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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

**6.776**

***High Speed Communication Circuits and Systems***

***Lecture 9***

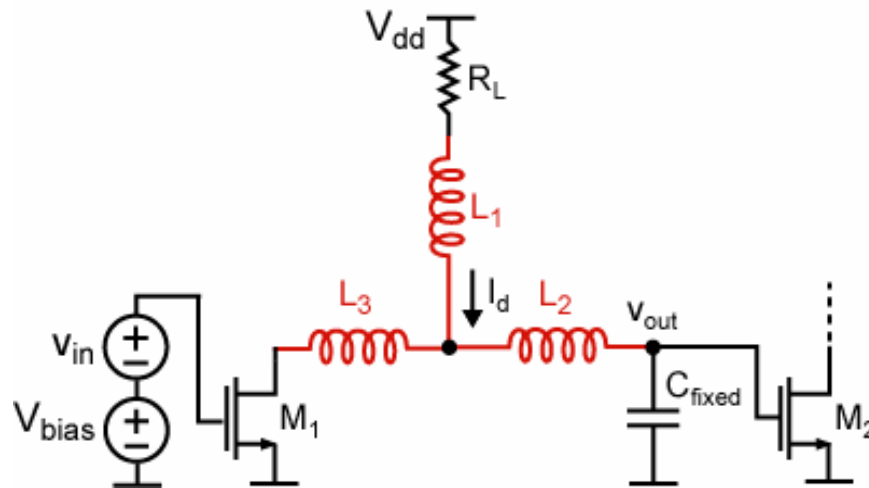
***Enhancement Techniques for Broadband Amplifiers,  
Narrowband Amplifiers***

**Massachusetts Institute of Technology**

**March 3, 2005**

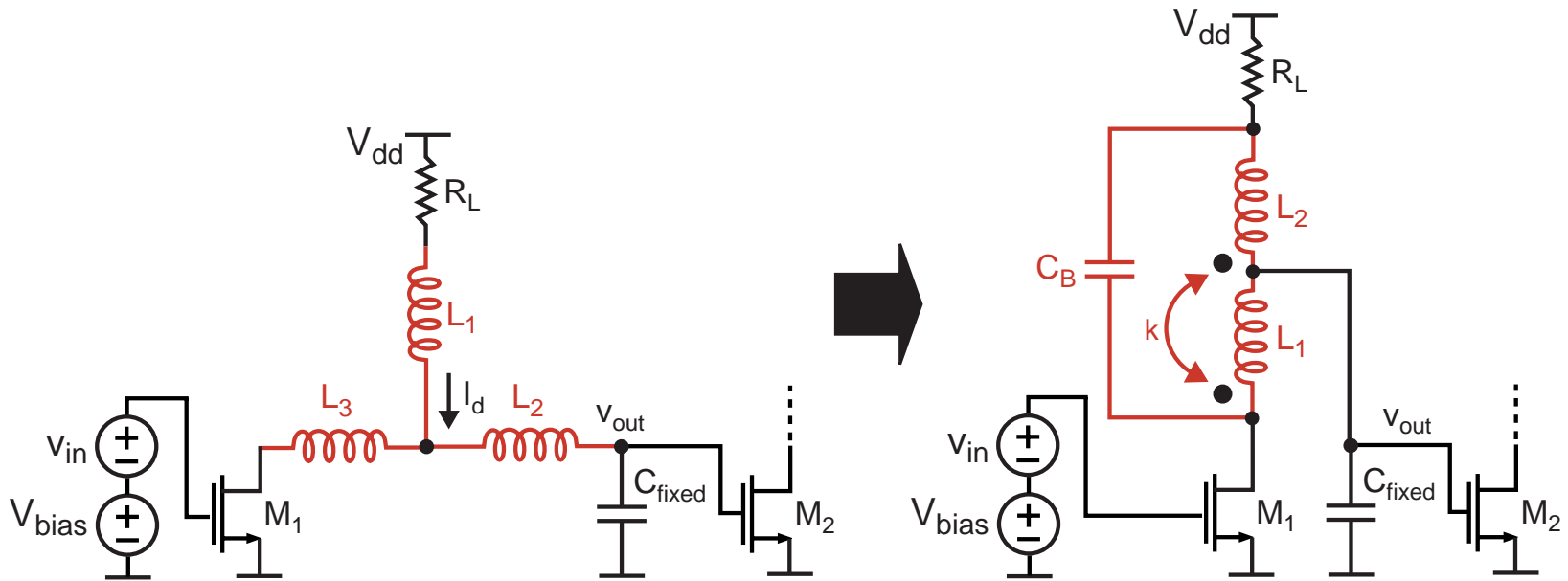
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Perrott**

# Shunt-Series Peaking



- Series inductors isolate load capacitance from  $M_1$ : delays charging of load capacitance
- Trades delay for bandwidth
- $L_1$ ,  $L_2$ ,  $L_3$  can be implemented by 2 coupled inductors with coupling coefficient of  $k$

# T-Coil Bandwidth Enhancement



- **Uses coupled inductors to realize T inductor network**
  - Works best if capacitance at drain of  $M_1$  is much less than the capacitance being driven at the output load
- $C_B$  provides parallel resonance to improve bandwidth further  
See Chap. 9 (Ch. 8, 1<sup>st</sup> ed.) of Tom Lee's book pp 279-282 (187-191)

## T-Coil Continued

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- The self inductance  $L$  (with the other winding open-circuited) must be

$$L = \frac{R_L^2 C_L}{2(1 + k)}$$

- The bridging capacitance

$$C_B = \frac{C_L(1 - k)}{4(1 + k)}$$

- Coupling coefficient

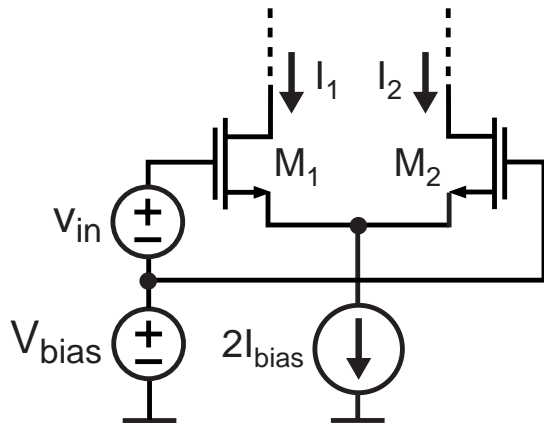
$k=1/3$  for Butterworth response

$k=1/2$  for maximally flat delay (linear phase)

**Bandwidth extension: approximately 2.8 (Butterworth)**

- See S. Galal, B. Ravazi, "10 Gb/s Limiting Amplifier and Laser/Modulator Driver in 0.18 $\mu$  CMOS", ISSCC 2003, pp 188-189 and "Broadband ESD Protection ...", pp. 182-183
- Also see "Circuit Techniques for a 40 Gb/s Transmitter in 0.13 $\mu$ m CMOS", J. Kim, et. al. ISSCC 2005, Paper 8.1

# Bandwidth Enhancement With $f_t$ Doublers



- A MOS transistor has  $f_t$  calculated as

$$2\pi f_t = \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{g_m}{C_{gs}}$$

- $f_t$  doubler amplifiers attempt to increase the ratio of transconductance to capacitance

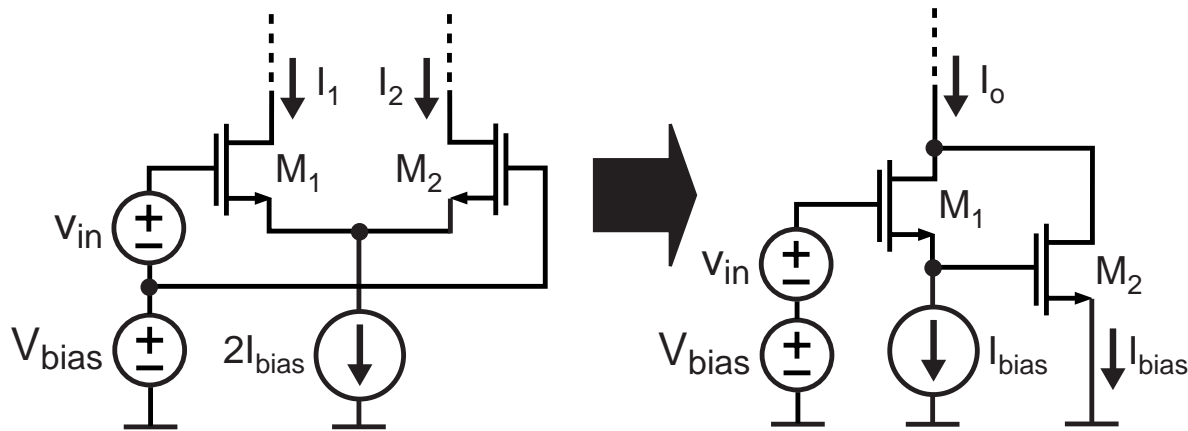
- We can make the argument that differential amplifiers are  $f_t$  doublers

- Capacitance seen by  $V_{in}$  for single-ended input:  $C_{gs}/2$
- Difference in current:

$$i_2 - i_1 = \frac{v_{in}}{2}g_m - \left(-\frac{v_{in}}{2}\right)g_m = v_{in}g_m$$

- Transconductance to Cap ratio is doubled:  $\frac{2g_m}{C_{gs}}$

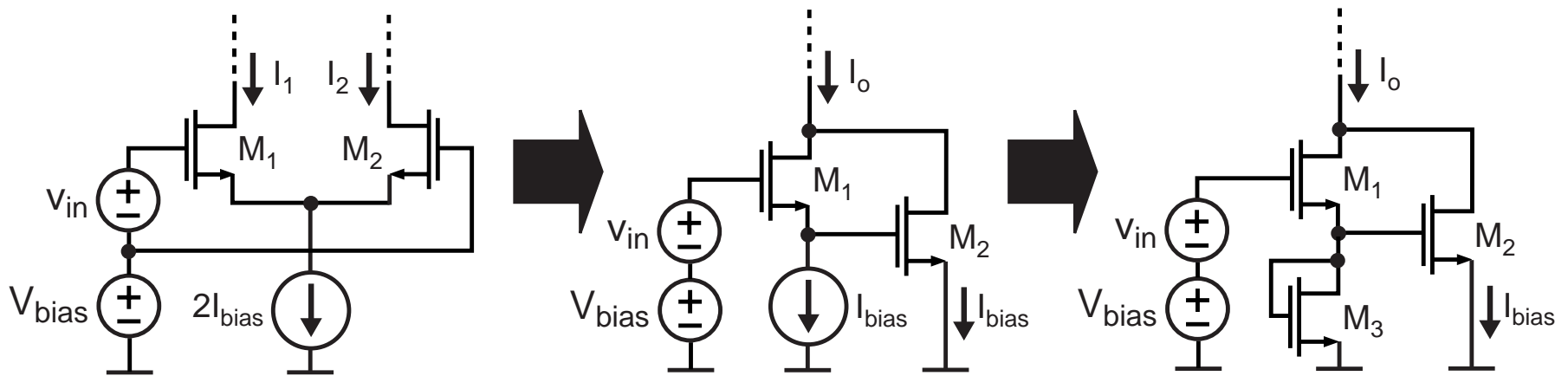
# Creating a Single-Ended Output



- **Input voltage is again dropped across two transistors**
  - Ratio given by voltage divider in capacitance
    - Ideally is  $\frac{1}{2}$  of input voltage on  $C_{gs}$  of each device
- **Input voltage source sees the series combination of the capacitances of each device**
  - Ideally sees  $\frac{1}{2}$  of the  $C_{gs}$  of  $M_1$
- **Currents of each device add to ideally yield ratio:**

$$\frac{2g_m}{C_{gs}}$$

## Creating the Bias for $M_2$



- **Use current mirror for bias (Battjes  $f_t$  doubler)**
  - Inspired by bipolar circuits (see Tom Lee's book, pp288-290 (197-199))
- **Need to set  $V_{bias}$  such that current through  $M_1$  has the desired current of  $I_{bias}$** 
  - The current through  $M_2$  will ideally match that of  $M_1$

