Engineering Risk Benefit Analysis

1.155, 2.943, 3.577, 6.938, 10.816, 13.621, 16.862, 22.82, ESD.72, ESD.721

DA 4. Introduction to Utility

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The Concept of utility

- **Utility of a consequence: A quantification of a person's relative preference for that consequence**
- **A simple extension of the indifference probability (or "preference value") concept (Axiom 2a)**
- **Utility function: Expresses a person's relative preferences among a set of consequences (often defined over a continuous range)**

Axiom 2a: Quantification of preferences

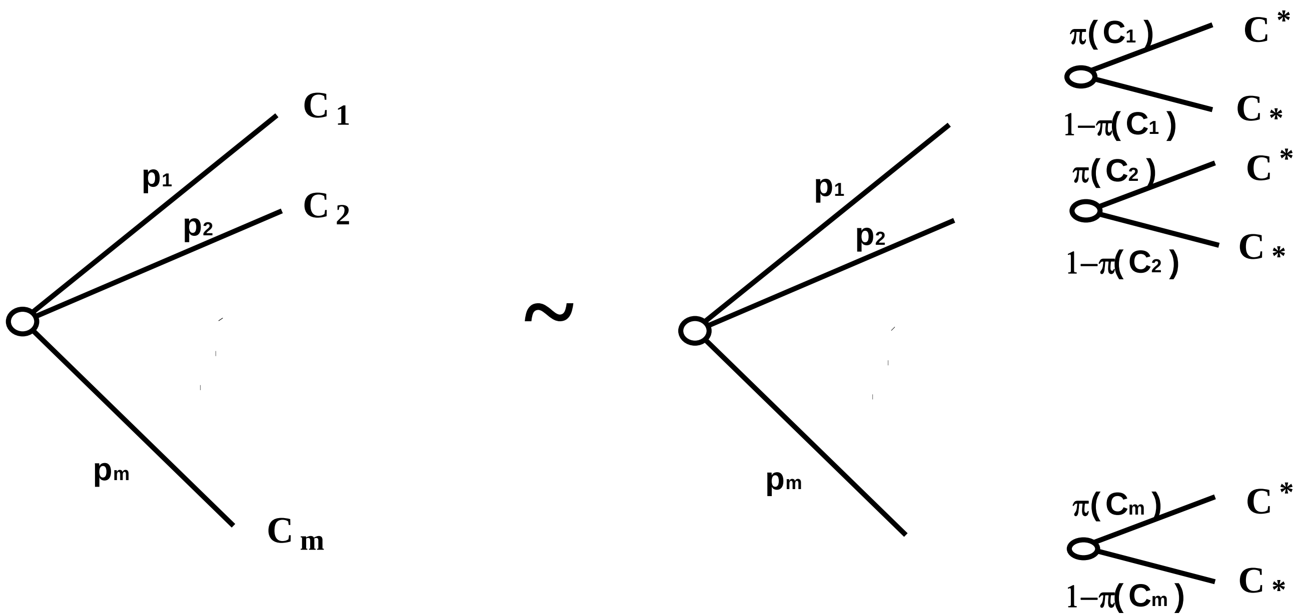
- **For each C_i, the DM can specify a number $\pi(C_i)$, with $0 \leq \pi(C_i) \leq 1$, such that the DM is indifferent between**

  - **possessing C_i with certainty**

and

  - **possessing the lottery L (C^*, C_*; $\pi(C_i)$, $1 - \pi(C_i)$)**

Schematic representation



Preference measures for lotteries

- Having established the relative preferences, i.e., the $\pi(C_i)$, $i = 1,\ldots,m$, we can derive the relative preferences for lotteries.

