\overline{E}}\left(\overline{\Psi}, t\right)\right|^2\right\} \propto I\left(\overline{\Psi}\right), T_A\left(\overline{\Psi}\right)$$

Simple Adding Interferometers

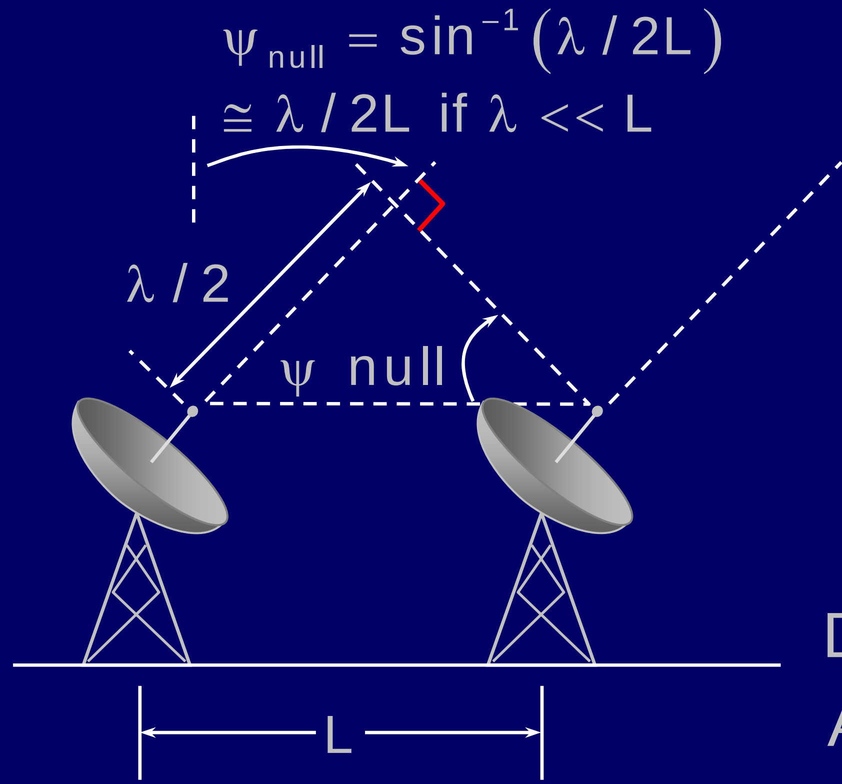
$$\overline{v_o}\left(\vec{\psi}\right) = \overline{a^2} + \overline{b^2} + 2\overline{ab}$$

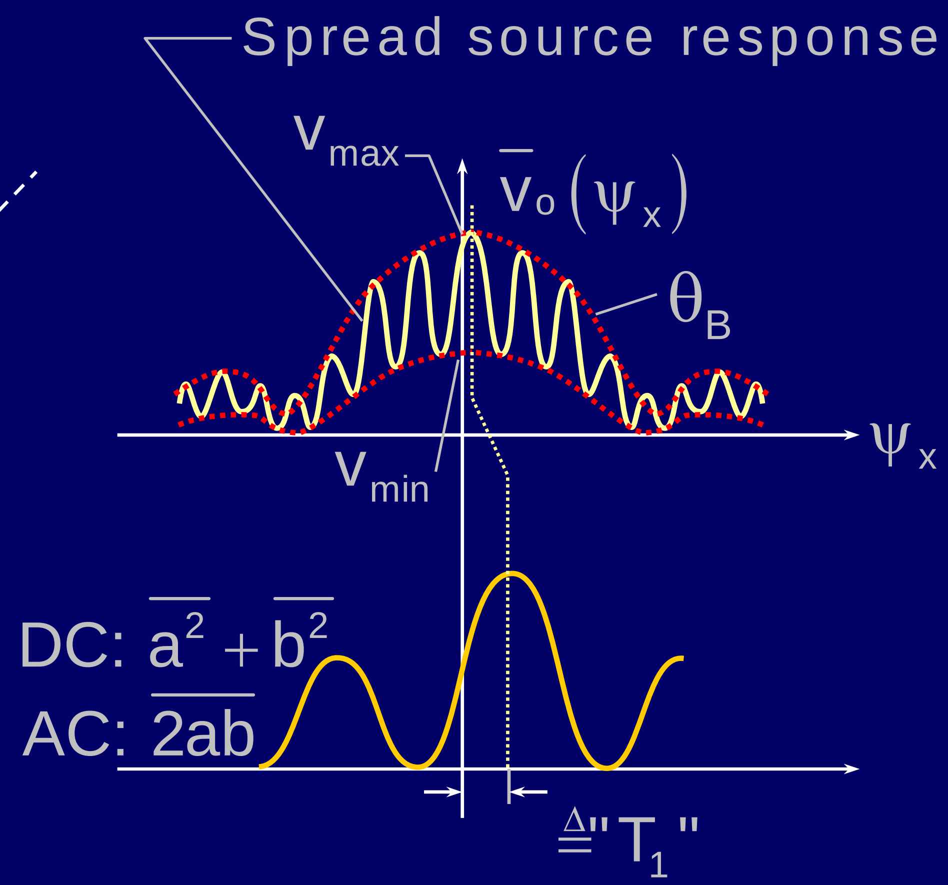
(overbars mean "time average" here)





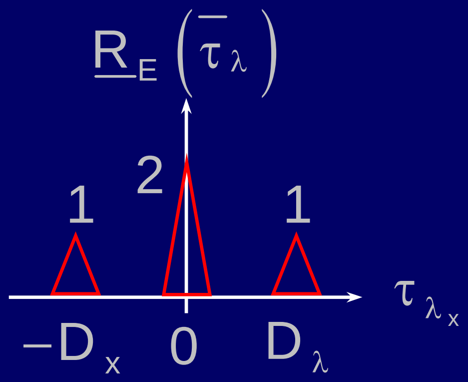
Simple Adding Interferometers





Complex fringe visibility $\underline{v} \triangleq \frac{v_{max} - v_{min}}{v_{max} + v_{min}} e^{j2\pi T_1/T} \propto \underline{E}\left(\overline{\tau_\lambda}\right) \leftrightarrow T_A\left(\overline{\psi}\right)$

Interferometry as Fourier Analysis



Recall: $\phi_A(\bar{\tau}_\lambda) = \underline{R}_E(\bar{\tau}_\lambda) \bullet \phi_E(\bar{\tau}_\lambda)$

$\updownarrow \quad \sim\updownarrow \quad \sim\updownarrow \quad \sim\updownarrow$

$T_A(\bar{\psi}) = G(\bar{\psi}) * T_B(\bar{\psi})$

$\updownarrow \quad \updownarrow\sim \quad \updownarrow\sim \quad \updownarrow\sim$

$\underline{T}_A(f_\psi) = \underline{G}(f_\psi) \bullet \underline{T}_B(f_\psi)$