# Problems of $f_t$ Doubler in Modern CMOS RF Circuits

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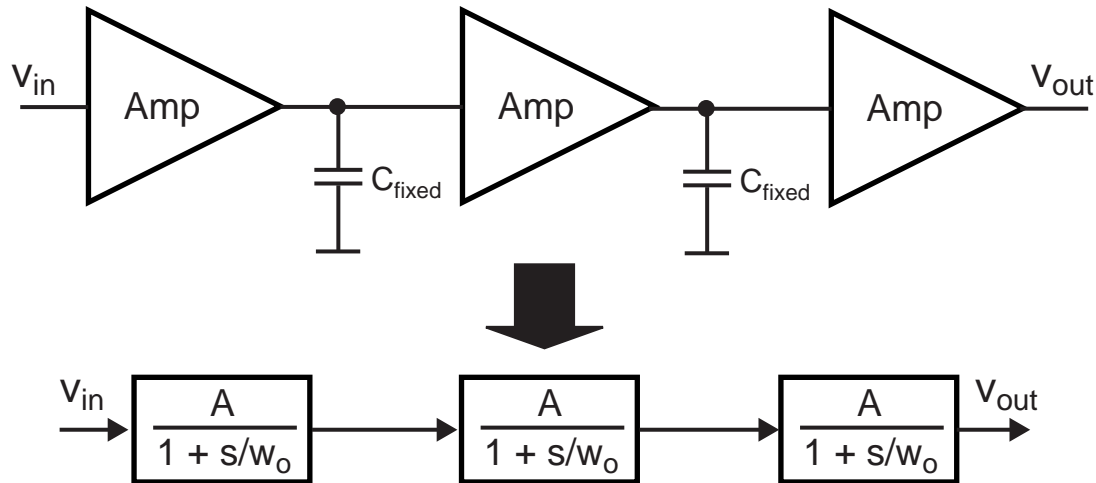
- Problems:
  - Works if  $C_{gs}$  dominates capacitance, but in modern CMOS, this is not the case (for example,  $C_{gd}=0.45C_{gs}$  in  $0.18\ \mu$  CMOS)
  - achievable bias voltage across  $M_1$  (and  $M_2$ ) is severely reduced by 2x! (thereby reducing effective  $f_t$  of device)
  - Input capacitance degrades due to  $C_{gs}$ ,  $C_{db}$  of  $M_3$ : at most 1.5x improvement in transconductance/capacitance ratio

Assuming zero  $C_{db}$ :

$$C_{in} = C_{gs} || 2C_{gs} = \frac{1}{1.5} C_{gs}$$



# Increasing Gain-Bandwidth Product Through Cascading



- We can significantly increase the gain of an amplifier by cascading  $n$  stages

$$\Rightarrow \frac{v_{out}}{v_{in}} = \left( \frac{A}{1 + s/w_0} \right)^n = A^n \frac{1}{(1 + s/w_0)^n}$$

- Issue – bandwidth degrades, but by how much?

## Analytical Derivation of Overall Bandwidth

---

- The overall 3-db bandwidth of the amplifier is where

$$\left| \frac{v_{out}}{v_{in}} \right| = \left| \frac{A}{1 + jw_1/w_o} \right|^n = \frac{A^n}{\sqrt{2}}$$

- $w_1$  is the overall bandwidth
- $A$  and  $w_o$  are the gain and bandwidth of each section

$$\Rightarrow \left( \frac{A}{\sqrt{1 + (w_1/w_o)^2}} \right)^n = \frac{A^n}{\sqrt{2}}$$

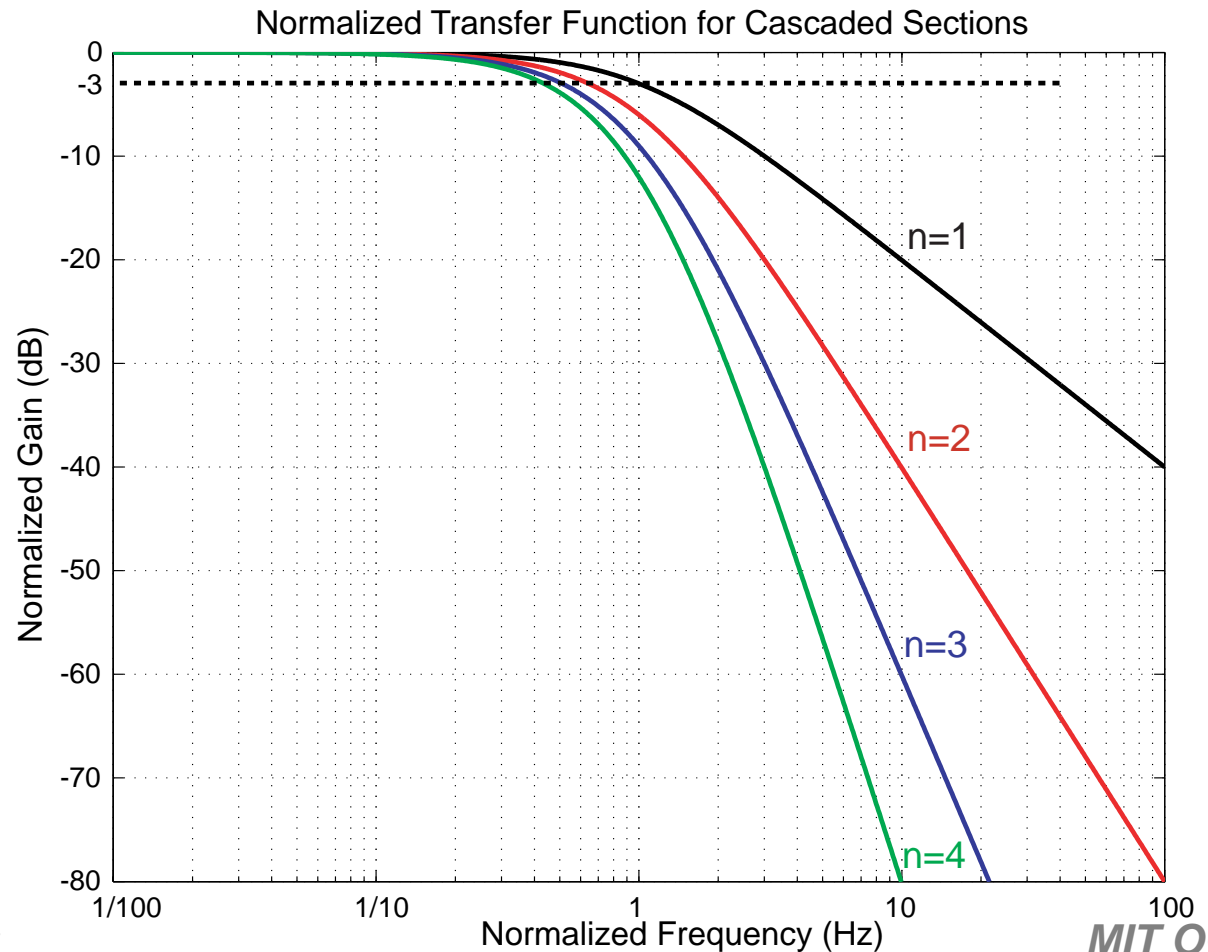
$$\Rightarrow \left( 1 + (w_1/w_o)^2 \right)^n = 2$$

$$\Rightarrow w_1 = w_o \sqrt{2^{1/n} - 1}$$

- Bandwidth decreases much slower than gain increases
  - Overall gain bandwidth product of amp can be increased

# Transfer Function for Cascaded Sections

$$H(f) = \left| \frac{1}{1 + j2\pi f} \right|^n$$



## Choosing the Optimal Number of Stages

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- To first order, there is a constant gain-bandwidth product for each stage

$$\Rightarrow Aw_o = w_u \Rightarrow w_o = w_u/A$$

- Increasing the bandwidth of each stage requires that we lower its gain
- Can make up for lost gain by cascading more stages
- We found that the overall bandwidth is calculated as

$$w_1 = w_o \sqrt{2^{1/n} - 1} = \frac{w_u}{A} \sqrt{2^{1/n} - 1}$$

- Assume that we want to achieve gain G with n stages

$$\Rightarrow A = G^{1/n} \Rightarrow w_1 = \frac{w_u}{G^{1/n}} \sqrt{2^{1/n} - 1}$$

