2.372J/6.777J Design and Fabrication of Microelectromechanical Devices
Spring Term 2007

Massachusetts Institute of Technology

PROBLEM SET 7 (18 pts)

Issued 4/11/07 Due 4/20/07

Problem 15.7 (4 pts): Design a PID controller

We are trying to control an electrostatic electrode actuator with a lumped mass of 1 μg, a resonant frequency of 50 kHz, and a Q of 10. We will model the linearized system as a spring-mass-damper 2nd-order system. The plant transfer function $H(s)$ will be $X(s)/F(s)$.

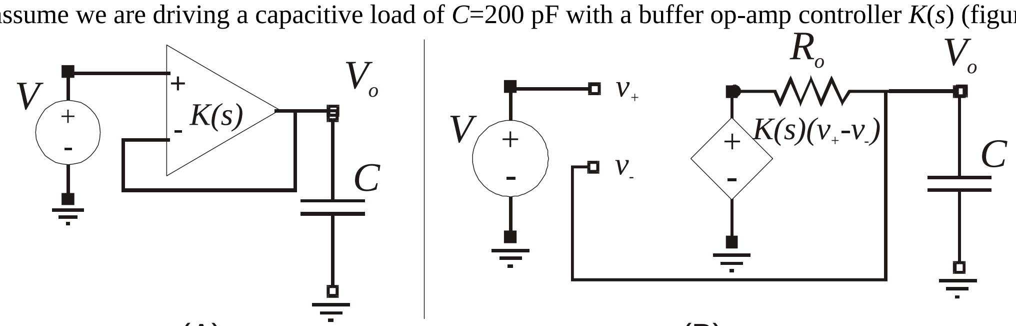
a. First, recast the transfer function in terms of the non-dimensional complex frequency $\hat{s} = s/\omega_0$.

b. Now, assume that you are using a single-pole controller of the form in equation (15.14), with non-dimensionalized time constant $\hat{\tau} = 0.1$. Using a root-locus plot for the overall control system transfer function $X_{out}(s)/X_{in}(s)$, determine the maximum controller gain (K_0) at which the system is stable.

c. Instead of using a single-pole controller, design a PID controller that achieves overall critically damped system response with no DC error. Demonstrate results with SIMULINK or MATLAB simulations.

Problem 15.8 (4 pts): Stability and capacitive loads

We are often faced with the task of driving capacitive loads including electrostatic transducers such as parallel-plate actuators. Such loads, when driven at high frequencies, can create circuit instabilities without proper care. In this example, assume we are driving a capacitive load of C=200 pF with a buffer op-amp controller $K(s)$ (figure, A).



(A) (B)

a. Assume that the amplifier (like ALL amplifiers) has a finite output resistance R_0=40 Ω, using the circuit model in (B), with the load capacitor C, determine analytically the loop transmission function $H(s)K(s)$ for the circuit, where the loop transmission function is defined as $V_0 = H(s)K(s)(V - V_0)$.

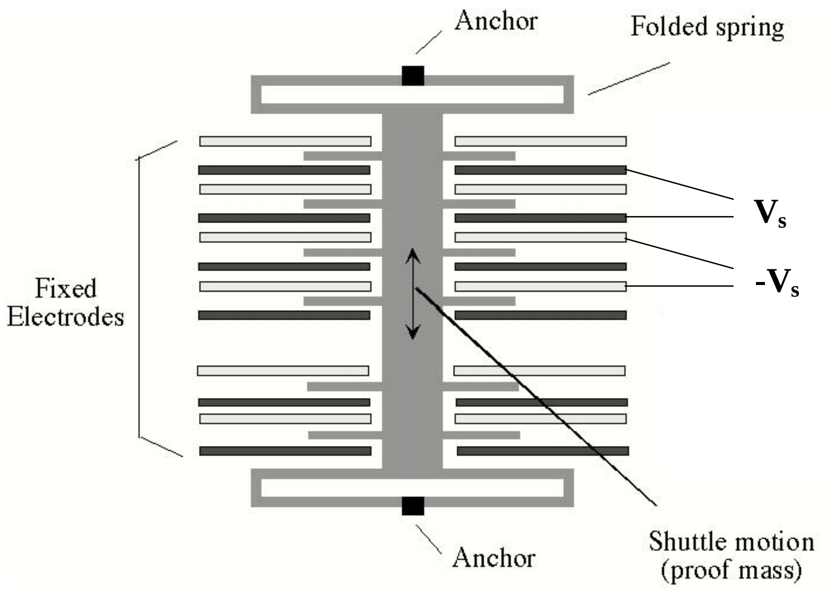
b. Now assume that $K(s)$ is a 2nd-order transfer function of the form

$$K(s) = \frac{10^{3.5}}{\left(1 + {s}/{2\pi \cdot 10^5}\right)\left(1 + {s}/{2\pi \cdot 10^8}\right)}$$