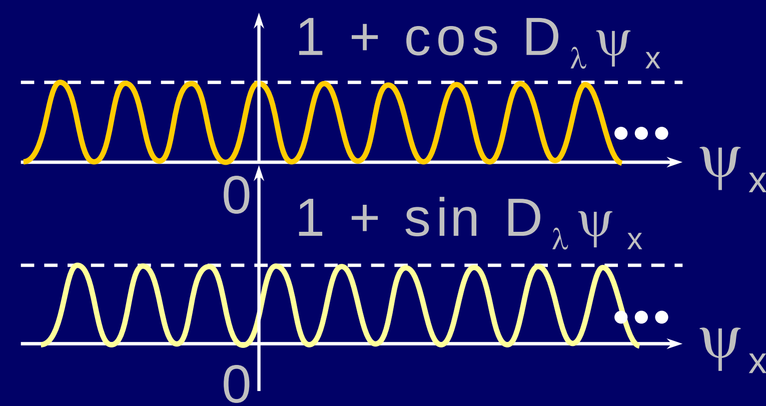
observed = antenna times signal

Example: 2 small duplicate antennas separated by D_λ in the x direction

2-element adding interferometer

$G_{cos}(\psi_x) \tilde{\propto}$

$G_{sin}(\psi_x) \tilde{\propto}$

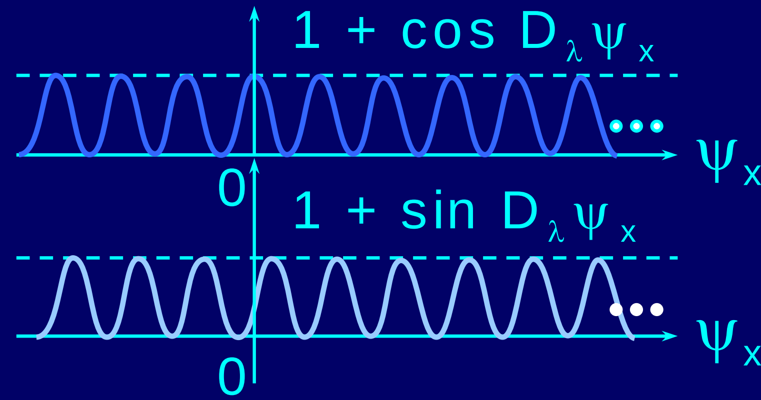


Interferometry as Fourier Analysis

2-element adding interferometer

$$G_{cos}(\psi_x) \tilde{\propto}$$

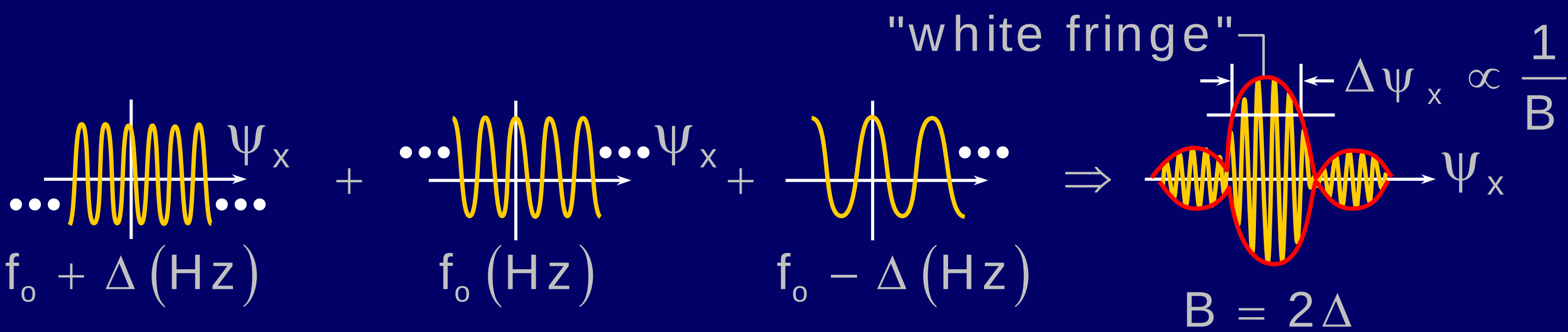
$$G_{sin}(\psi_x) \tilde{\propto}$$



Interferometer directly measures Fourier components of source

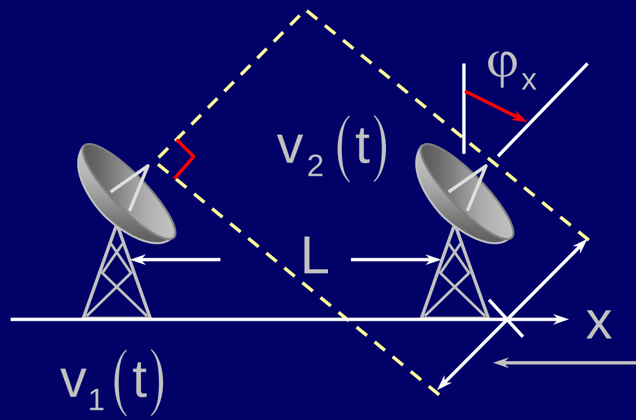
Effect of finite bandwidth:

Fringe patterns for all frequencies (colors) add in phase to create



A delay line in one interferometer arm can redirect the strong white fringe in other directions.

Broad-Bandwidth Effects in Interferometers



$\underline{E}(x,y,t)$ (the "slowly varying" part) may vary rapidly enough that the offset time $L \sin \varphi_x / c$ is significant

$\dfrac{L \sin \phi_x}{c}$ seconds

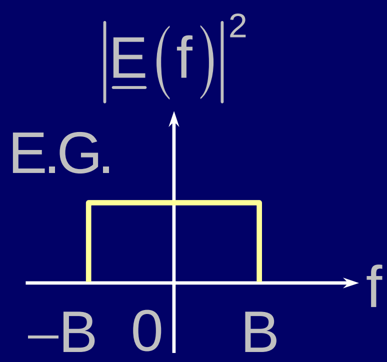
$$E\left[v_1(t)v_2(t)\right] = \frac{k_1k_2}{2} G_{12}\left(\bar{\phi}_x\right) E\left[R_e \left\{ \underline{E}\left(x, y, t + \frac{L \sin \varphi_x}{2c} \right) \right.\right.$$

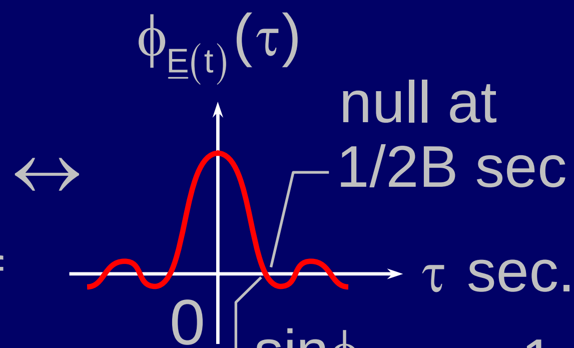
$$\left.\left. \underline{E}^*\left(x - \tau_x, y - \tau_y, t - \frac{L \sin \varphi_x}{2c} \right) \bullet \underbrace{e^{j\omega L \sin \varphi_x / c}}\, e^{-j\gamma} \right\} \right]$$

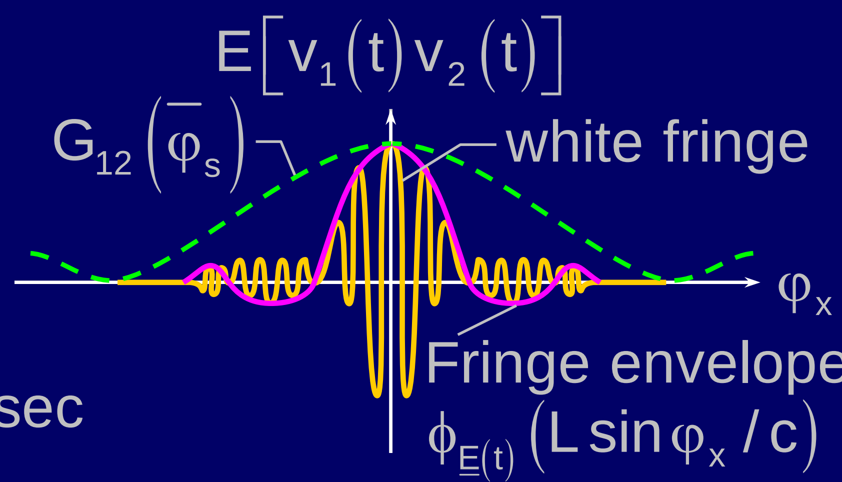
$$\left[e^{j\omega\left(t + L \sin \varphi_x / 2c\right)} e^{-j\omega\left(t - L \sin \varphi_x / 2c\right)} = e^{j\omega L \sin \varphi_x / c} \right]$$

$$E\left[v_1(t)v_2(t)\right] = \frac{k_1k_2}{2} \underbrace{G_{12}\left(\bar{\varphi}_s\right)}_{\text{cross-gain}} R_e \left\{ e \underbrace{^{-j^\gamma} e^{j2\pi L \sin \varphi_x / \lambda}}_{\text{monochromatic fringe}} \bullet \underbrace{\phi_{\underline{E}(t)}\left(\frac{L \sin \varphi_x}{c} \right)}_{\text{fringe envelope}} \right\}$$

Bandwidth-Limited Angular Response







$$\text{Null @ } L\frac{\sin\phi_{x_{null}}}{c} = \frac{1}{2B}\text{ sec}$$

In broadband optical interferometer all colors contribute to central "white" fringe; sidelobe fringes appear colored.

Therefore $\varphi_{x_{null}} = \sin^{-1}(c/2LB) \approx c/2LB$ for $\varphi_x \cong 1$

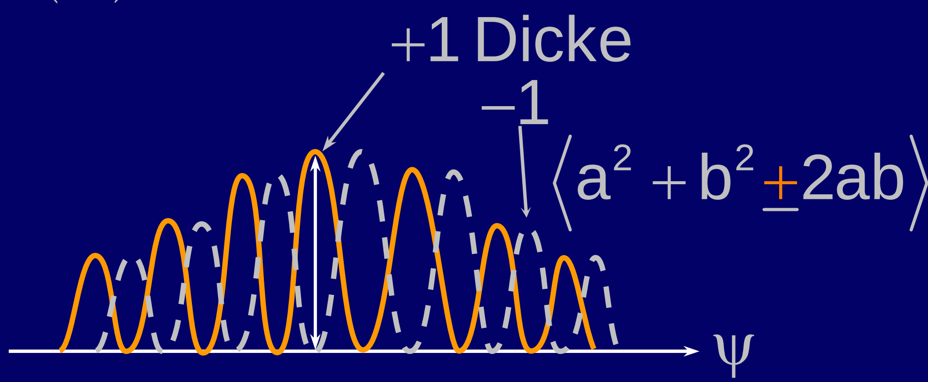
e.g. $\varphi_{x_{null}} \cong \dfrac{3\times10^8}{2\times\underset{\uparrow\ 10\text{-m}}{10}\times\underbrace{10^7}_{10\text{ MHz}}} = 1.5$ radians for $L = 10$ m, $B = 10$ MHz

If $B = 1$ GHz, and $L = 100$ m, then $\varphi_{x_{null}} = 1.5$ mrad $\cong 5$ arc min

$B = 3\times10^{14}$ Hz and $L = 100$ m, then $\varphi_{x_{null}} = 10^{-3}$ arc sec.

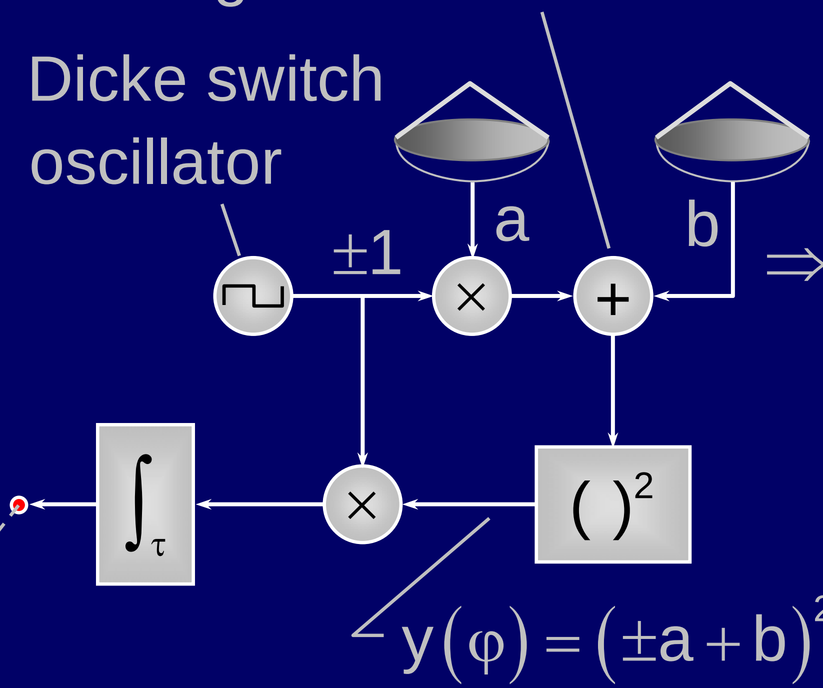
Dicke Adding Interferometer

$y(\psi)$ point source response



(Also called "lobe-switching" interferometer)