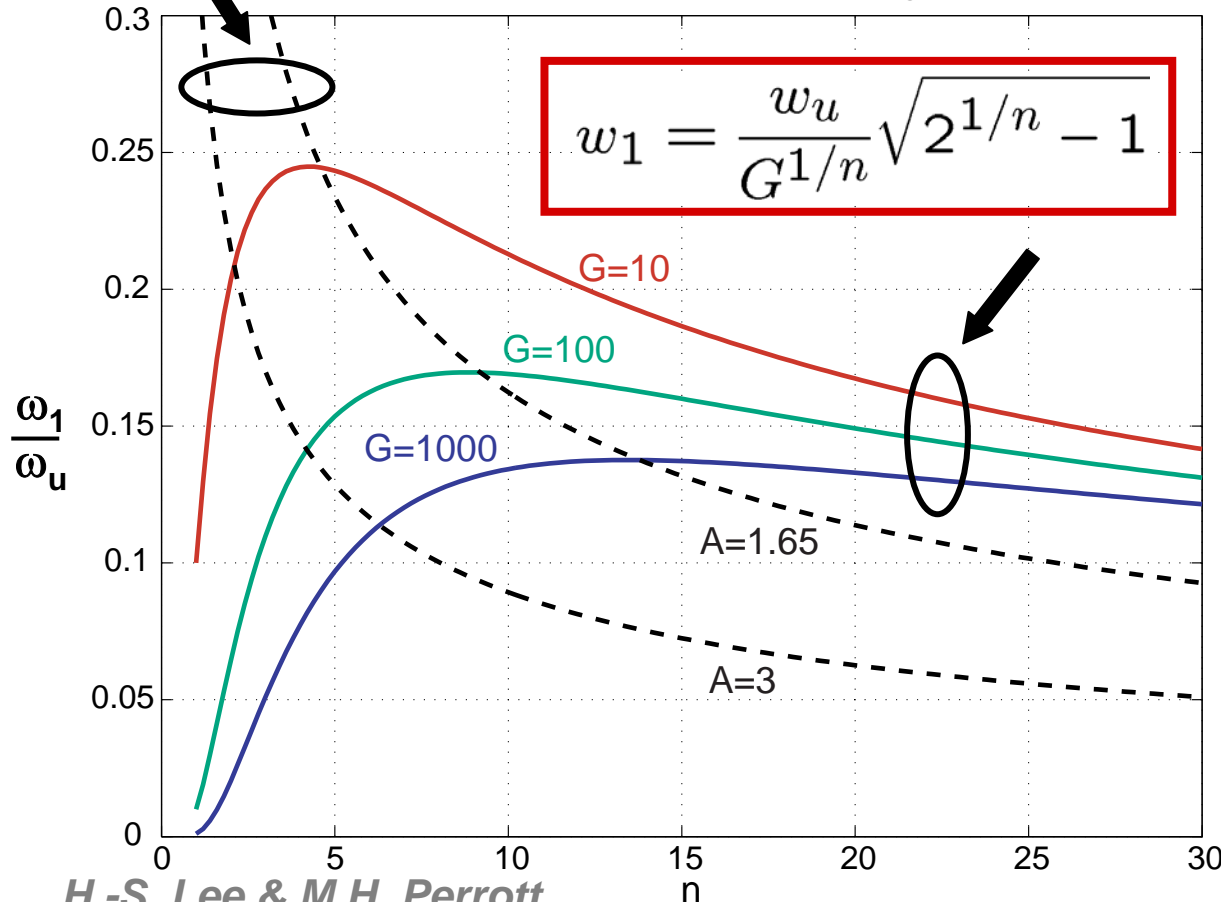
- From this, optimum gain/stage  $\approx \text{sqrt}(e) = 1.65$

- See Tom Lee's book, pp 299-302 (207-211, 1<sup>st</sup> ed.)

# Achievable Bandwidth Versus $G$ and $n$

$$\frac{w_u}{A} \sqrt{2^{1/n} - 1}$$

Achievable Bandwidth (Normalized to  $\omega_u$ )  
Versus Gain ( $G$ ) and Number of Stages ( $n$ )



- Optimum gain per stage is about 1.65

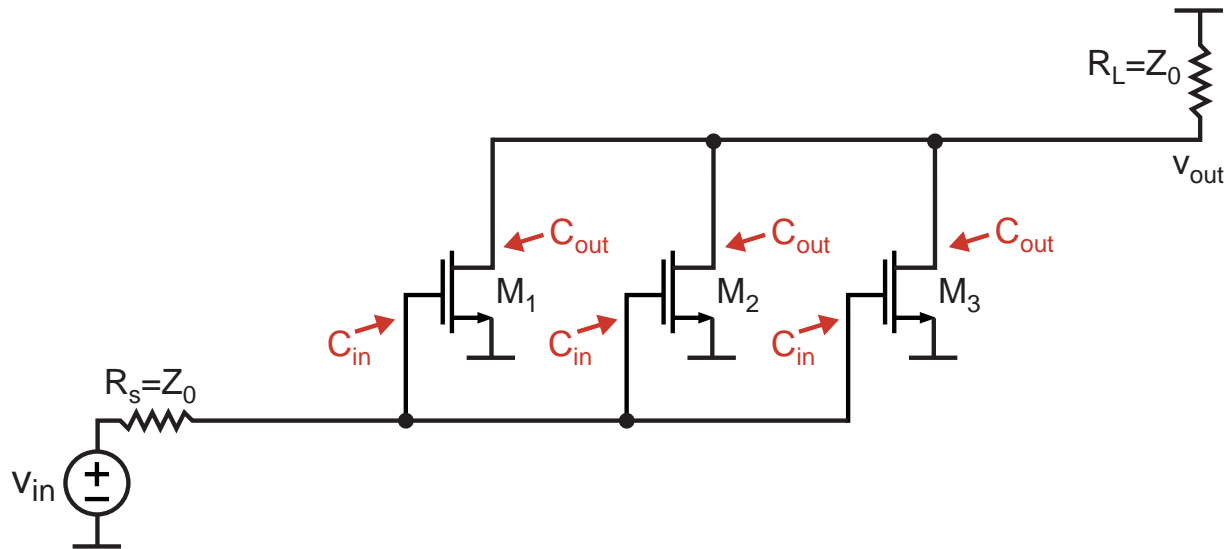
- Note that gain per stage derived from plot as

$$A = G^{1/n}$$

- Maximum is fairly soft, though

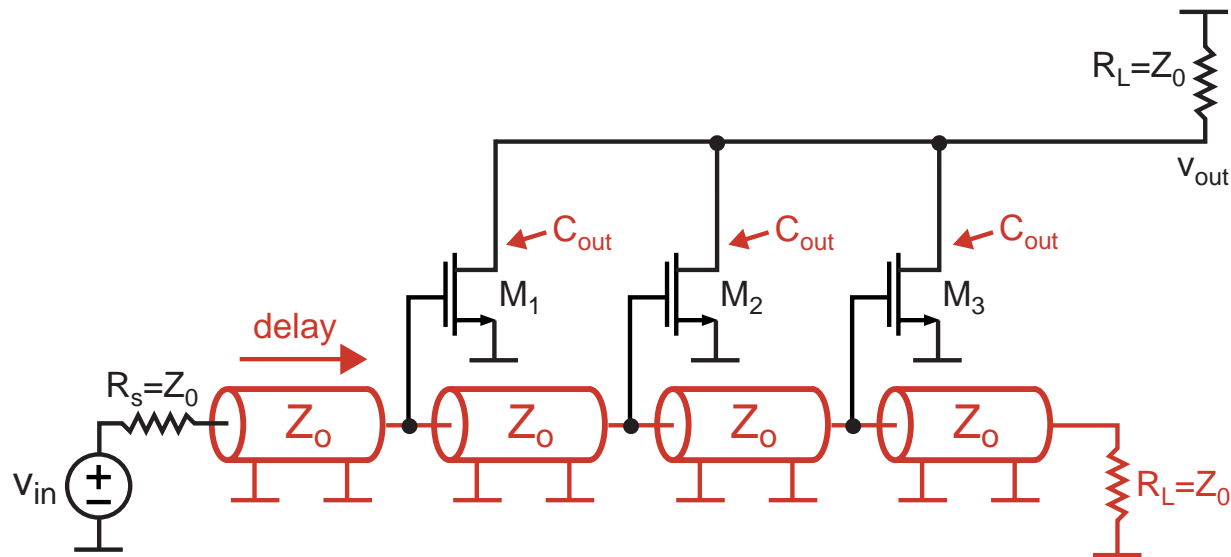
- Can dramatically lower power (and improve noise) by using larger gain per stage

# Motivation for Distributed Amplifiers



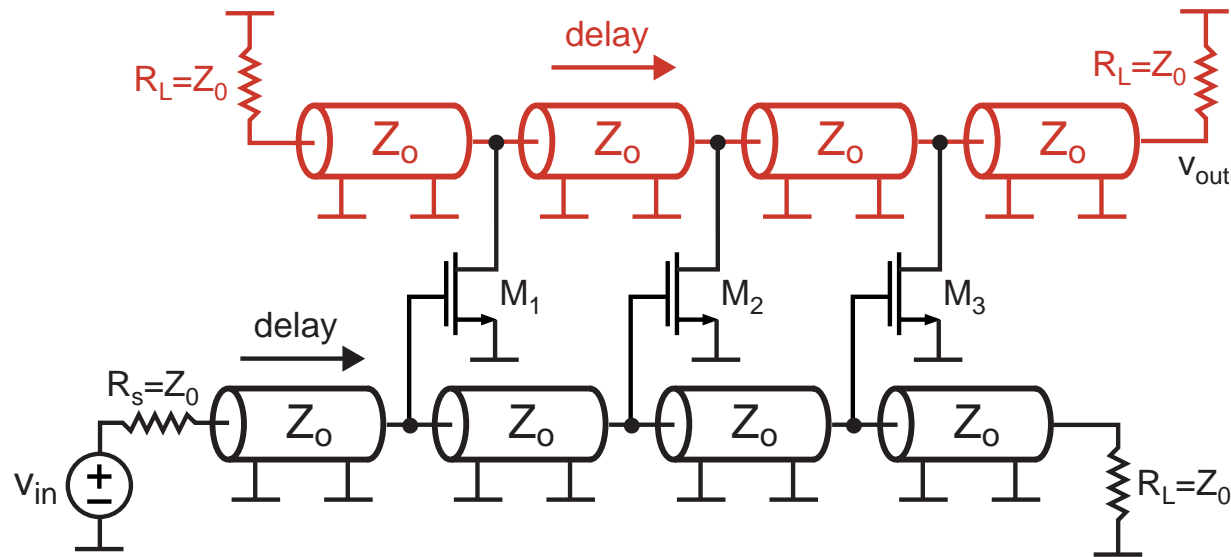
- We achieve higher gain for a given load resistance by increasing the device size (i.e., increase  $g_m$ )
  - Increased capacitance lowers bandwidth
    - We therefore get a relatively constant gain-bandwidth product
- We know that transmission lines have (ideally) infinite bandwidth, but can be modeled as LC networks
  - Can we lump device capacitances into transmission line?

