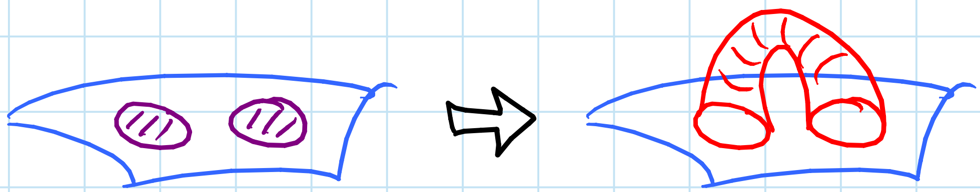
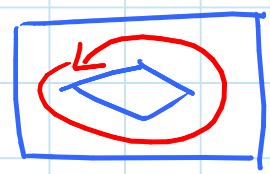
6.849	Class 15	Nov. 1, 2012

- What's a handle?
  - transformation on surface with 2 disks:



  - every orientable manifold without boundary = sphere + some handles

- Holes in unfoldings:



  - Gauss-Bonnet Theorem:
    turn angle along closed curve
    + curvature enclosed by curve
    = 360°

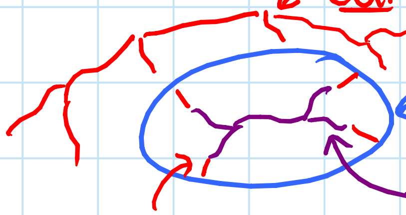
  ($\Rightarrow$ total curvature of polyhedron = 720°)
  - loop around hole has turn angle 360°

  $\Rightarrow$ enclosed curvature = 0
  - convex polyhedron $\Rightarrow$ no vertices

  $\Rightarrow$ no need for cuts (suture up slits)

- Leaves of ridge tree:
  - indeed, have unique shortest path to x
  - limit of nonunique points though

- Sun unfolding: [Demaine & Lubiw 2011]
  star unfold inside & source unfold outside convex curve C on convex polyhedron
  ⇒ no overlap!



  - generalization of source unfolding from a point or from a geodesic

OPEN: no overlap if source/star roles reversed? (i.e., C is a reflex curve)

- Zipper unfolding: [Demaine, Demaine, Lubiw, Shallit, Shallit 2010/2011]
  - goal: cutting is a path
  - edge zipper unfolding possible for Platonic & Archimedean
    - OPEN: Johnson solids
  - but not for e.g. rhombic dodecahedron

OPEN: does every convex polyhedron have a general zipper unfolding?

- **Ununfoldable polyhedra:**
  - 2 "witch's hat" constructions
    [Bern, Demaine, Eppstein, Kuo, Mantler, Snoeyink 1999]
    - triangular faces: $4 \cdot 9 = 36$ faces
    - convex faces: $4 \cdot 6 = 24$ faces

  OPEN: hat × which polyhedra = ununfoldable?

  - pointy cube: $6 + 8 \cdot 3 = 30$ faces
    [Tarasov 1999]
  - starshaped pointy dodecahedron:
    [Grünbaum 2001] $12 + 20 \cdot 3 = 72$ faces
  - smallest: $4 + 3 \cdot 3 = \underline{13}$ faces
    [Grünbaum 2002]

OPEN: 12 faces impossible "ununfoldable"

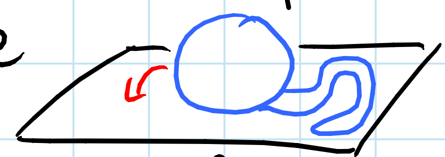
Strongly NP-complete to decide edge unfoldability of topologically convex orthogonal polyhedra
[Abel & Demaine - CCCG 2011]
- reduction from packing □s into a □

Heuristics: Pepakura

- **Band unfolding:**
  - bad cut can overlap
    [Demaine, Demaine, Lubiw 1999]
  - always a good cut avoiding overlap
    [Aloupis, Demaine, Langerman, Morin, O'Rourke, Streinu, Toussaint — CGTA 2008; Aloupis — PhD 2005]
  - also blooms by squishing

$\boxed{\text{OPEN}}$: attach top/bottom face, even for prismoid!
easy?

- Continuous blooming: [Demaine, Demaine, Hart, Iacono, Langerman, O'Rourke 2009/2011]
  - every unfolding can be refined & bloomed
    - cut along spanning tree of dual
    - ⇒ Hamiltonian dual; cut to make path
    - unroll one face at time



    - at all times:
      subset of unfolding in xy plane
      + subset of polyhedron in $z \geq \emptyset$
    - ⇒ no intersection (allowing layering at xy)
  - avoid 2D layering via "two step":
    1. unfold f_i to almost coplanar with f_{i-1}
    2. finish unfolding f_{i-1}
    3. repeat
  - avoid 1D layering via "waltz"
    1. unfold f_i to almost coplanar with f_{i-1}
    2. unfold f_{i+1} slightly
    3. finish unfolding f_{i-1}
    4. repeat

  - source unfolding unfolds by postorder traversal
    - imagine unrolling each shortest path separately
    - imagine unfolding as growing the polyhedron
    - claim: as long as the shortest path remains on surface, it remains shortest
    - tree just interleaves these path unrolls

PROJECT: implement these algorithms

OPEN: continuous blooming of

- star unfolding?
- sun unfolding?
- all edge unfoldings?
- all unfoldings?
- nonconvex polyhedra?