"Adding" interferometer

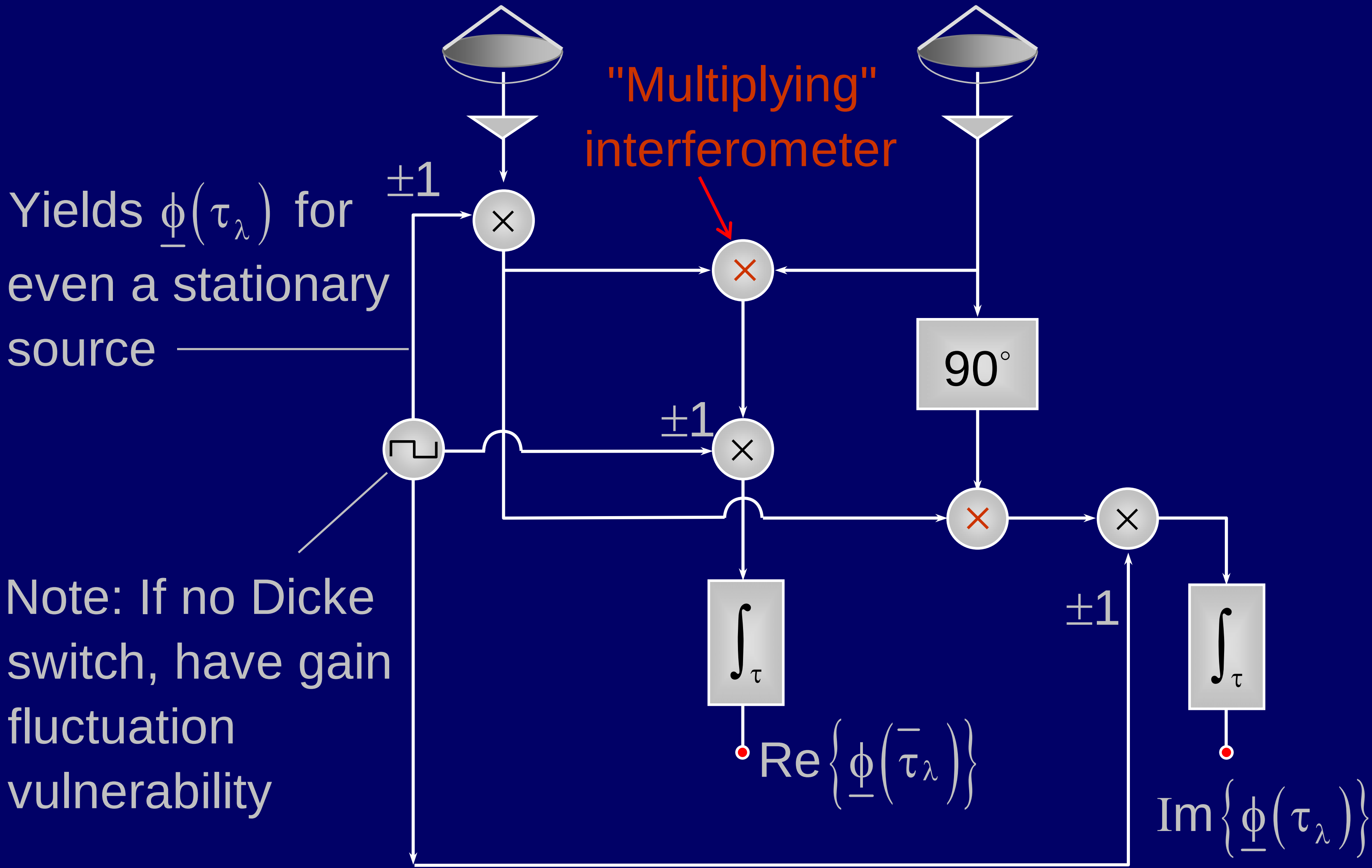


$$\left[\overline{a^2} + \overline{b^2} + \overline{2ab}\right] - \left[\overline{a^2} - \overline{2ab} + \overline{b^2}\right] = \overline{4ab}$$

This circuit cancels D.C. term, leaving only $\langle 4ab \rangle$ as source traverses beam ⇒

Can add second adder and square-law device operating on a and –jb to yield sine terms in Fourier expansion

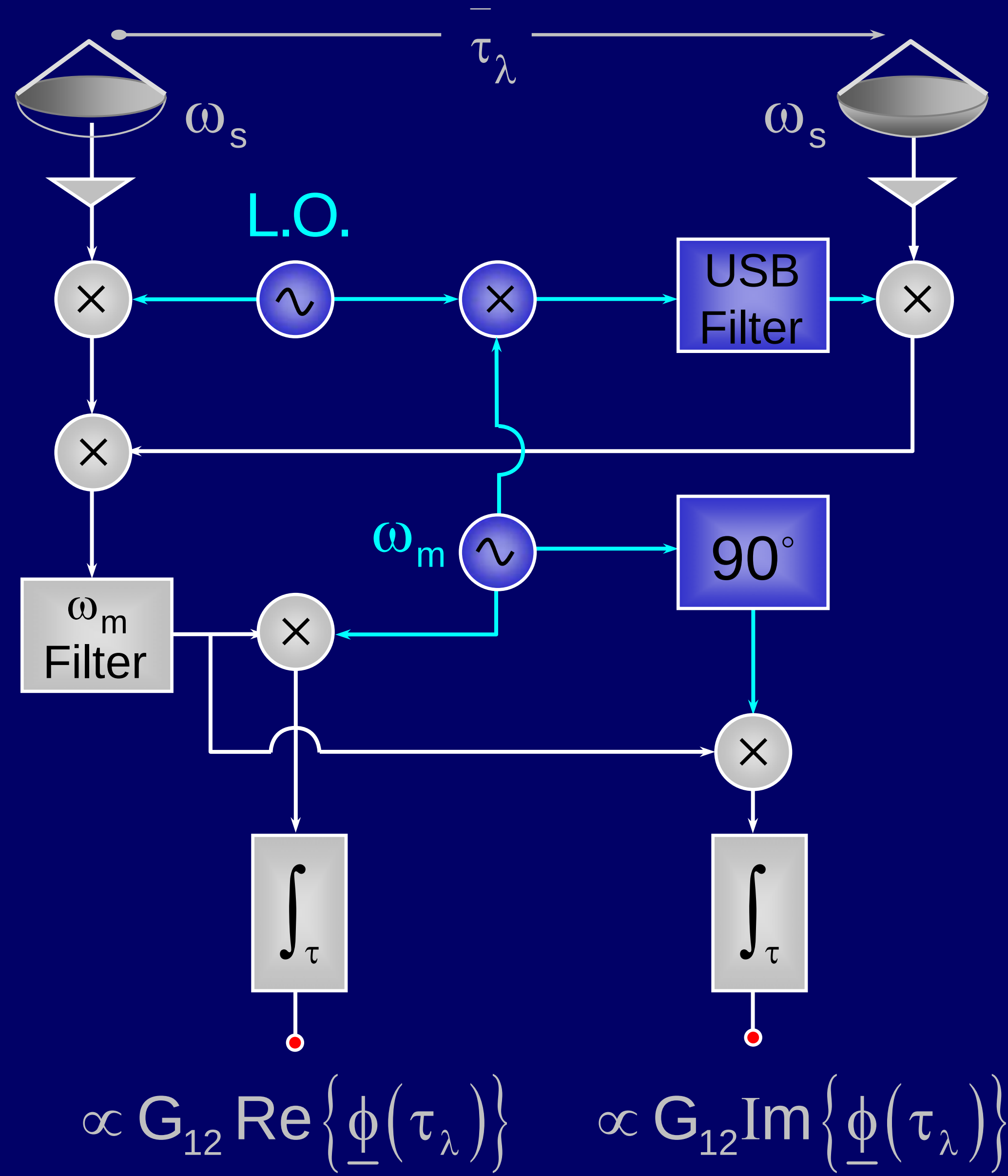
Dicke Adding Interferometer



"Lobe-Scanning" Interferometer

Lobes are scanned at ω_m, demodulated, and averaged to yield $\underline{\phi}(\tau_\lambda)$