# Distributing the Input Capacitance



- **Lump input capacitance into LC network corresponding to a transmission line**
  - Signal ideally sees  $Z_0 = R_L$  rather than an RC lowpass
  - Often implemented as lumped networks such as T-coils
  - We can now trade delay (rather than bandwidth) for gain
- **Issue: outputs are delayed from each other**

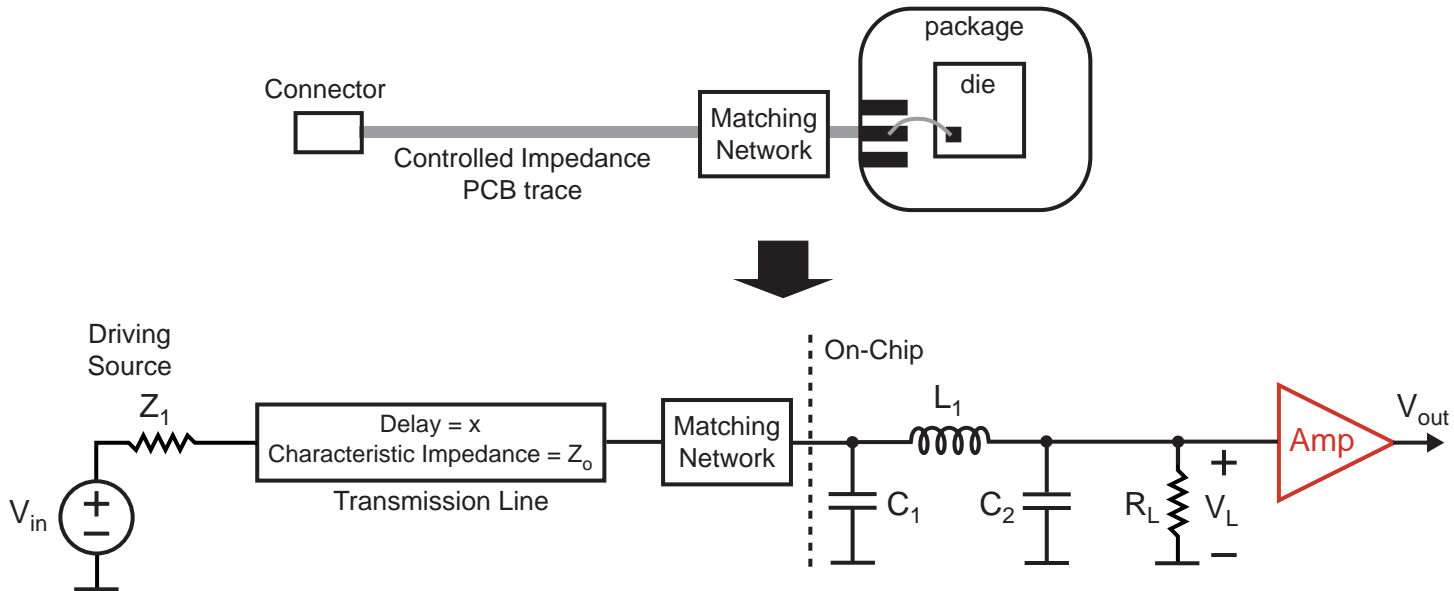
# Distributing the Output Capacitance



- **Delay the outputs same amount as the inputs**
  - Now the signals match up
  - We have also distributed the output capacitance
- **Benefit – high bandwidth**
- **Negatives – high power, poorer noise performance, expensive in terms of chip area**
  - Each transistor gain is adding rather than multiplying!

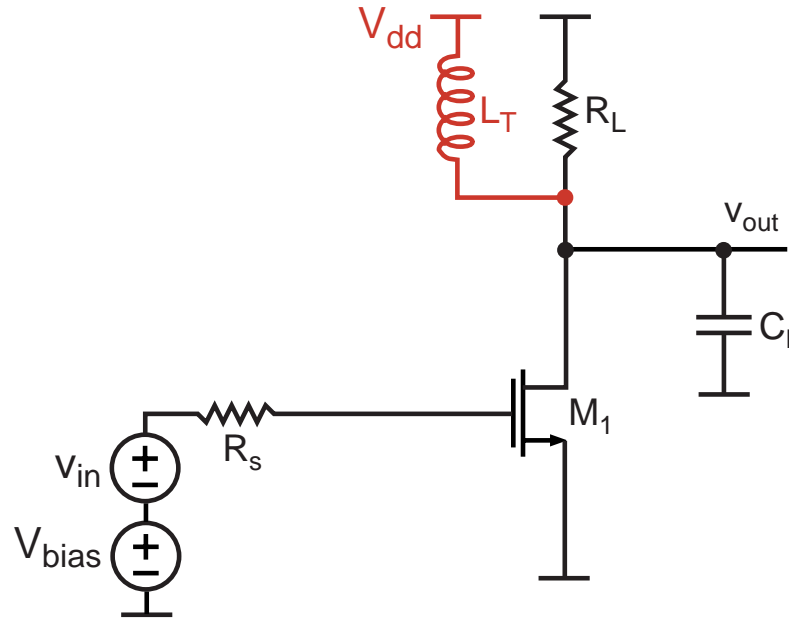


# Narrowband Amplifiers



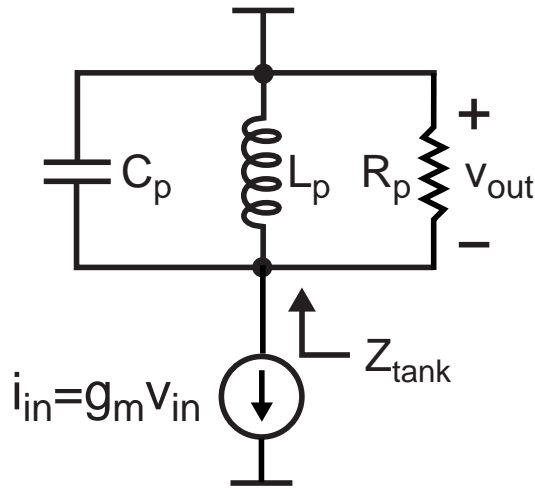
- For wireless systems, we are interested in conditioning and amplifying the signal over a narrow frequency range centered at a high frequency
  - Allows us to apply narrowband transformers to create matching networks
- Can we take advantage of this fact when designing the amplifier?

# Tuned Amplifiers



- Put inductor in parallel across  $R_L$  to create bandpass filter
  - It will turn out that the gain-bandwidth product is roughly conserved regardless of the center frequency
- To see this and other design issues, we must look closer at the parallel resonant circuit

# Tuned Amp Transfer Function About Resonance



- Amplifier transfer function

$$\frac{v_{out}}{v_{in}} = g_m Z_{tank}(s) = \frac{g_m}{Y_{tank}(s)}$$

- Note that conductances add in parallel

$$Y_{tank}(s) = \frac{1}{R_p} + \frac{1}{sL_p} + sC_p$$

- Evaluate at  $s = j\omega$

$$Y_{tank}(\omega) = \frac{1}{R_p} - \frac{j}{\omega L_p} + j\omega C_p = \frac{1}{R_p} + \frac{j}{\omega L_p} (-1 + \omega^2 L_p C_p)$$

- Look at frequencies about resonance:  $\omega = \omega_o + \Delta\omega$

$$\begin{aligned} \Rightarrow Y_{tank}(\Delta\omega) &= \frac{1}{R_p} + \frac{j}{(\omega_o + \Delta\omega)L_p} (-1 + (\omega_o + \Delta\omega)^2 L_p C_p) \\ &\approx \frac{1}{R_p} + \frac{j}{\omega_o L_p} (-1 + \omega_o^2 L_p C_p + 2\omega_o \Delta\omega L_p C_p) \end{aligned}$$

# Tuned Amp Transfer Function About Resonance (Cont.)

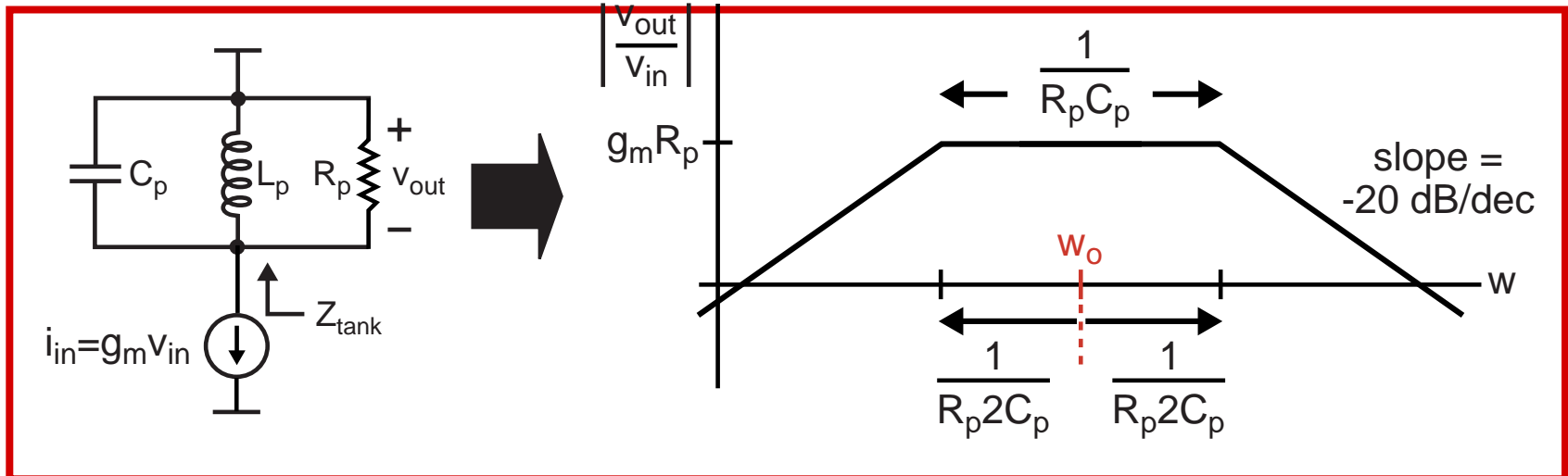
- From previous slide

$$Y_{tank}(\Delta\omega) \approx \frac{1}{R_p} + \frac{j}{\omega_o L_p} \left( \underbrace{-1 + \omega_o^2 L_p C_p}_{=0} + 2\omega_o \Delta\omega L_p C_p \right)$$

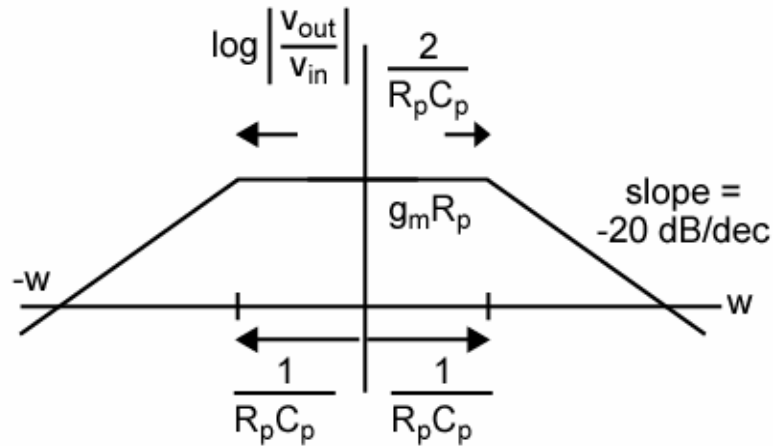
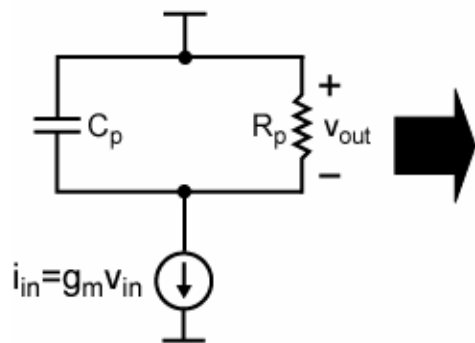
$$\approx \frac{1}{R_p} + \frac{j}{\omega_o L_p} (2\omega_o \Delta\omega L_p C_p) = \frac{1}{R_p} + j\Delta\omega 2C_p$$