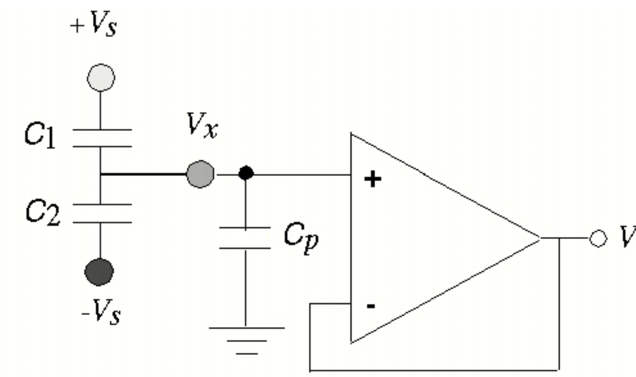
which approximates the National Semiconductor LM359 high-speed op-amp. Determine the overall loop transmission function $H(s)K(s)$. Make a Bode plot for the loop transmission function and determine the phase margin with and without the capacitor. What is the maximum capacitive load that this amplifier can drive and be stable?

Problem 16.6 (10 pts): Noise in a (vastly simplified) capacitive accelerometer

In this problem, we will compare noise sources for a differential capacitive accelerometer. The accelerometer employs interdigitated capacitive fingers, which are read differentially. When the proof mass moves, one capacitance increases and the other decreases.



The circuit assumed here is vastly simplified; we will learn more about accelerometer readouts in the case studies. In the meantime, we assume that voltages are applied to the fingers in pulses with an amplitude $V_s = 2.5$ V. We will look at the noise from various sources *while the pulse is applied* (in other words, we don't look very long, so it looks like DC.) However, we will assume a finite bandwidth due to the remainder of the circuit, which is not shown here.



a. First we will examine thermomechanical noise. Assume that the microaccelerometer has a proof mass of 300 ng, a resonant frequency of 25 kHz, and the bandwidth of the measurement is 1 kHz. Also assume that the accelerometer is critically damped and is in thermal equilibrium at room temperature. What is the total thermomechanical noise in the bandwidth of interest?
b. Assume that at zero displacement the inter-finger gaps are 1.3μm and the inter-finger capacitances $C_1 = C_2 = 100$ fF (purely parallel plate capacitances). Also assume that the parasitic capacitance $C_p = 100$ fF. What is the effective variation in the capacitances C_1 and C_2 due to the fluctuation of the position of the proof mass? Find the relationship between V_x (the voltage between the capacitors) and V_s, and use it to calculate an effective voltage noise source from the thermomechanical noise at the point labeled V_x.

c. We saw in class that a capacitor connected to a thermal reservoir has a voltage noise associated with it. In this case, the result is modified because the bandwidth of interest is limited to 1 kHz. Estimate the total voltage noise introduced by the capacitors in the bandwidth of interest. You may assume that the dominant resistor to which the capacitors are connected is the resistance of the long interdigitated fingers (assume 25 fingers in parallel). Each finger is made of heavily-doped polysilicon and has a resistivity of about 400 μΩ-cm. Each finger is 100 μm long and has a cross-sectional area of 25 μm^2.

d. We will now consider the noise sources for the amplifier. Write an equation for the amplifier noise at the inputs, and calculate the spectral density as a function of frequency. Assume a transconductance of 300 μSiemens, a channel length L = 2 μm, a channel width W = 30 μm, and a gate oxide thickness of 15 nm; also take $K_f = 10^{-24}$ V^2F. Calculate the total voltage noise at the amplifier inputs over the bandwidth of interest, assuming a low frequency cut-off of at 10 Hz.

e. Redraw the accelerometer circuit diagram to include all of the noise sources. Analyze the circuit to find V_0 in terms of V_s, the effective noise voltages, and the zero-noise component values. Assume that the noise sources are uncorrelated, and calculate the total voltage noise at the output V_0. Which term is dominant? Does the form of the transfer function for this circuit (with the unity gain buffer) have a significant effect on which terms are dominant?