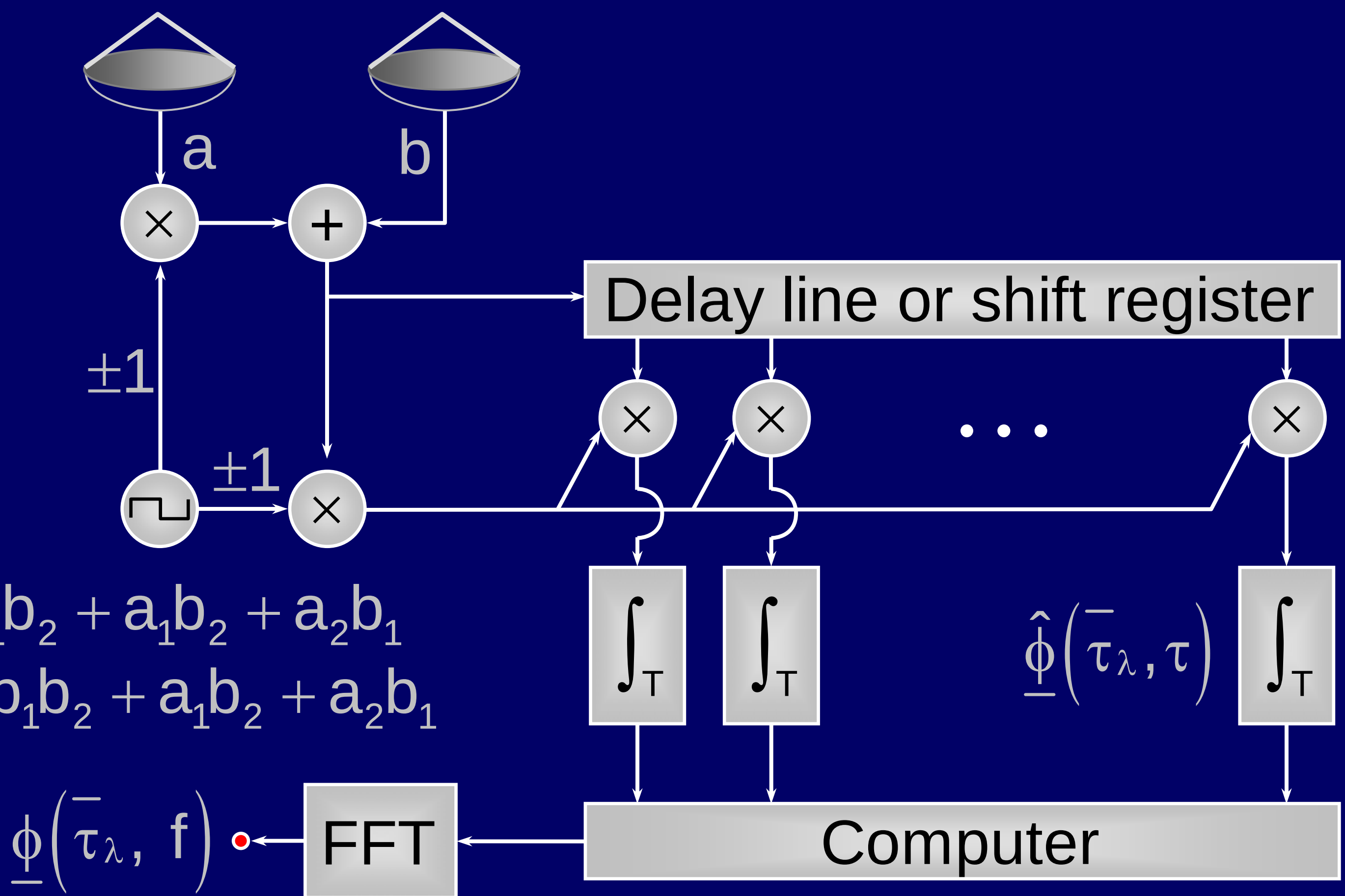
Because all bias and large-scale sources yield no fine-scale response, can integrate long times seeking fine structure, e.g. 10^{-3} Jansky point sources like stars (can measure stellar diameters at ~10^{-3} arc sec)



Cross-Correlation Interferometer Spectrometer

$$\text{Let } a(t) \triangleq a_1$$

$$a(t-\tau) \triangleq a_2$$



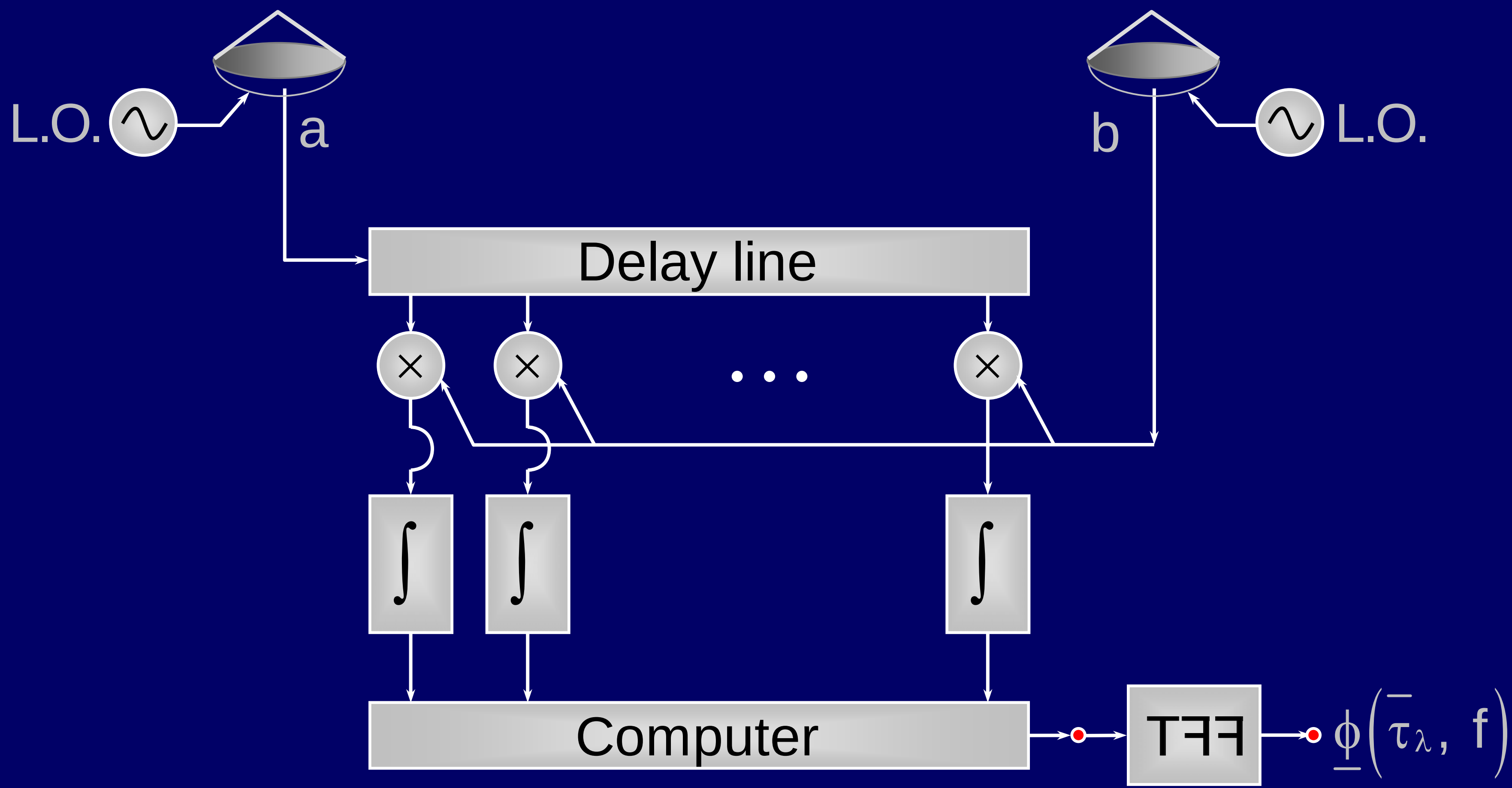
$$c_+ = a_1a_2 + b_1b_2 + a_1b_2 + a_2b_1$$

$$c_- = -a_1a_2 - b_1b_2 + a_1b_2 + a_2b_1$$

Note: if $a = b$, $\underline{\phi}\left(\bar{\tau}_\lambda,\ f\right) \rightarrow \underline{\phi}\left(0,\ f\right) = S\left(f\right)$

if $a \perp b$, $\underline{\phi}\left(\bar{\tau}_\lambda,\ f\right) \rightarrow 0$

Alternate Cross-Correlation Interferometer Spectrometer

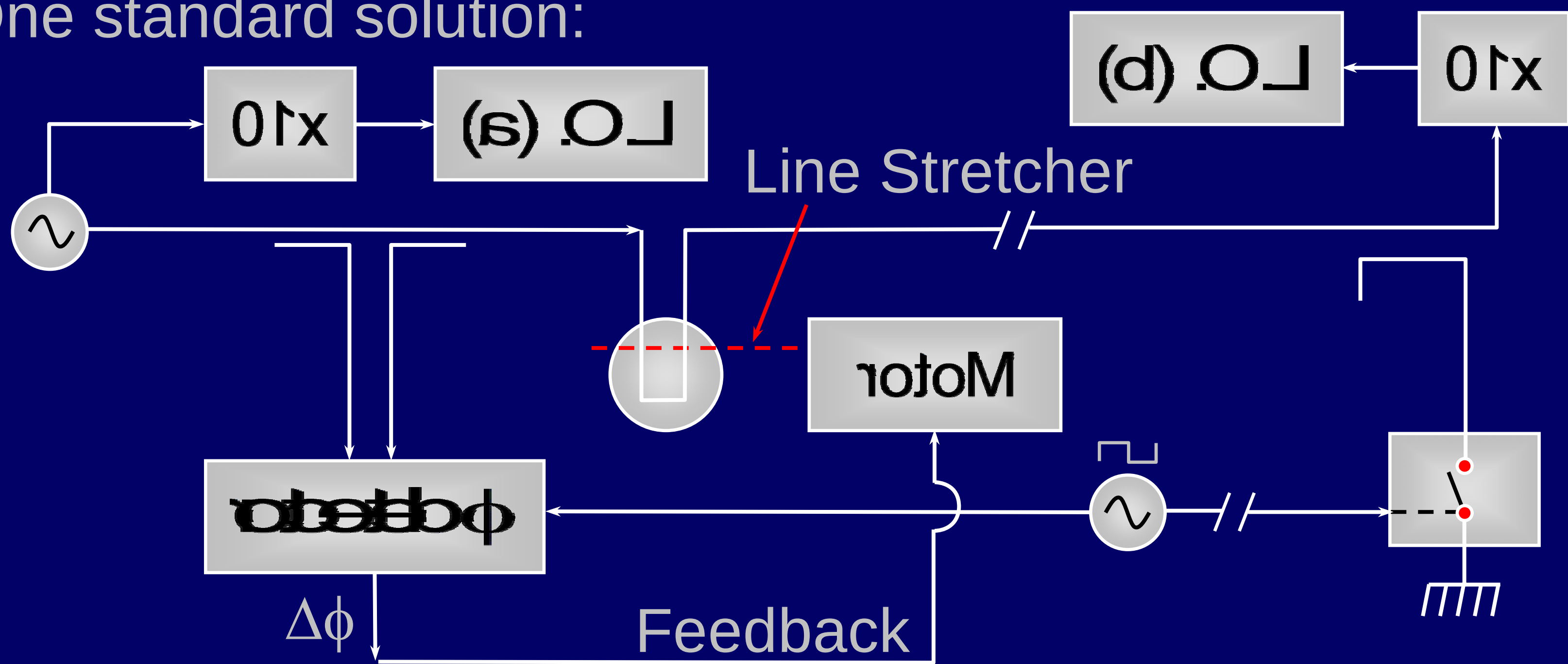


Note: Figures omit down-converters and bandlimiting filters

Mechanical Long Distance Phase Synchronization

For L >> λ, L.O. synchronization can be degraded by random phase variations in path length L between two (or more) sites (due principally to thermal and acoustic variations)