- Simplifies to RC circuit for bandwidth calculation

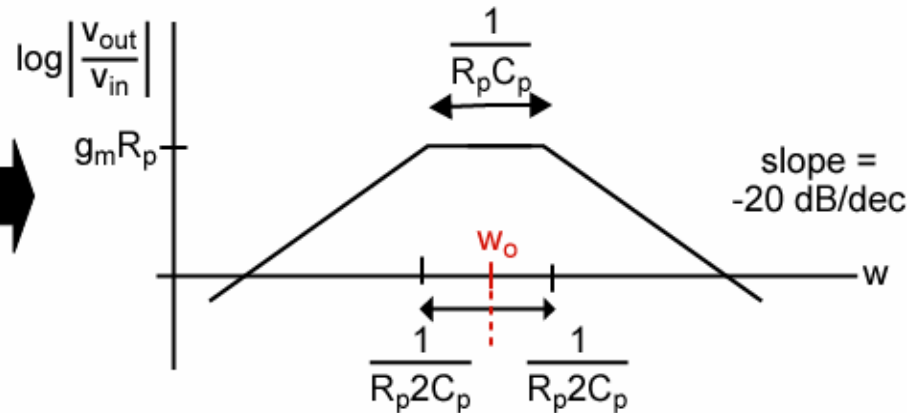
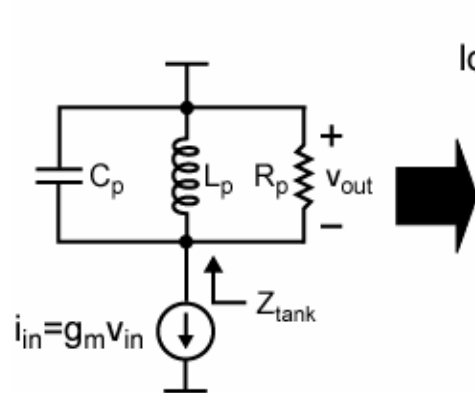
$$Z_{tank}(\Delta\omega) \approx R_p \parallel \frac{1}{j\Delta\omega 2C_p}$$



# Comparison between Low-Pass and Band-Pass



**low-pass  
amplifier**



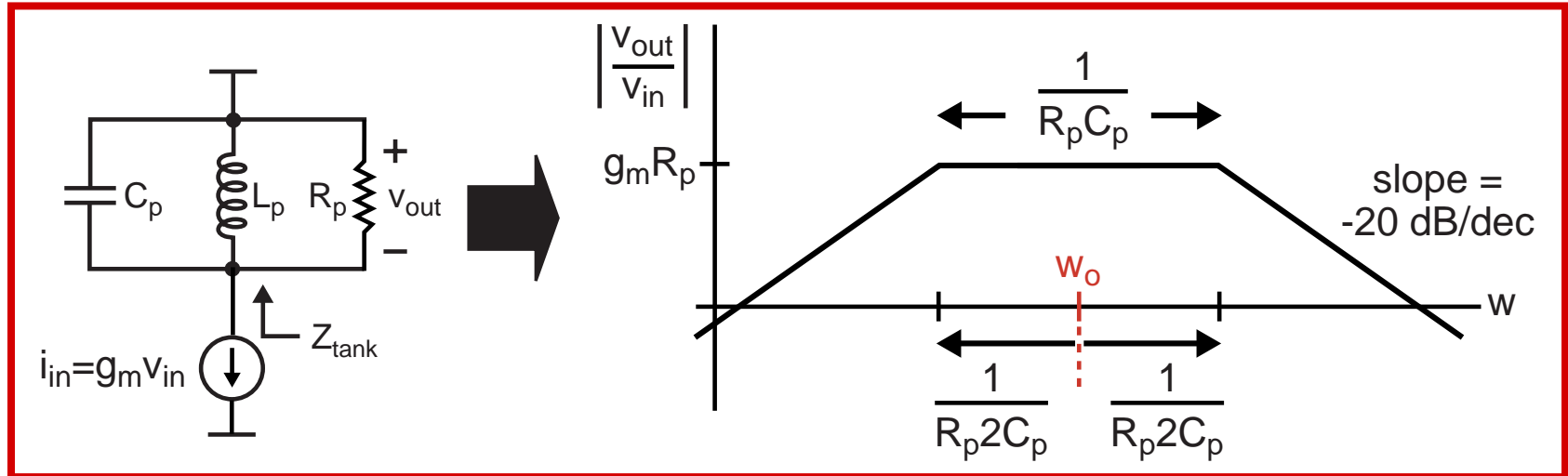
**band-pass  
(tuned)  
amplifier**

## Comparison between Low-Pass and Band-Pass

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- Tuned amplifier characteristic is a frequency translated version of the low-pass amplifier
- The band-pass bandwidth is equal to the low-pass bandwidth (the band-pass *shape* is 2x narrower but upper and lower sidebands give the same bandwidth as LP)
- We are *tuning out* the effect of capacitor (parallel LC looks like an open circuit at resonance, so C doesn't load the amplifier)
- This is often called low-pass to band-pass transform in filter design:
  - Replace C with parallel LC tank
  - Replace L with series LC tank

# Gain-Bandwidth Product for Tuned Amplifiers

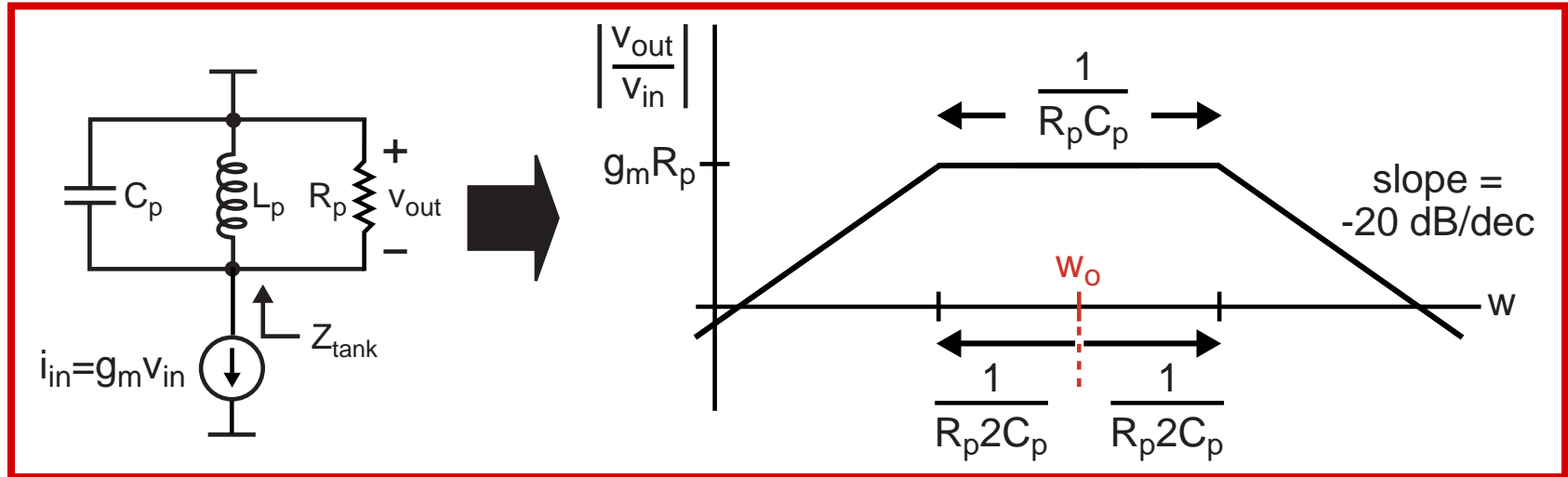


- The gain-bandwidth product:

$$G \cdot BW = g_m R_p \frac{1}{R_p C_p} = \frac{g_m}{C_p}$$

- The above expression is just like the low-pass and independent of center frequency!
  - In practice, we need to operate at a frequency less than the  $f_t$  of the device

# The Issue of Q



- **By definition**  $Q = w \frac{\text{energy stored}}{\text{average power dissipated}}$

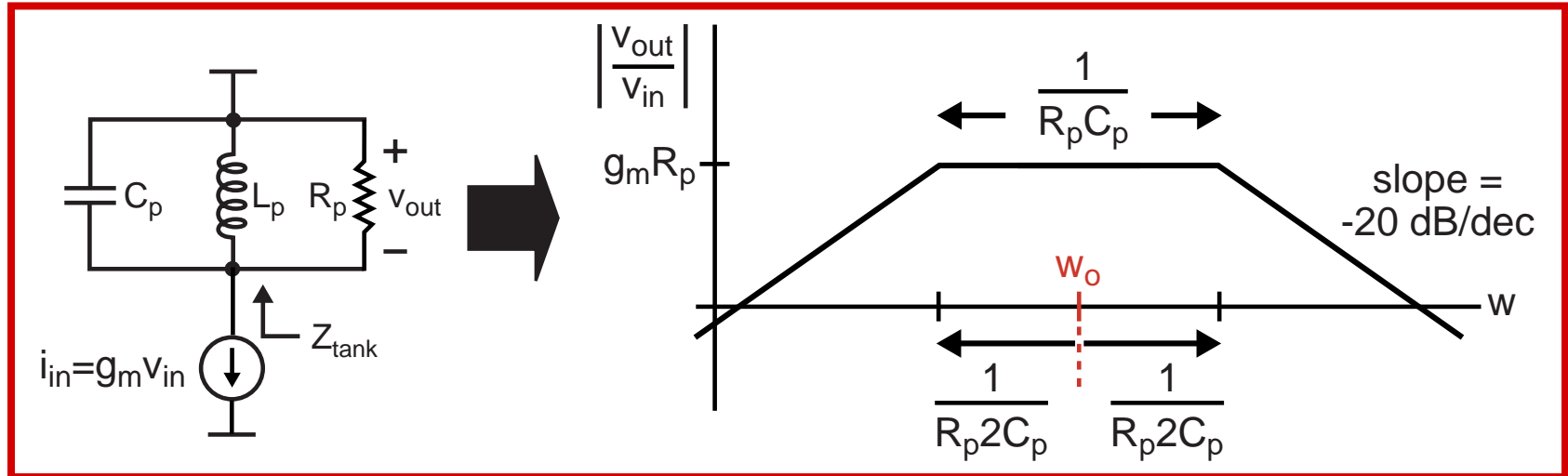
- **For parallel tank**

at resonance:  $Q = \frac{R_p}{w_o L_p} = w_o R_p C_p$

- **Comparing to above:**  $Q = w_o R_p C_p = \frac{w_o}{1/(R_p C_p)} = \boxed{\frac{w_o}{BW}}$



# Design of Tuned Amplifiers



## ■ Three key parameters

- Gain =  $g_m R_p$
- Center frequency =  $\omega_o$
- $Q = \omega_o / \text{BW}$

