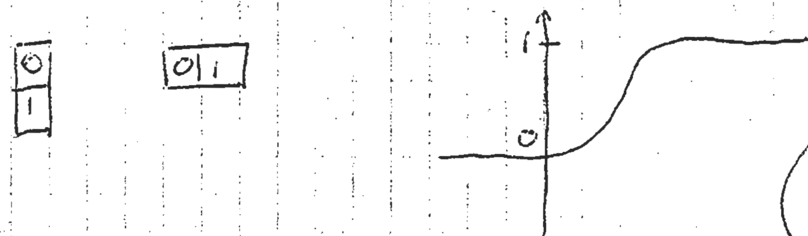
Quiz #1 Reminder in continuous images



2D

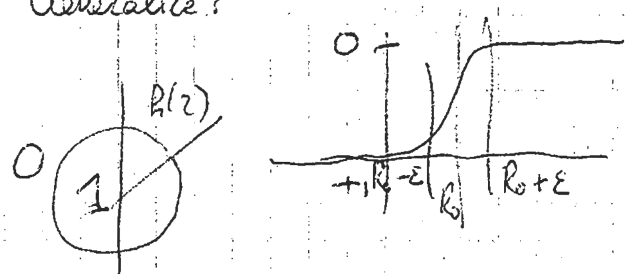
$E(x,y) = h(x)$

$$P = \iint_{-L}^{+L} |E_x|\,dx\,dy = \int_{-\infty}^{+\infty} |E_x|\,dx \cdot 2L = 2L\,(E_{\infty} - E_{-\infty}) = 2L\,V$$

independent of edge curvature.

If edge is curved? Perimeter

Generalize:



$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sqrt{E_x^2 + E_y^2}\,dx\,dy$$

$$\int_{-\pi}^{\pi}\int_{0}^{\infty} \sqrt{E_x^2 + E_y^2}\; r\,dr\,d\theta$$

$$\frac{\partial E}{\partial x} = \frac{\partial E}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial E}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} = h'(r)\cos\theta$$

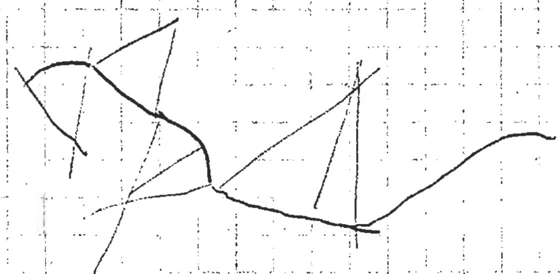
$$r = \sqrt{x^2+y^2} \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} = \frac{r\cos\theta}{r}.$$

$$\int_{-\pi}^{\pi}\int_{R_0-\varepsilon}^{R_0+\varepsilon} |h'(r)|\; r\,dr\,d\theta$$

as $\varepsilon \to 0$: $\approx \int_{-\pi}^{\pi} |h'(r)| R_0 \, d\theta = \int_{-\pi}^{\pi} R_0 \left(h(R_0+\varepsilon) - h(R_0-\varepsilon) \right) d\theta$

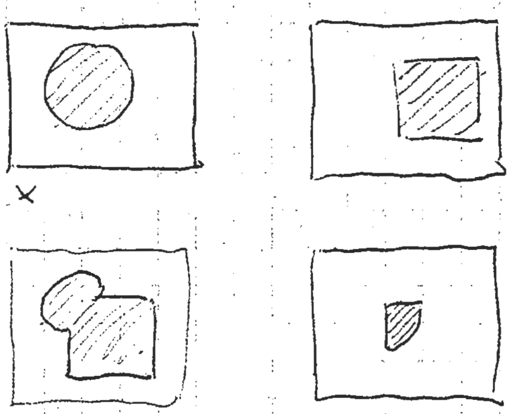
$2\pi R_0 = p$

Circular arc tangent continuous approximation



Area: $A = \iint E(x,y)\,dx\,dy$

<u>Addition Set Property</u>



$A(x) + A(y) = A(x \cup y) + A(x \cap y)$

Area satisfies ASP.