One standard solution:

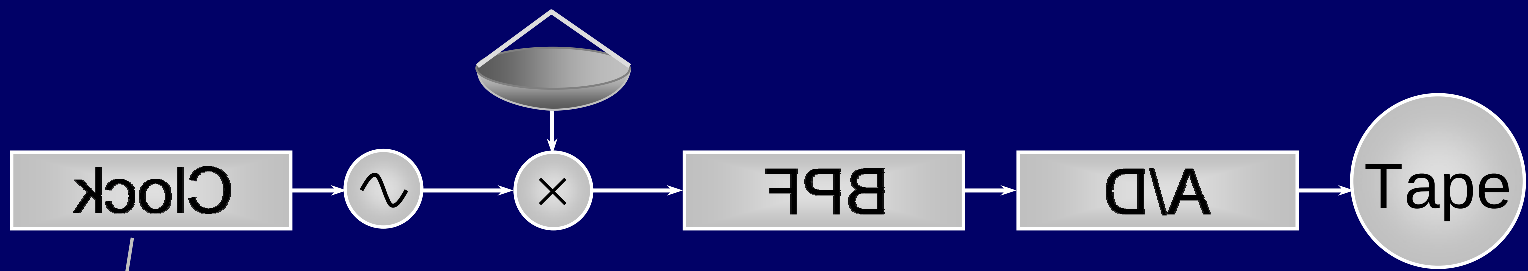


"Line stretcher" varies path so that $\Delta\phi \cong 0$

Communications and telemetry systems can similarly be phase synchronized

Synchronizing with Remote Atomic Clocks

If distance L is too great to synchronize L.O.'s, then we can use a remote clock, e.g. "very-long baseline" interferometry, "VLBI"



- Hydrogen maser $\Rightarrow \sim 10^{-14},\ 10^{-15}$
- Cesium beam $\Rightarrow \sim 10^{-12}$
- Global Positioning System (GPS) $\Rightarrow \sim 10^{-8}$ sec
- Loran – C $\Rightarrow \sim 10^{-6}$ sec
- Temperature-stabilized crystal oscillators $\Rightarrow \sim 10^{-5} - 10^{-8}$
- Crystal oscillators $\Rightarrow \sim 10^{-4} - 10^{-5}$

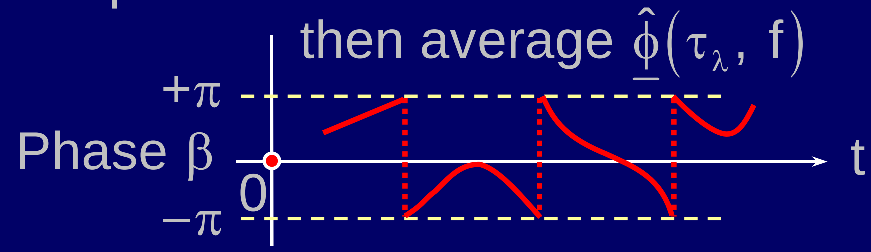
If clocks perfect: cross-correlate to find time offset, correct for it, then correlate the signals, albeit with an unknown fixed phase offset ϕ_0 [unless reference source (in the sky) or phase is available].

If clocks imperfect and delays each way are identical: at site A measure delay between clock B and A; do the same at B, and subtract results to yield twice the clock offsets. Use this offset to align A and B data streams.

Synchronizing with Remote Atomic Clocks

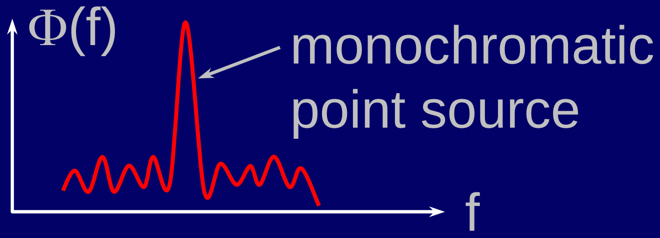
Alternative approaches if clocks and transmissions are imperfect:

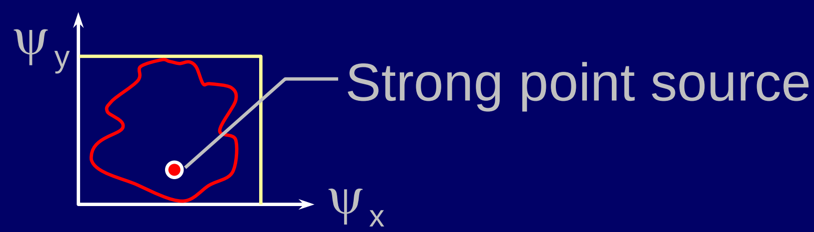
a) Track and correct phase shifts:



Example: A 10^{-12} cesium clock drifts 2π in ~ 100 sec, so $\underline{\hat{\phi}}(\tau_\lambda, f)$ might be computed for 5 – 10 sec blocks before averaging; then only $|\underline{\hat{\phi}}|$ is known.

b) Same, but set phase using strong resonant line point source,

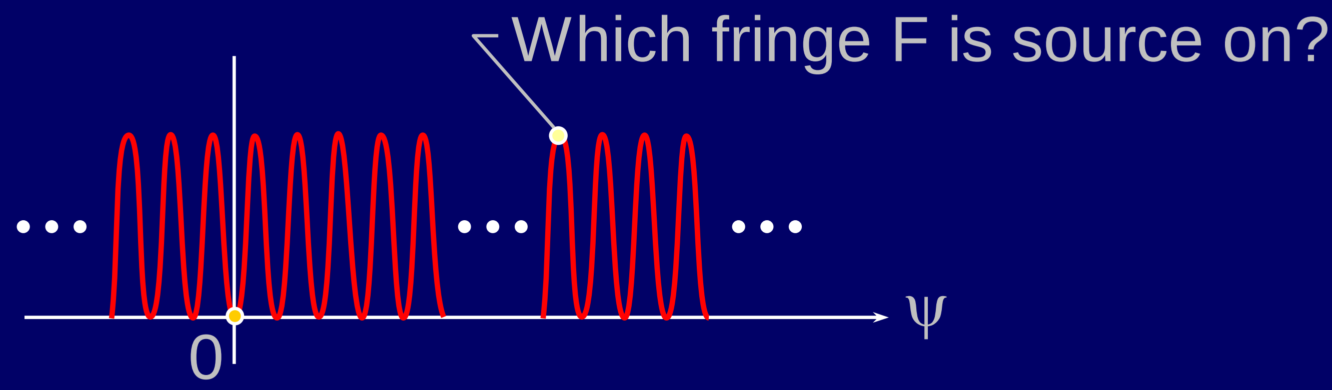




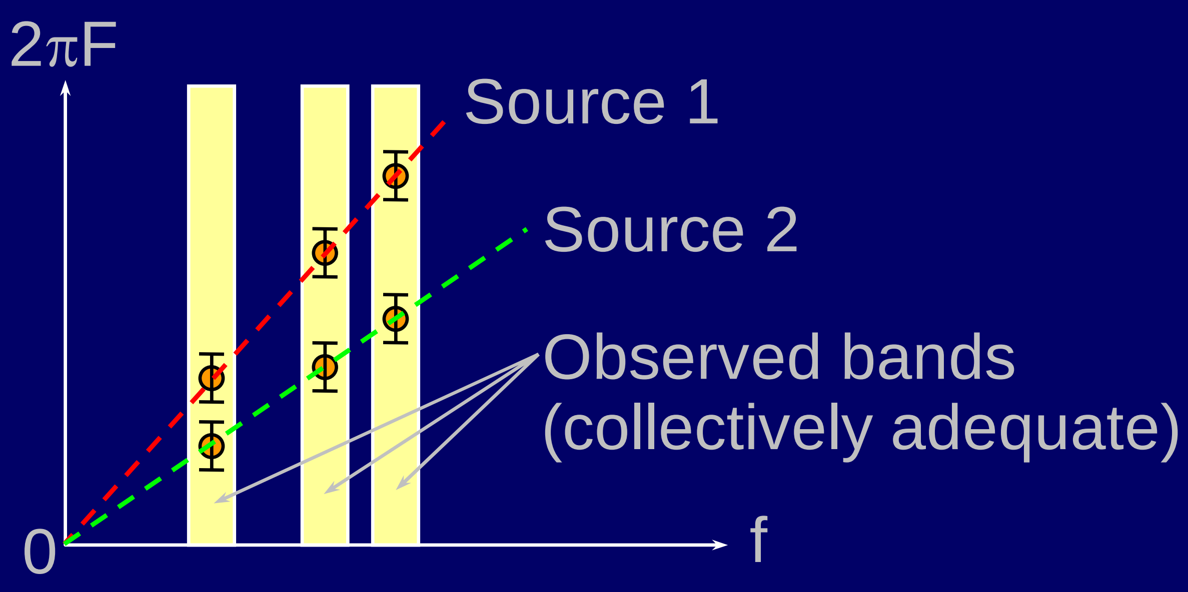
c) or separable point source in space, modulation, etc.

Multiband Synchronization of Clocks

Use multiple frequencies for wideband sources:



Switch across all f's within coherence time of clock.