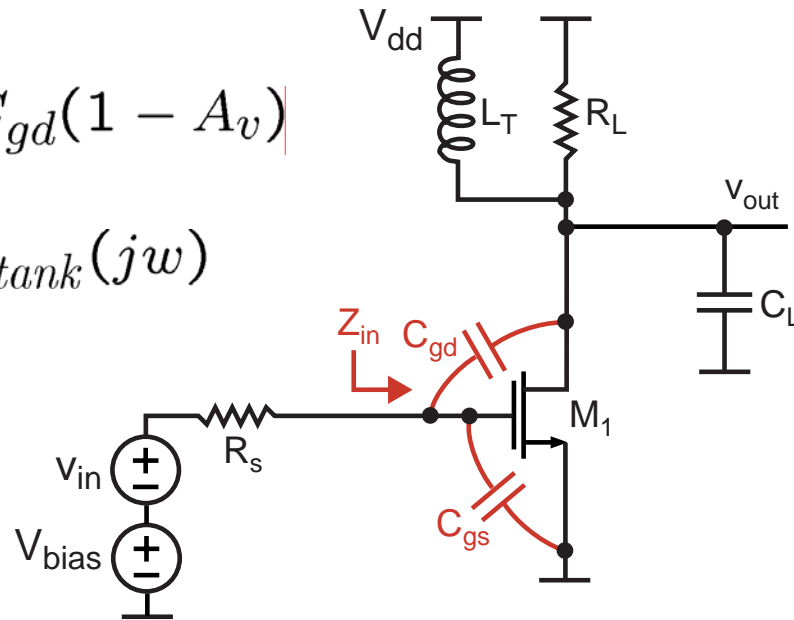
## ■ Impact of high Q

- Benefit: allows achievement of high gain with low power
- Problem: makes circuit sensitive to process/temp variations

## Issue: $C_{gd}$ Can Cause Undesired Oscillation

$$Y_{in}(w) = jwC_{gs} + jwC_{gd}(1 - A_v)$$

where  $A_v = -g_m Z_{tank}(jw)$



- At frequencies below resonance, tank looks inductive

$$A_v \approx -g_m(jwL) \Rightarrow Y_{in}(w) \approx jwC_{gs} + jwC_{gd}(1 + g_m(jwL))$$

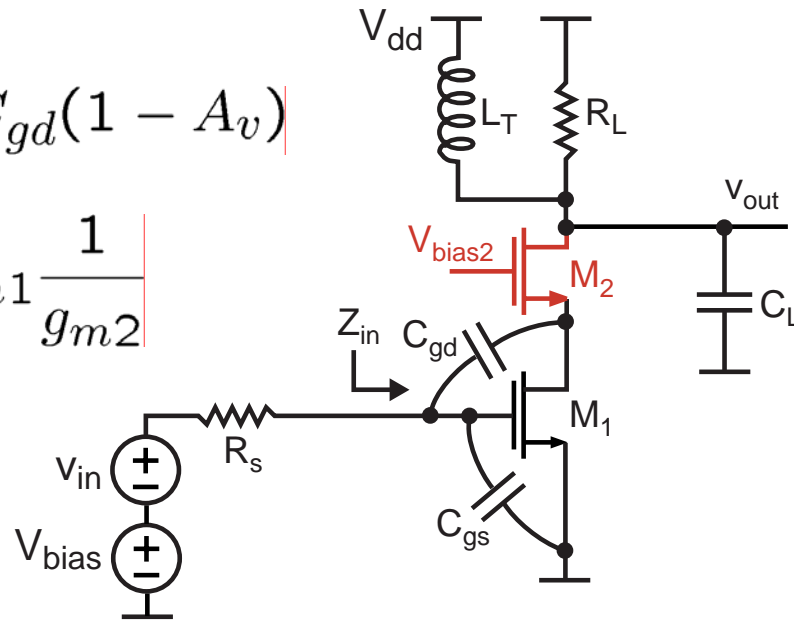
$$\Rightarrow Y_{in}(w) \approx jwC_{gs} + jwC_{gd} - w^2 g_m C_{gd} L$$

**Negative  
Resistance!**

## Use Cascode Device to Remove Impact of $C_{gd}$

$$Y_{in}(w) = jwC_{gs} + jwC_{gd}(1 - A_v)$$

$$\text{where } A_v = -g_{m1} \frac{1}{g_{m2}}$$



- At frequencies above and below resonance

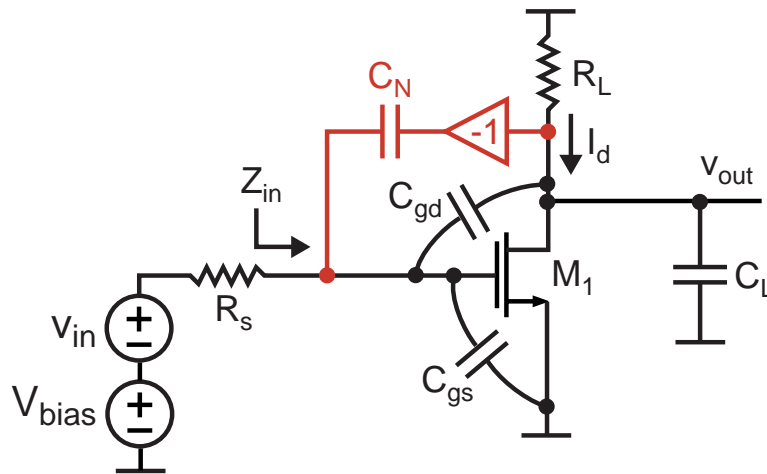
$$Y_{in}(w) = jwC_{gs} + jwC_{gd}(1 + g_{m1}/g_{m2})$$

**Purely  
Capacitive!**

# Neutralization in Tuned Amplifier

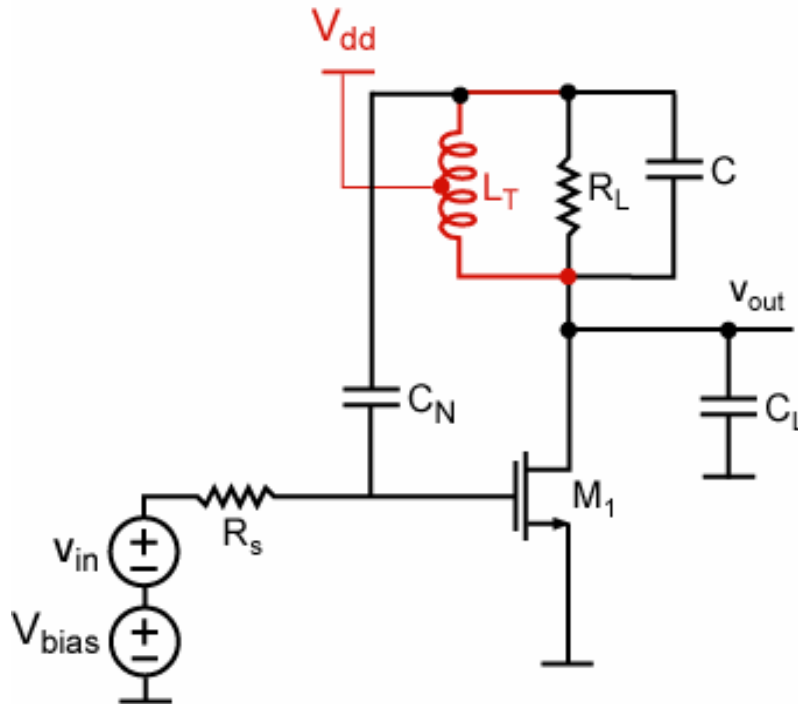
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Recall the neutralization for broadband amplifier



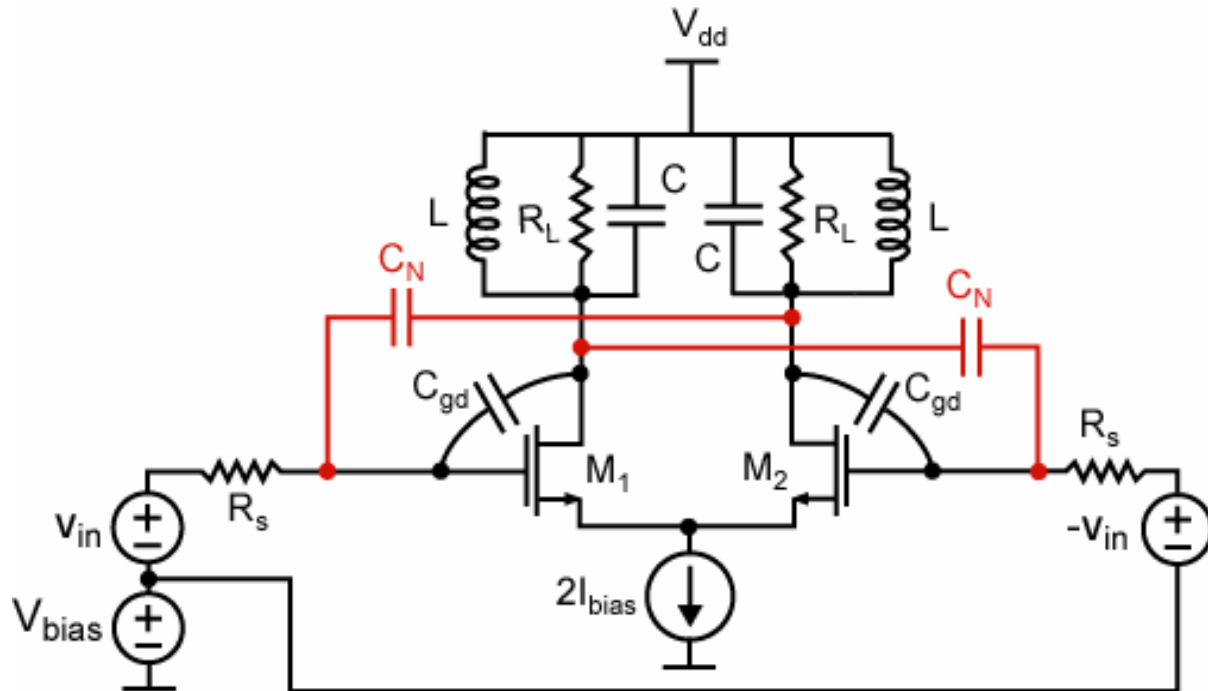
For narrowband amplifier, the inverting signal can be generated by a tapped transformer

# Neutralization with Tapped Transformer



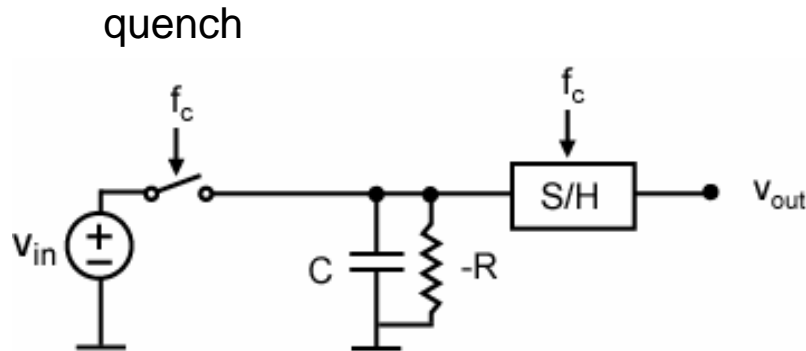
**Problems: Area and quality of on chip transformer**  
The neutralization cap  $C_N$  must be matched to  $C_{gd}$

# Differential Neutralization for Narrow Band Amplifier



- Same principle as differential neutralization in broadband amp
- Only issue left is matching  $C_N$  to  $C_{gd}$ 
  - Often use lateral metal caps for  $C_N$  (or CMOS transistor)

# Superregenerative Amplifier



$$v_{out}(t) = v_{in}(0)e^{\frac{t}{RC}}$$

$V_{in}$  is sampled at the rate  $f_c$  (in actual implementation  $f_c$  may be input level dependent)

The sampled output voltage

$$v_{out}(T) = v_{in}(0)e^{\frac{T}{RC}} \quad T = \frac{1}{f_c}$$

Can be used for both broadband and narrowband – we'll do a simple analysis for broadband amp.