$A(x \cup y) - A(x) = A(y) - A(x \cap y)$

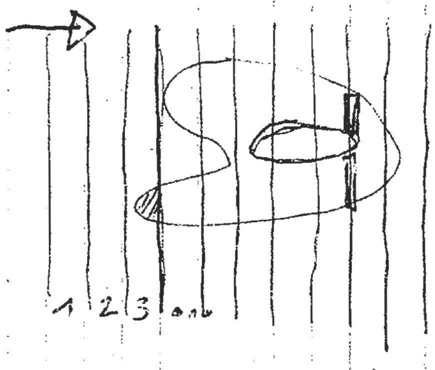
$A(x \cup \Delta x) - A(x) = A(\Delta x) - A(x \cap \Delta x)$

$A(x^n \cup \Delta x^n) - A(x^n) = A(\Delta x^n) - A(x^n \cap \Delta x^n)$

$\Delta A^n = A(\Delta x^n) - A(x^n \cap \Delta x^n)$

ex: Δx^n simple pixel

Stream direction - and slice the image into this direction



Area: $x^n \cap \Delta x^n = 0$.

$$A = \sum \Delta A^n = \sum A(\Delta x^n)$$

Perimeter:

$$P = \sum \Delta P^n - \sum P^n \cap \Delta P^n$$

<u>Euler number</u> $E = n(\text{object}) - n(\text{holes})$

 $E = 1-2 = -1$

 $E = 3$

Algorithm

- $E = 0$

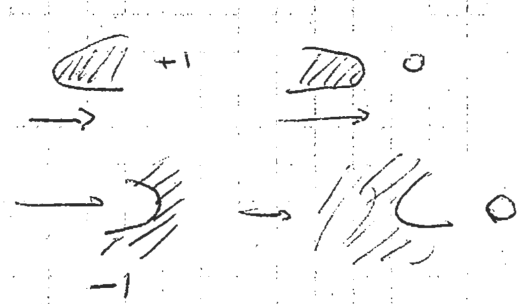
(1) $E_1 = 1 - 0 = 0$ (ASP)

(2) $E_2 = E_1 + 2 - 1 = 2$

(3) $E_3 = E_2 + 2 - 2 = E_2$ etc...

⋮

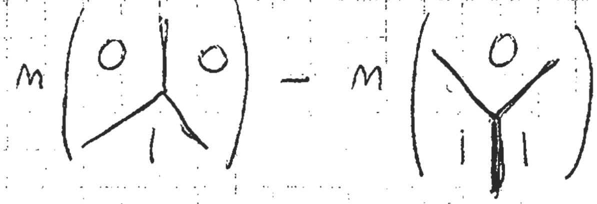
(8) $E_8 = 0$??? How to improve?

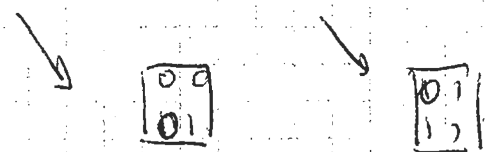


$E = n($  $) - n($  $)$

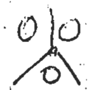
Back to discrete World

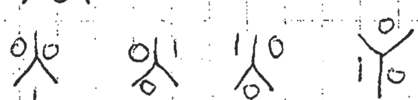
stream





6 - connectiveness

N_0 

N_1  ...

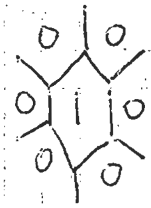
N_2  ...

N_3 

$$E = \frac{1}{6}(N_1 - N_2)$$

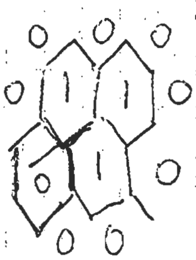
$$P = \cancel{\frac{1}{6}}(N_1 + N_2)$$

$$A = \frac{1}{6}(N_1 + 2N_2 + 3N_3)$$

ex: 