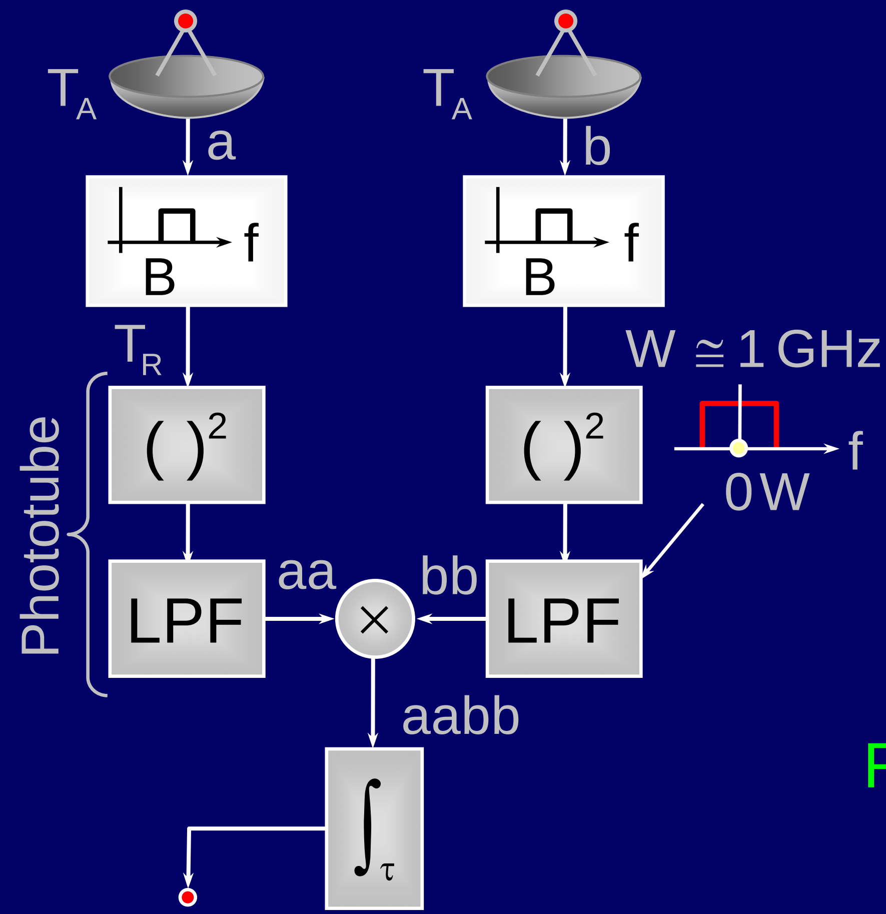


Phaseless Interferometry

Hanbury-Brown and Twiss

Visible interferometer at Narrabri, Australia

$$\left[\text{For } T_R \gg T_A, \frac{v_{o\,rms}}{\langle v_o \rangle} \cong \frac{T_R^2}{T_A^2\sqrt{2W\tau}}\right]$$



$\propto \left|\phi_E\left(\bar{\tau}_\lambda\right)\right|^2 \Rightarrow$ source size, etc.

One might (wrongly) think photodetectors would lose all phase information and ability to measure source structure at λ/D resolution.

Recall: $E\left[aabb\right] = \overline{a^2}\,\overline{b^2} + 2\overline{ab}^2$,

where $\overline{ab}$ is $\phi_{\underline{E}}\left(\bar{\tau}_y\right)$ here.

Phaseless Recovery of Source Structure

Recall: $E[aabb] = \overline{a^2}\,\overline{b^2} + 2\overline{ab}^2$, where $\overline{ab}$ is $\phi_{\underline{E}}\left(\bar{\tau}_y\right)$ here.

$$\text{Recall: } \underline{E}(x,\ y) \leftrightarrow \underline{E}\left(\bar{\psi}\right)$$

$$\downarrow \qquad\qquad \downarrow$$

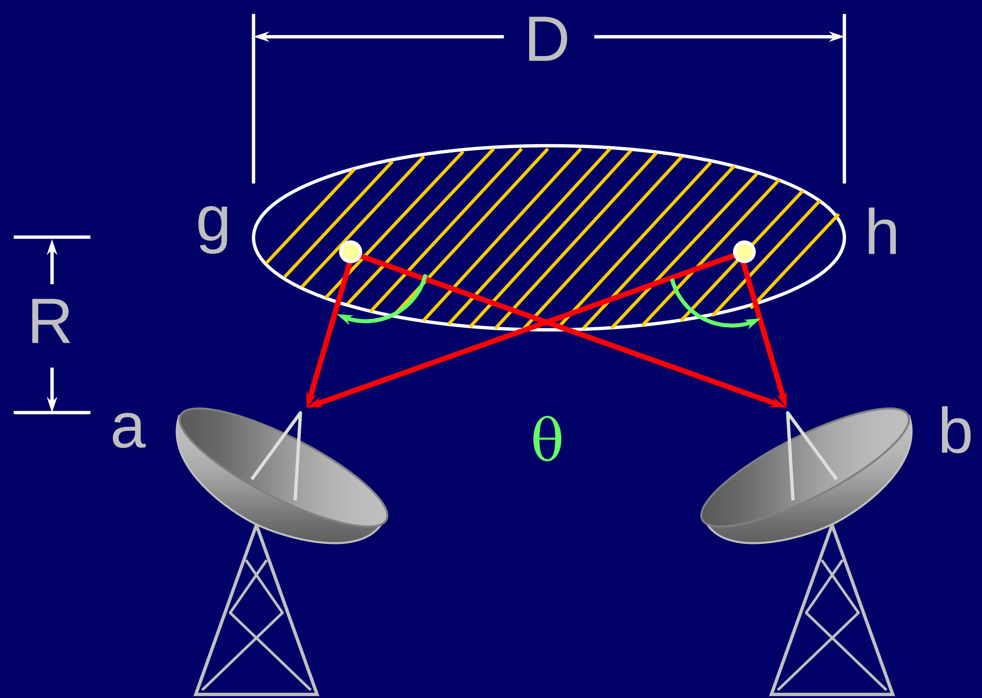
$$\underline{\phi}_{\underline{E}}\left(\bar{\tau}_\lambda\right) \leftrightarrow \left|\underline{E}\left(\bar{\psi}\right)\right|^2 \Rightarrow I\left(\bar{\psi}\right)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\left|\underline{\phi}_{\underline{E}}\left(\bar{\tau}_\lambda\right)\right|^2 \leftrightarrow R_{\left|\underline{E}\left(\bar{\psi}\right)\right|^2}\left(\Delta\bar{\psi}\right)$$

Purely real if source is even function of position, allowing perfect source reconstruction

Phaseless Interferometer Interpretation: Independent Radiators

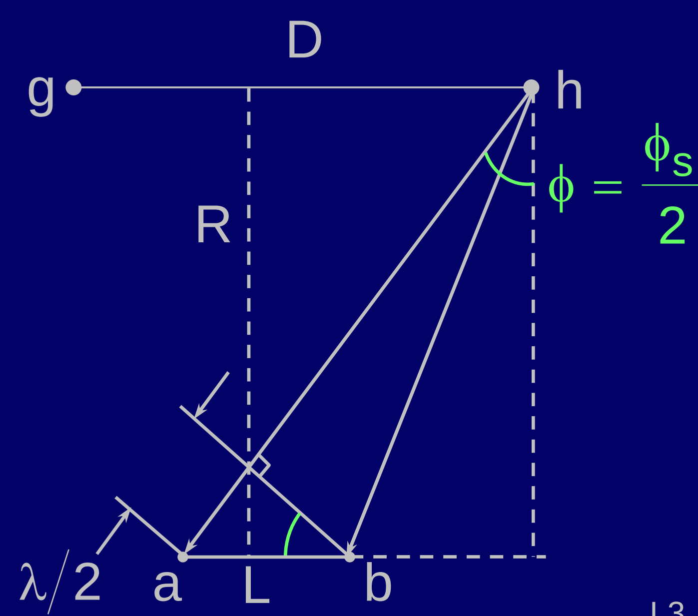


Source, independent thermal radiators g and h

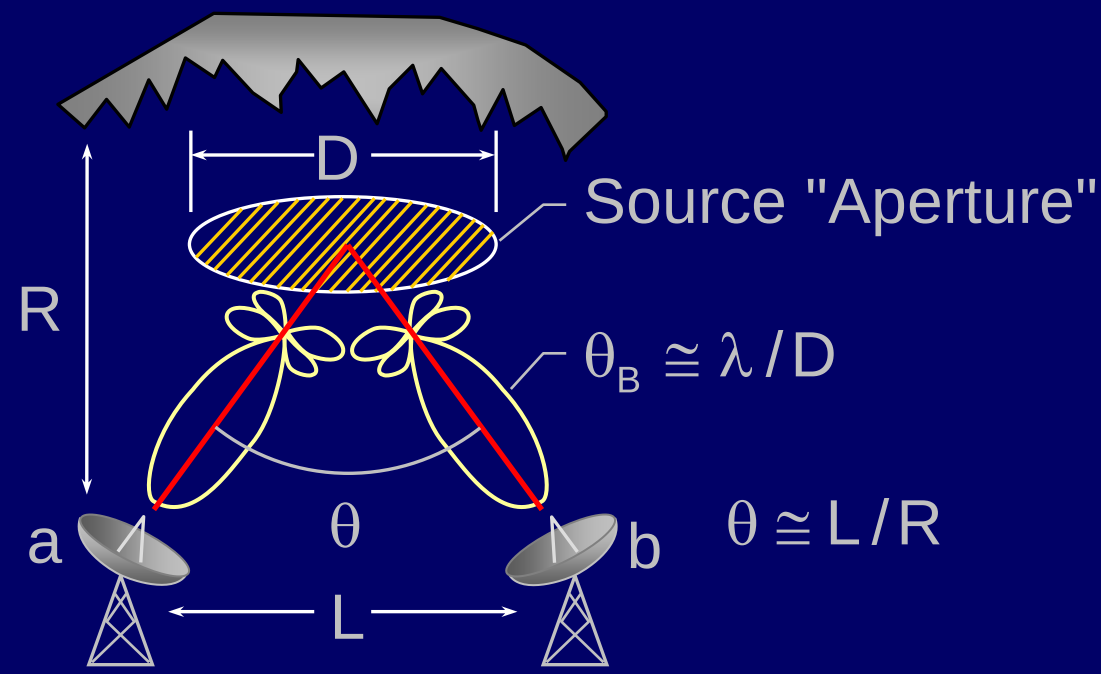
a,b are uncorrelated if $\Delta\phi_a - \Delta\phi_b \gtrsim 2\pi$ [$\Delta\phi_a$ is $\Delta\phi$ at "a" for rays g,h]; or if

$$\frac{\phi_s}{2} = \phi \gtrsim (\lambda/2)/L.$$

Thus a,b decorrelated if $\phi_s \gtrsim \lambda/L$.