# Superregenerative Amplifier

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## Gain calculation

$$v_{out}(T) = v_{in}(0)e^{\frac{T}{RC}}$$

$$A_v = \frac{v_{out}}{v_{in}} = e^{\frac{T}{RC}}$$

## Nyquist theorem

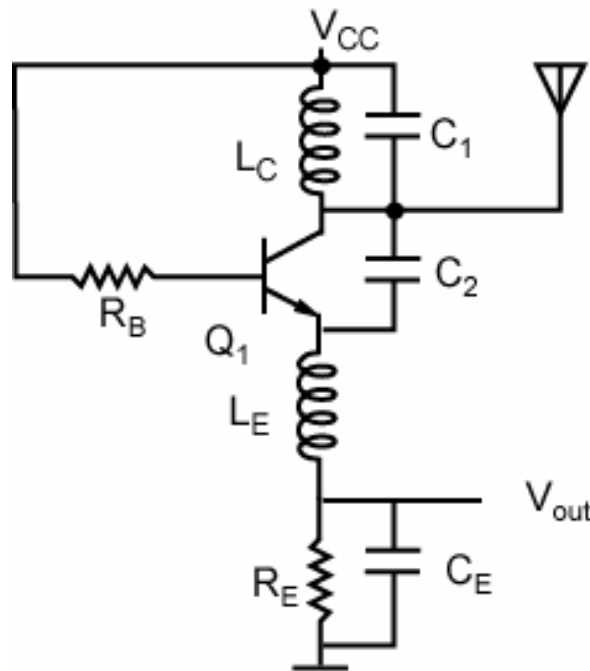
$$BW < \frac{f_C}{2} = \frac{1}{2T}$$

$$BW|_{max} \bullet \ln A_v = \frac{1}{2RC}$$

**Trades bandwidth only logarithmically with gain!**



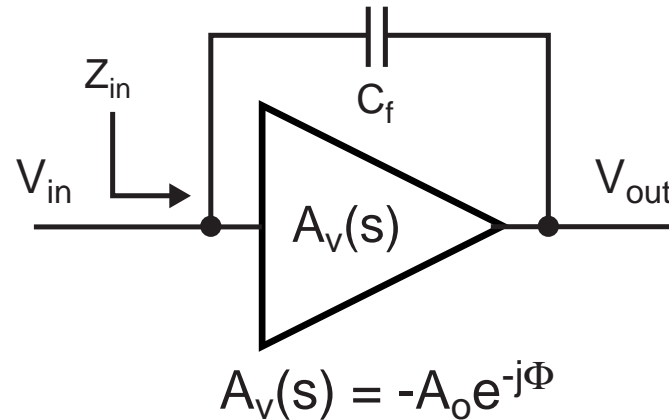
# Superregenerative Amplifier Example (Narrowband)



$R_B$ : DC bias resistor  
 $L_C, C_1$ : Tuning LC  
 $C_2$ : positive feedback  
 $L_E$ : RF choke (large inductance)

- When the RF amplitude becomes large, it is rectified at the emitter of Q1
- This raises the DC potential at the emitter Q1 eventually turning it off
- The RF oscillation dies (quenched), and the DC potential at emitter of Q1 returns
- Amplitude of oscillation grows again due to positive feedback

# Active Real Impedance Generator

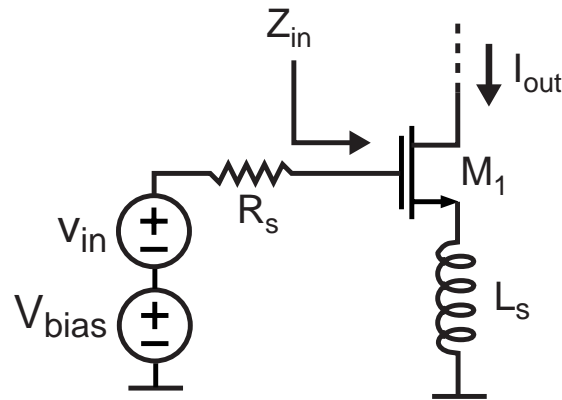


- **Input admittance:**

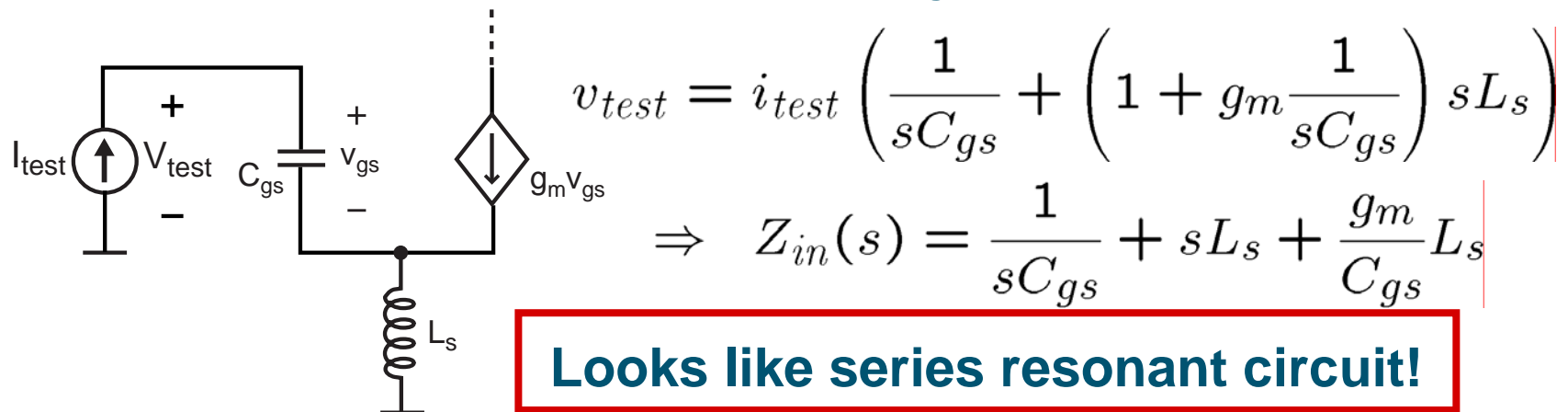
$$Y_{in}(\omega) = j\omega C_f(1 - A_v) = j\omega C_f(1 + A_o e^{-j\Phi})$$
$$= j\omega C_f(1 + A_o \cos \Phi) + A_o \omega C_f \sin \Phi$$

**Resistive component!**

# This Principle Can Be Applied To Impedance Matching

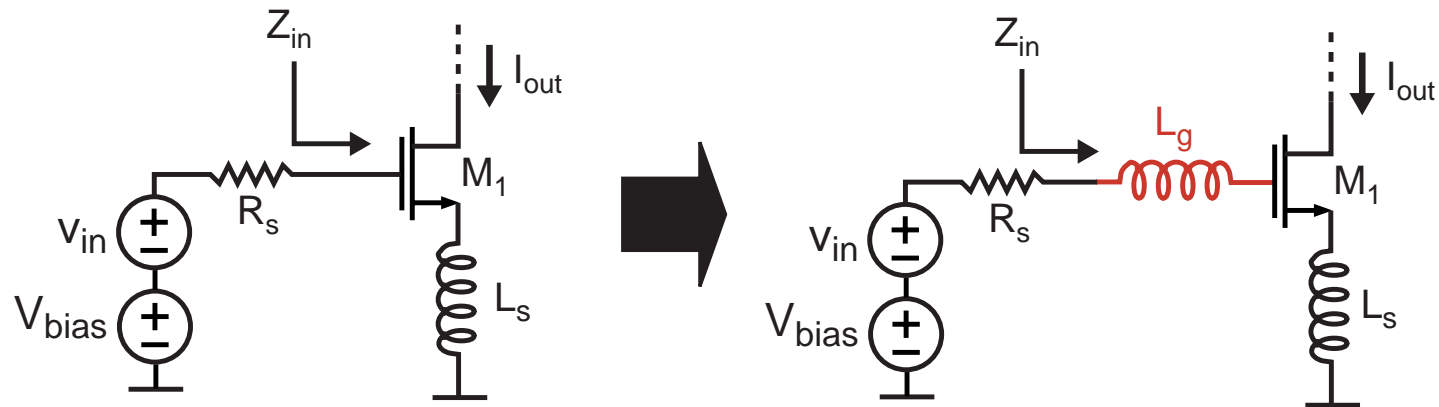


- We will see that it's advantageous to make  $Z_{in}$  real without using resistors
  - For the above circuit (ignoring  $C_{gd}$ )



**Looks like series resonant circuit!**

# Use A Series Inductor to Tune Resonant Frequency



- Calculate input impedance with added inductor (in order to choose resonance freq. and input resistance separately)

$$Z_{in}(s) = \frac{1}{sC_{gs}} + s(L_s + L_g) + \frac{g_m}{C_{gs}}L_s$$

- Often want purely resistive component at frequency  $\omega_0$ 
  - Choose  $L_g$  such that resonant frequency =  $\omega_0$