$N_1 = 6 \quad N_0 = N_2 = N_3 = 0 \qquad E = 1 \quad P = 1 \quad A = 1$

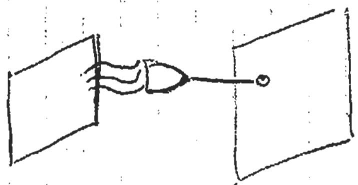
wrong! $P = 6$



$N_0 = 0 \quad N_1 = 9 \quad N_2 = 3 \quad N_3 = 1$

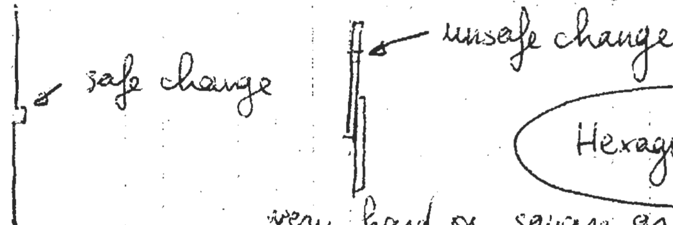
$E = 1 \quad P = 12 \quad A = 3.$

Iterative Motif Computation



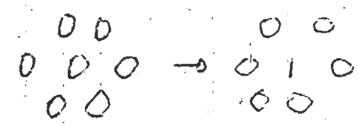
Euler differential.

How does E changes as one changes one pixel: $E^* = \dfrac{\Delta E}{\Delta I}$



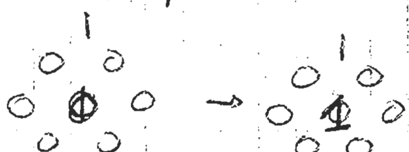
Hexagonal grid

very hard on square grid!

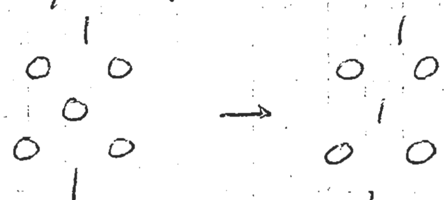


$E^* = +1$

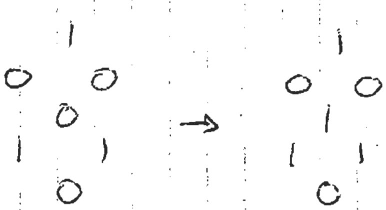
$2^6 = 64$ patterns + rotational symmetry + mirror images



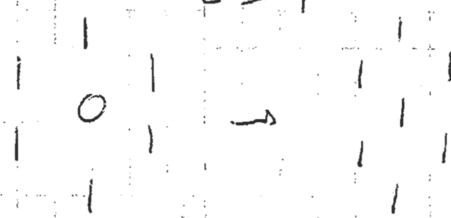
$E^* = 0$



$E^* = -1$

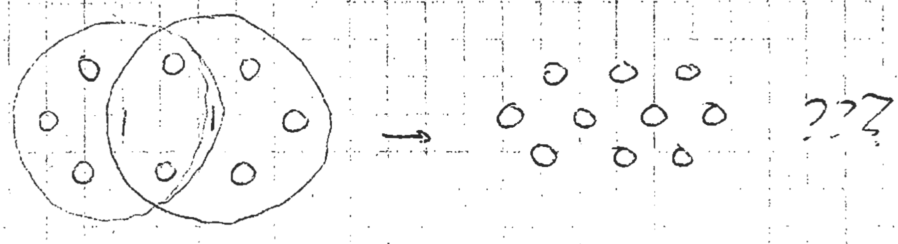


$E^* = -2$

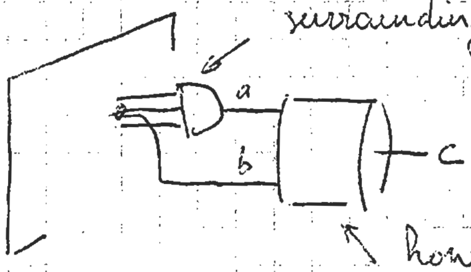


$E^* = +1$

Problem:



surrounding operator



how many possible functions? $2^4 = 16$

$a \to$ detect local features

$a \cdot b \leq b$ monotonic "thinning"

$a + b \geq b$ monotonic "swelling"

$a \otimes b$ not monotonic $\to$ Conway's game of life

Wolfram: A new kind of science