Phaseless Interferometer Diffraction-Limited Source



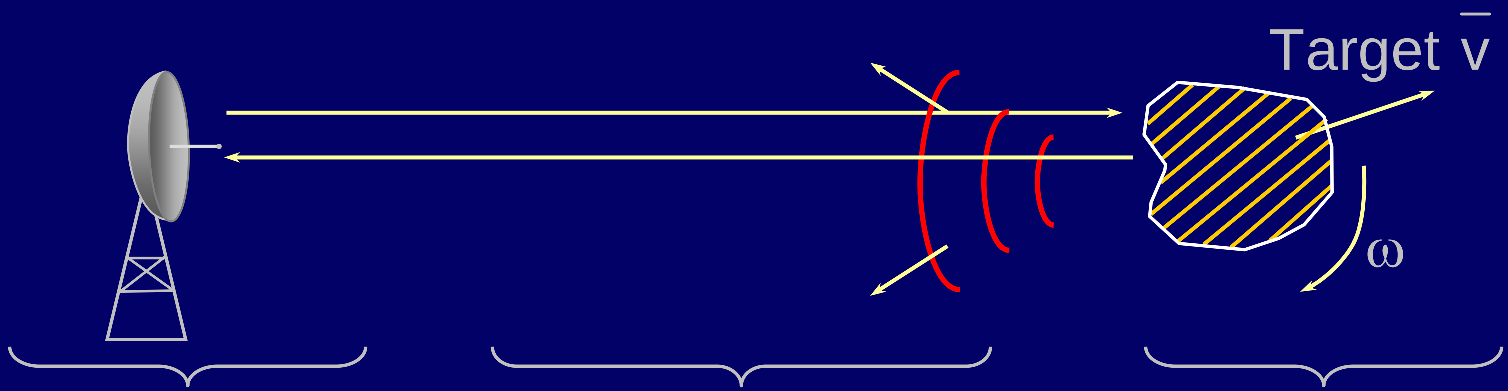
If $\theta \tilde{>} \theta_B \cong \lambda/D$, then a and b are ~uncorrelated

Therefore decorrelated if $D\theta \tilde{\geq} \lambda$

or if $DL/R \tilde{\geq} \lambda$ since $\theta \cong L/R$

or if $\phi_S \tilde{\geq} \lambda/L$ since $\phi_S \cong D/R$

Radar Equation



Issues: Signal design
Processor design
Antenna

Propagation, absorption, refraction, scintillation, scattering, multipath

Scattering

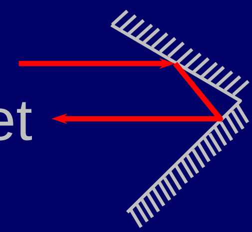
$$P_{rec} = \overbrace{\underbrace{\frac{P_t}{4\pi R^2} \bullet G_t}_{Wm^{-2}\ \text{at target}} \bullet \frac{\sigma}{4\pi R^2}}^{Wm^{-2}\ \text{at transmitter}} \bullet A_t = P_t \left(\frac{G\lambda}{4\pi R^2} \right)^2 \frac{\sigma}{4\pi}\ \text{Watts}$$

σ "scattering cross-section" is equivalent capture cross-section for a target scattering isotropically

Radar Scattering Cross-Section

σ "scattering cross-section" is equivalent capture cross-section for a target scattering isotropically

Note: Corner reflector can have $\sigma \gg$ size of target

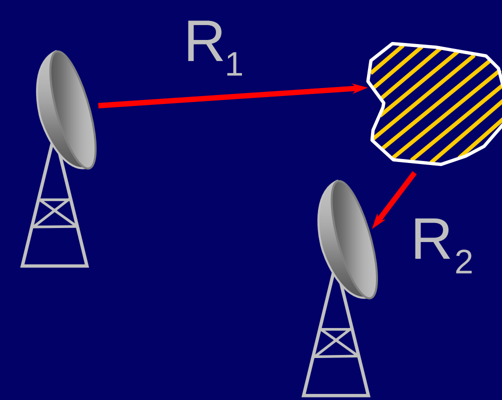


Biastatic radars:

If target is unresolved, $P_{rec} \propto 1/R_1^2 R_2^2$

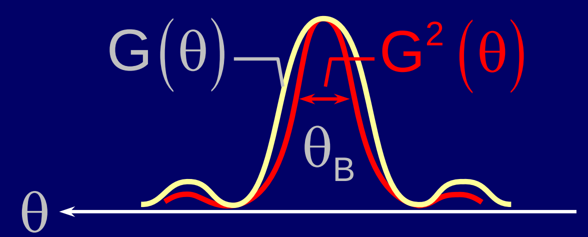
If target is resolved by the transmitter,

$P_{rec} \tilde{\propto} 1/R_2^2$



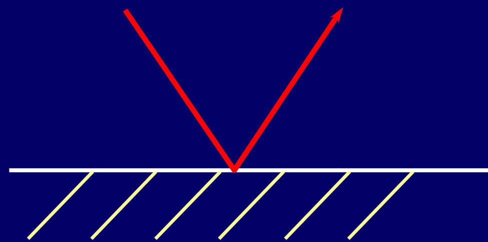
Note resolution enhancement:

$P_{rec} \propto R^{-4} G^2$ where $G^2(\theta)$ has a narrower beam than $G(\theta)$

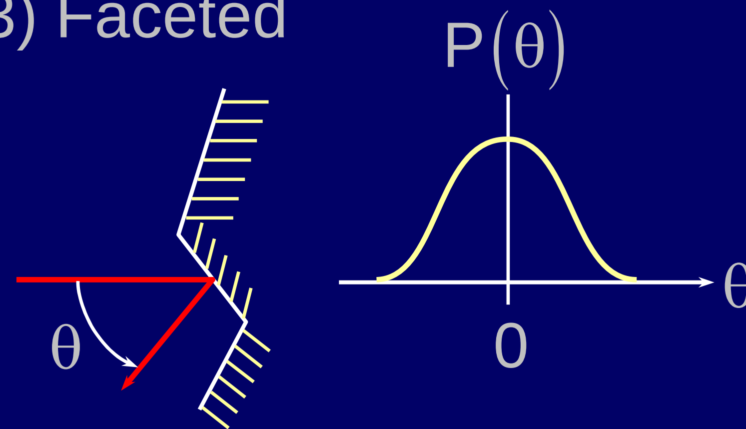


Target Scattering Laws

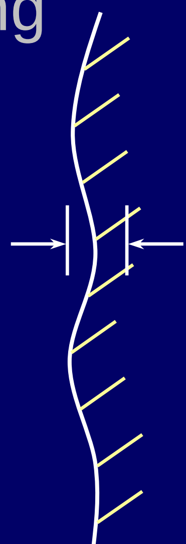
1) Specular



3) Faceted

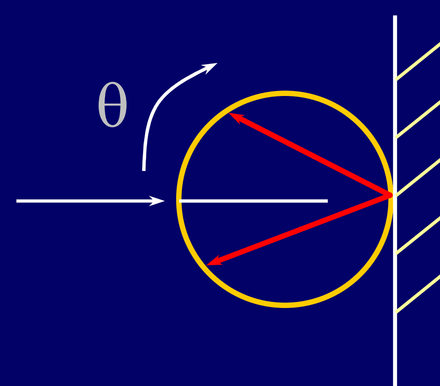


2) Scintillating



$\delta << \lambda \Rightarrow \sim$ nulls

4) Lambertian

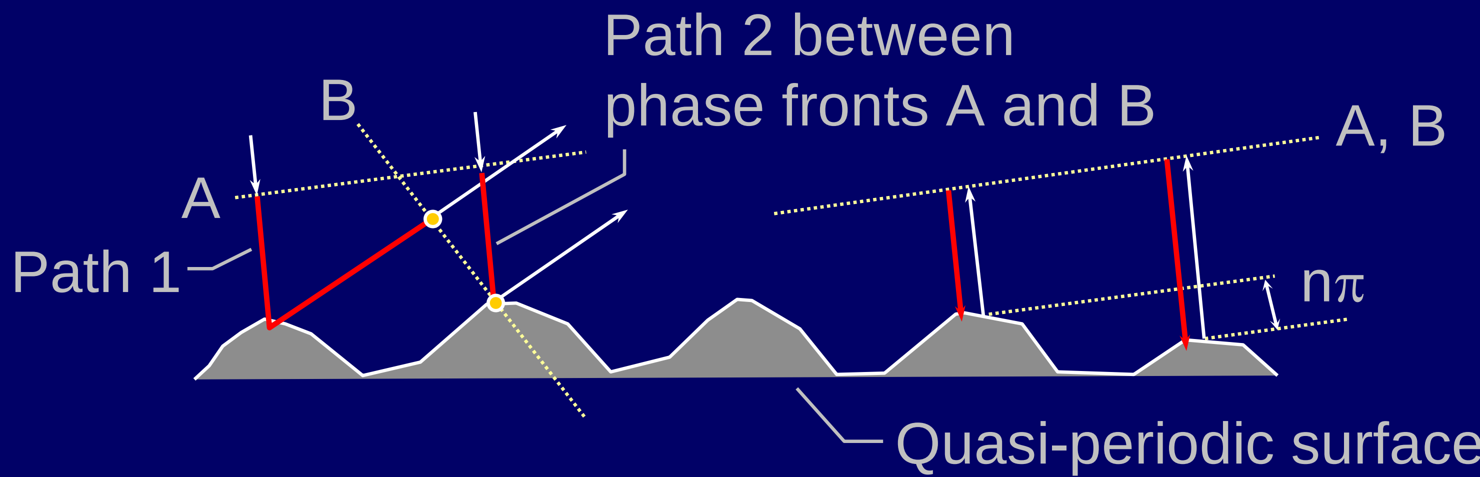


$\cos\theta \propto$ power scattered
(geometric projection only)