$$\text{i.e., want } \frac{1}{\sqrt{(L_s + L_g)C_{gs}}} = \omega_0$$

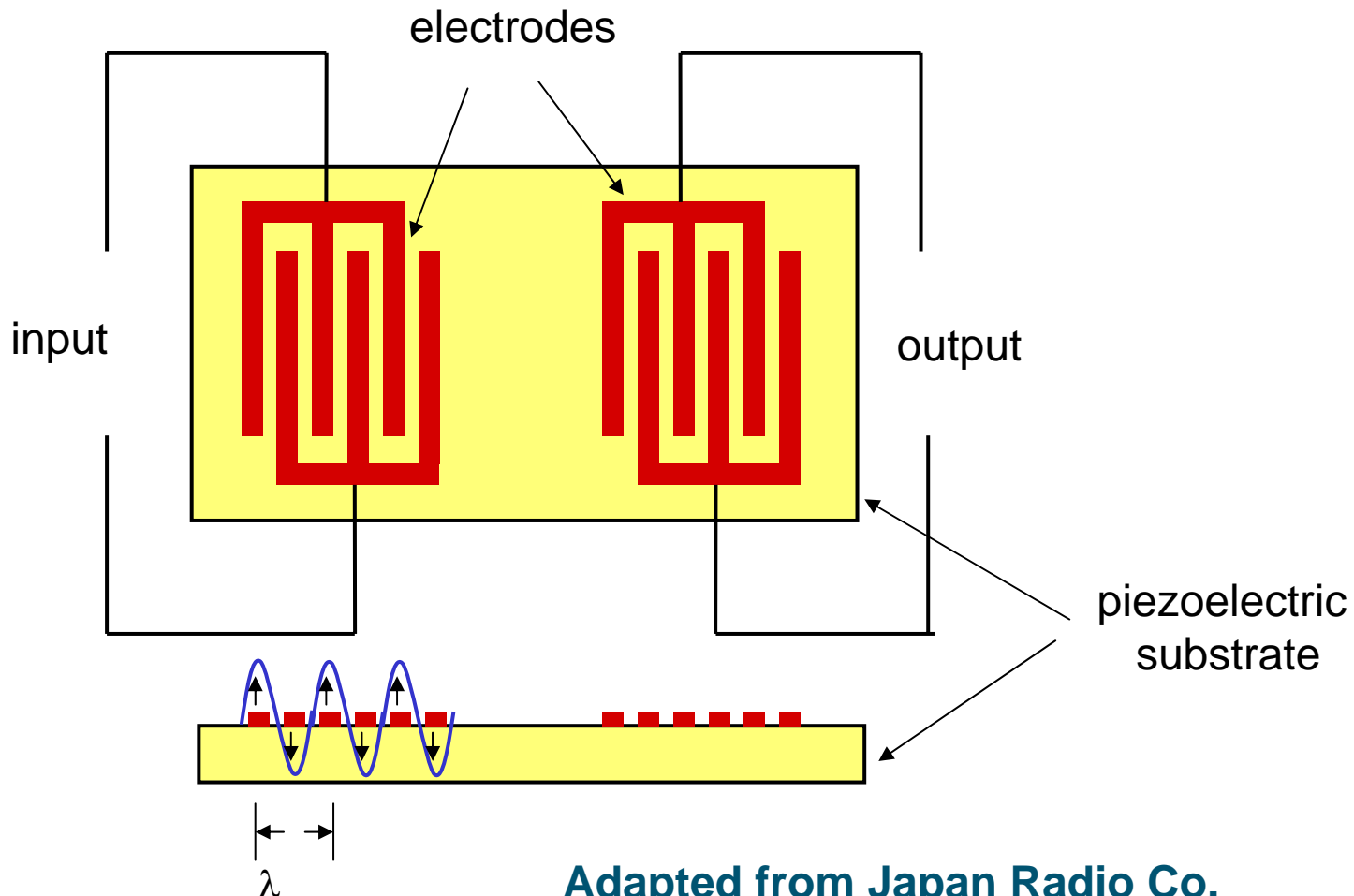
# Narrowband Alternative to LC

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- **On-chip inductors take up considerable die area and have relatively poor Q. Is there any other alternatives?**
  - Use quarter-wavelength transmission line (waveguide) resonator?
  - In Lecture 5 we found the  $\lambda/4$  waveguide with shorted load behave much like a parallel LC circuit, while with open load it behaves like a series LC
  - The problem is the dimension. For 900MHz mobile phone frequency,  $\lambda/4$  in free space is 3.25 inches!
  - With high permittivity dielectric material (ceramic), the size can be reduced to a reasonable dimensions. With  $\epsilon_r=10$ , the length of waveguide is only about inch.
  - Different configurations of filters can be built by combining sections of series and parallel LC equivalents
  - More appropriate at frequencies over GHz
- **SAW (Surface Acoustic Wave) filters are another popular alternative**

# SAW Filters

- SAW filters use piezoelectric substrate to generate surface acoustic wave



# SAW Filters

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- **Piezoelectric substrate**
  - $\text{LiTaO}_3$
  - $\text{LiNbO}_3$
  - Quartz
- **Filter Structures**
  - Longitudinal Filter
  - Transversal Filter
  - Ladder Filter
- **Saw filters have high selectivity and low insertion loss (down to a fraction of % fractional bandwidth, ~2dB insertion loss)**
- **Wide range of enter frequency (few 100 kHz-GHz)**
- **At 1-2 GHz, the dimensions of SAW filters are 1-2mm**
- **For more information on SAW filters look over [www.njr.com](http://www.njr.com)**

# SAW Filters in Mobile Phones

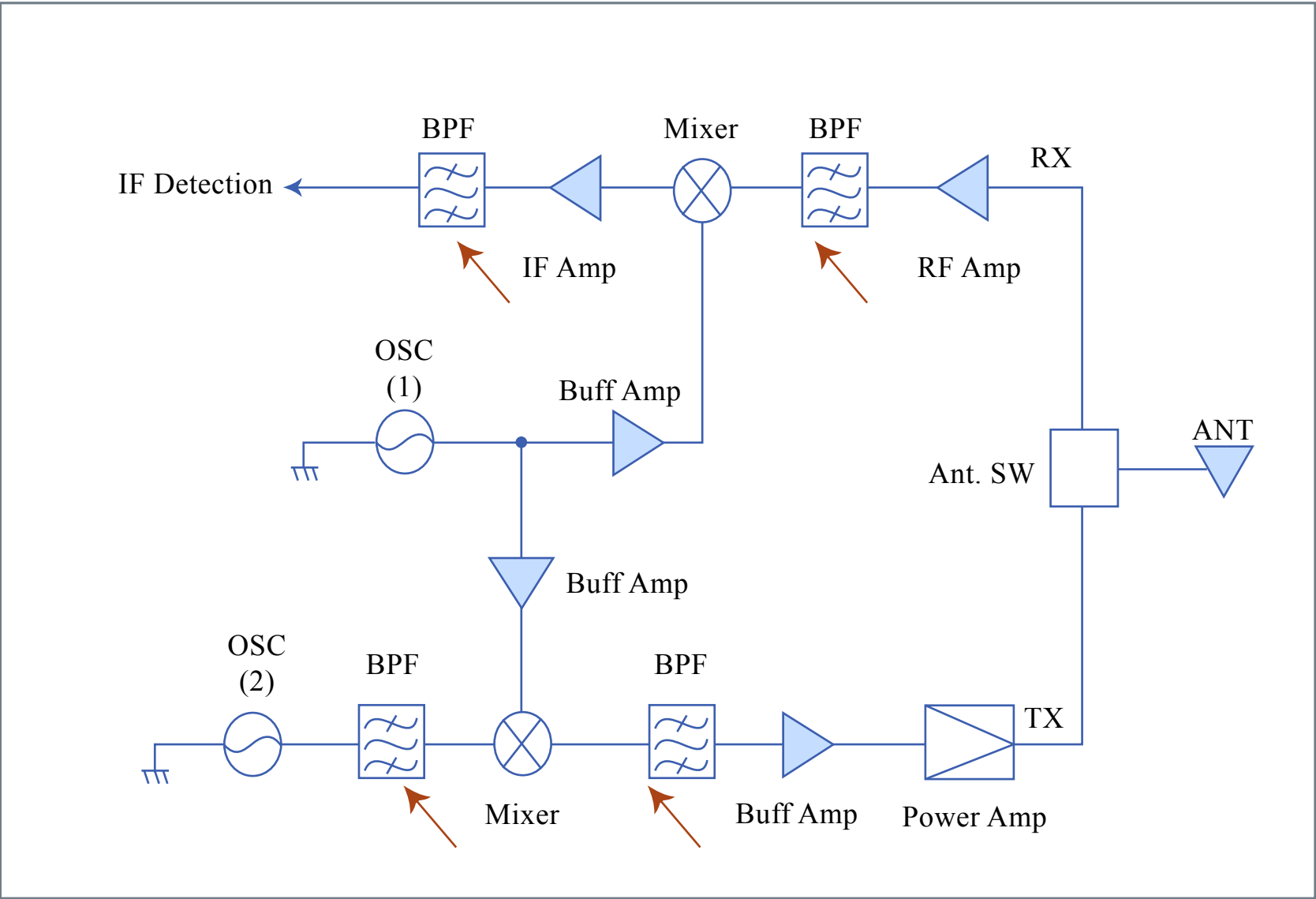


Figure by MIT OCW.



# SAW Filter Example

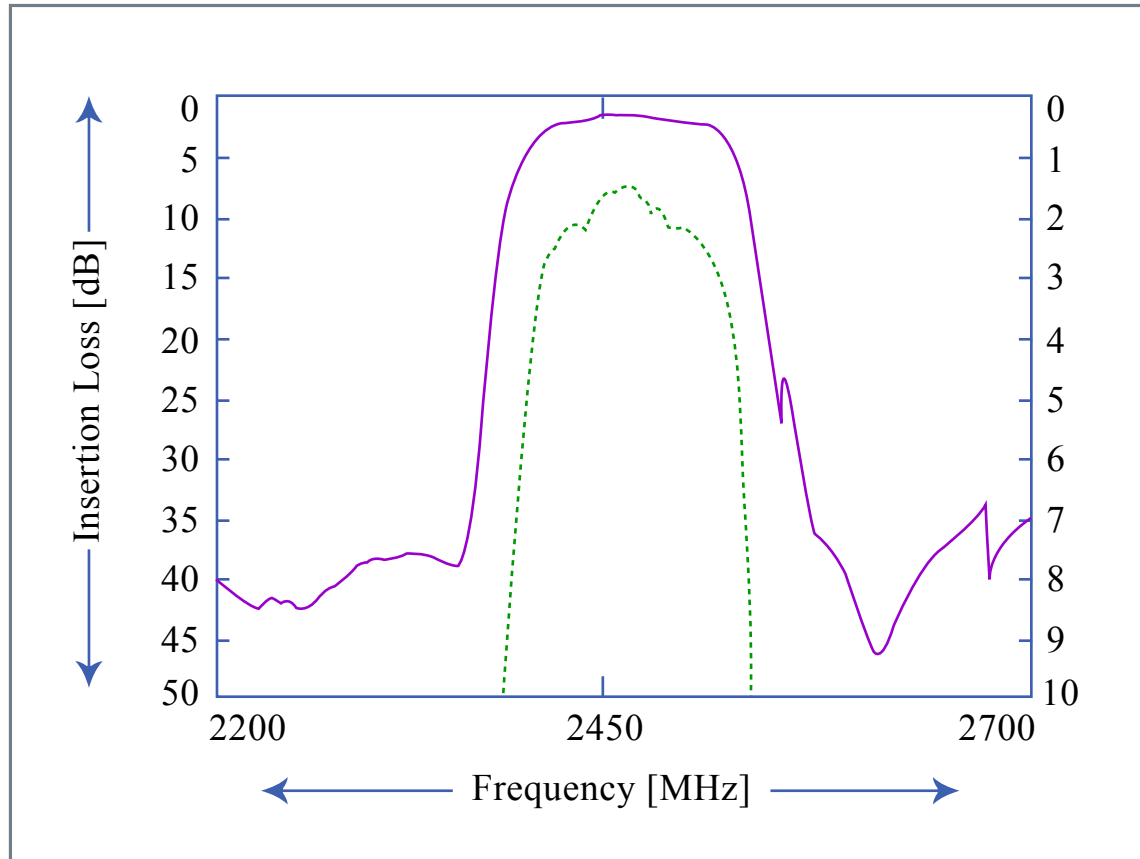


Figure by MIT OCW.

## JRC NSVS754 2.4 GHz RF SAW Filter Characteristics

Adapted from Japan Radio Co.