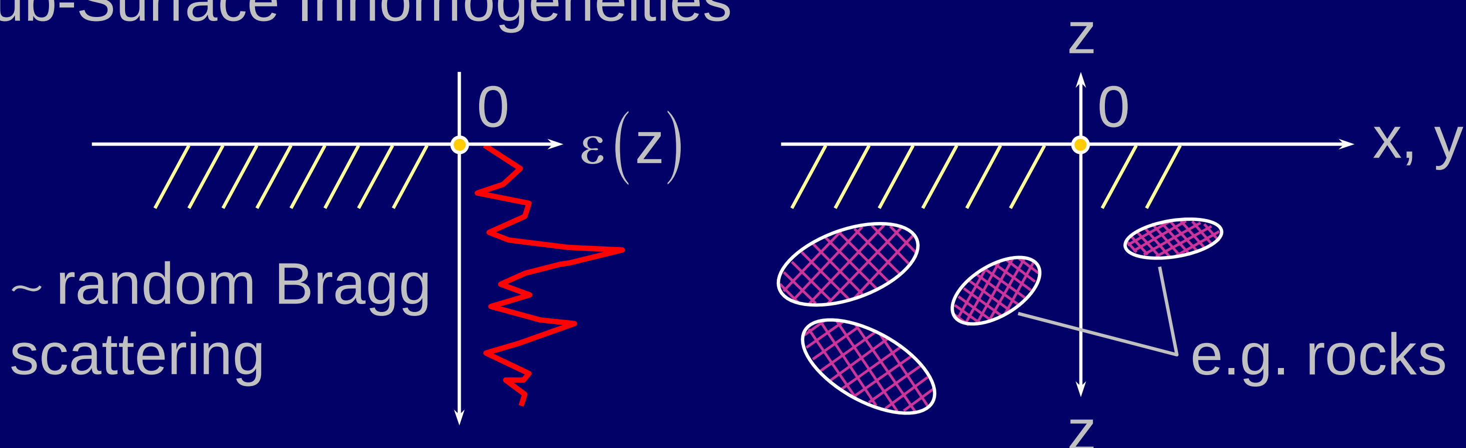
Target Scattering Laws

5) Random Bragg Scattering (frequency selective)

At Bragg angles $\Delta \text{Path}_{1,2} = n \bullet 2\pi \qquad n = 0, \ \pm 1, \ \pm 2, \ldots$

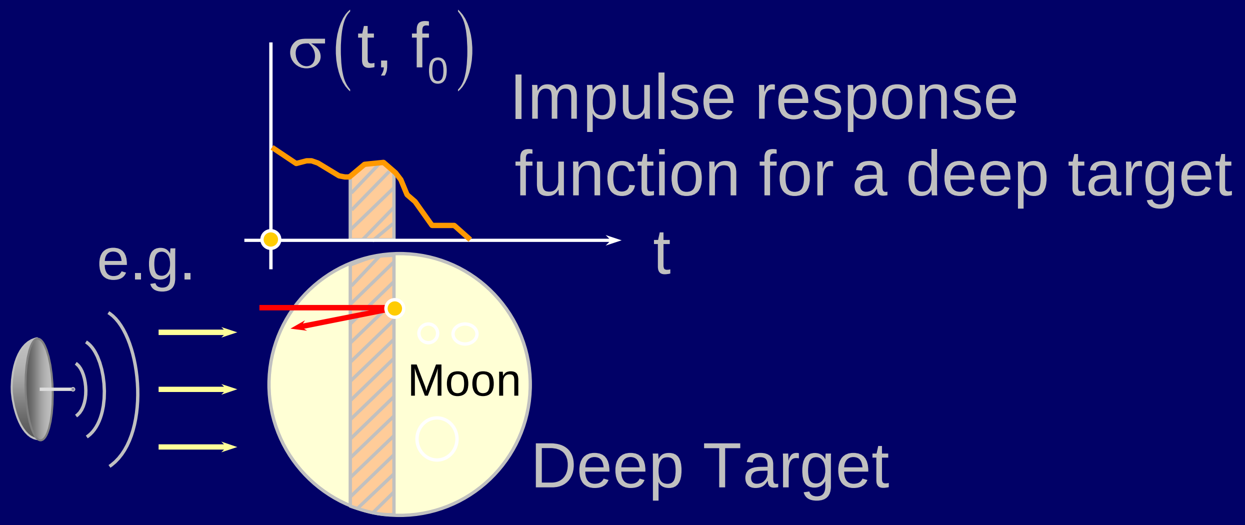


6) Sub-Surface Inhomogeneities

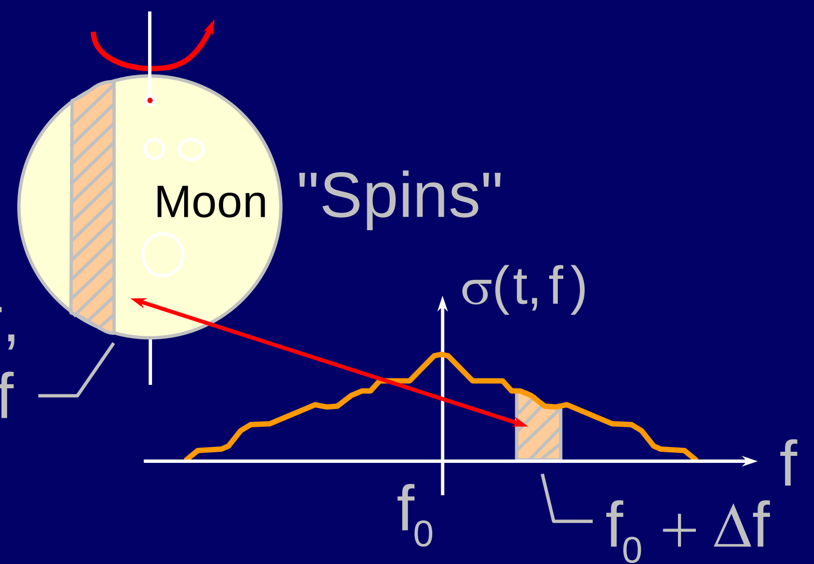


Target Range-Doppler Response

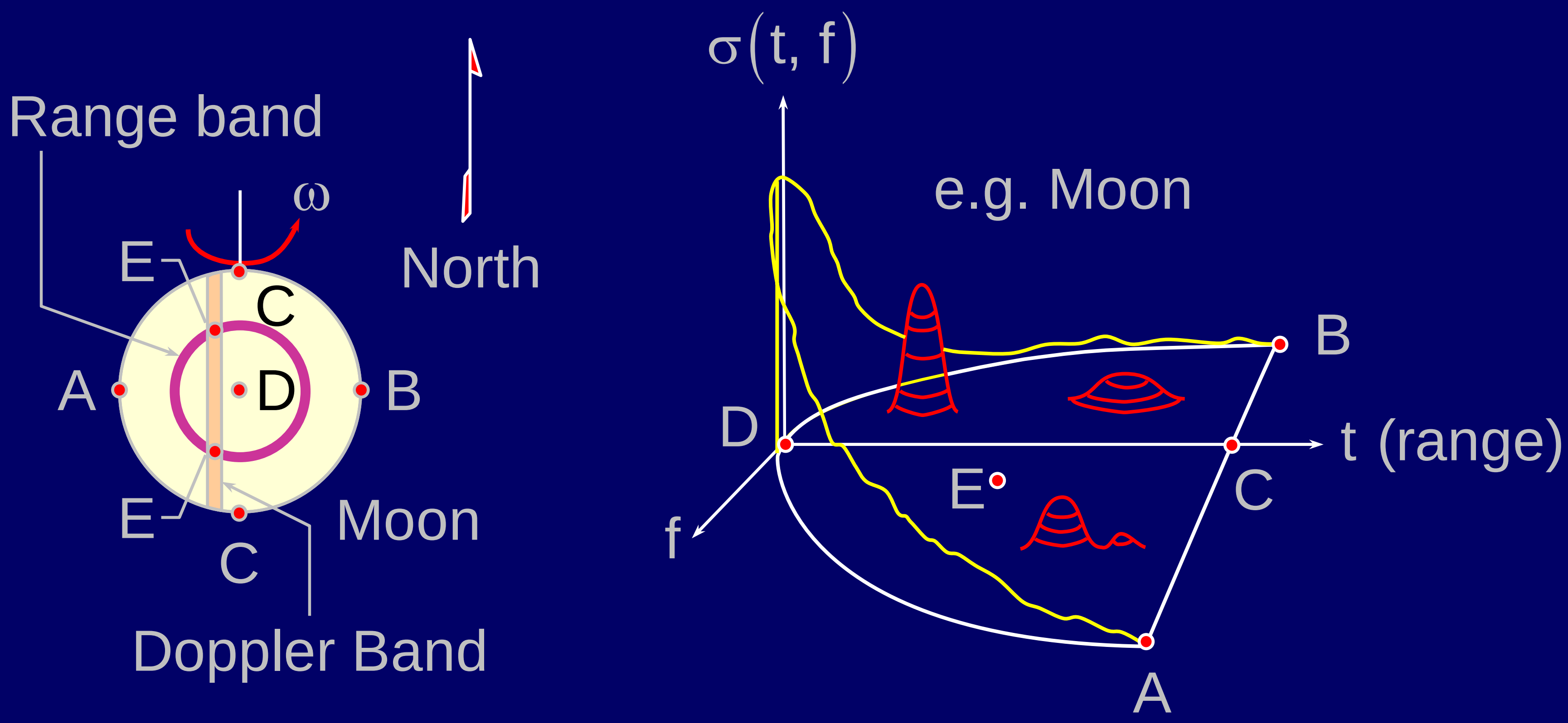
Narrowband Pulsed Radar



CW Radar



Range-Doppler Response for a CW Pulse



Note north-south ambiguity