Recall Axiom 1: Given

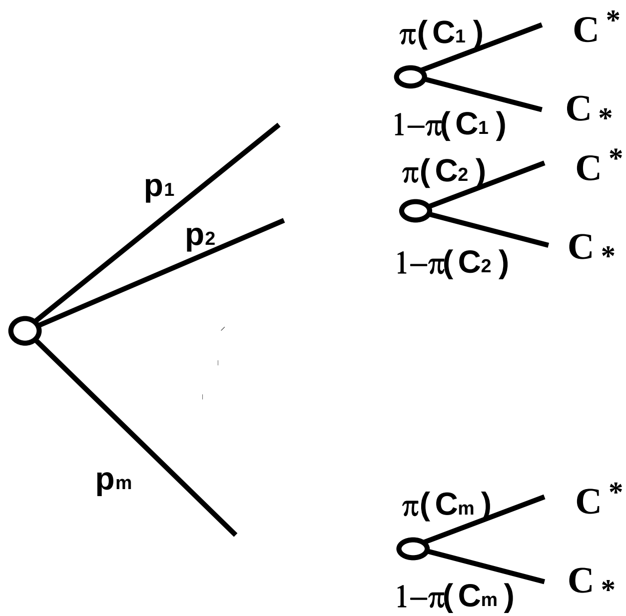
$L_1 = L\,(C^*, C_*; p_1\,, 1 - p_1)$ and $L_2 = L\,(C^*, C_*; p_2\,, 1 - p_2)$

Then: $L_1 \succ L_2$ if $p_1 > p_2$

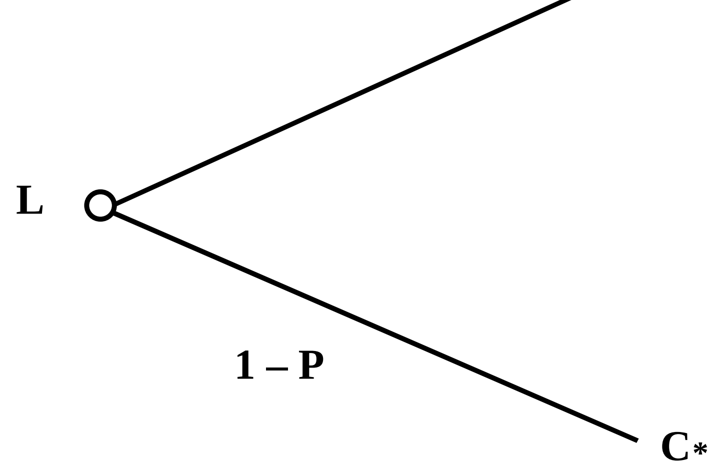
$L_1 \prec L_2$ if $p_1 < p_2$; $L_2 \sim L_1$ if $p_1 = p_2$

$\Rightarrow$ The probabilities p_1 and p_2 are the preference measures of the two lotteries.

Schematic representation (cont'd)



$$P = p_1 . \pi(C_1) + \ldots + p_m . \pi(C_m)$$



$$P = \sum_{i=1}^{m} p_i . \pi(C_i)$$ **expected value of preference values of possible outcomes of L**

Preference value of a lottery

The DM is indifferent between the original lottery on slide 4 and the last lottery on slide 6. The preference value of the latter is

$$\mathbf{P = \sum_{i=1}^{m} p_i \pi(C_i)}$$

Conclusion: **The preference value for a lottery is the expectation of the preference values of the possible outcomes of the lottery.**

Utility

- **The indifference probability or any positive linear transformation of the form**

$$U(x) = a\,\pi(x) + b \qquad a>0$$

is said to be a *utility function* over the set of consequences.

Convention: **$x = 0$ represents the present assets of the DM.**

Certainty Equivalent of a Lottery

- **The certainty equivalent (CE) of a lottery L is a consequence such that the DM is indifferent between the certainty equivalent and the lottery.**

[Note: U(CE) = U(L)]

Risk Premium for a Lottery

- **The risk premium (RP) for a single-attribute lottery is given by**

$$\mathbf{RP = EMV - CE}$$

- **For risk-averse individuals, RP is a positive quantity. Often, in practice, RP also decreases as additional wealth is acquired.**

Example: If a DM has a CE of \$40 for the lottery L(\$100, \$0; 0.5, 0.5), then his RP is \$50 - \$40 = \$10.

Utility of outcomes

Let the utility of payoffs be

$U(x) = 1.18 \ln(x+5) - 1.29$

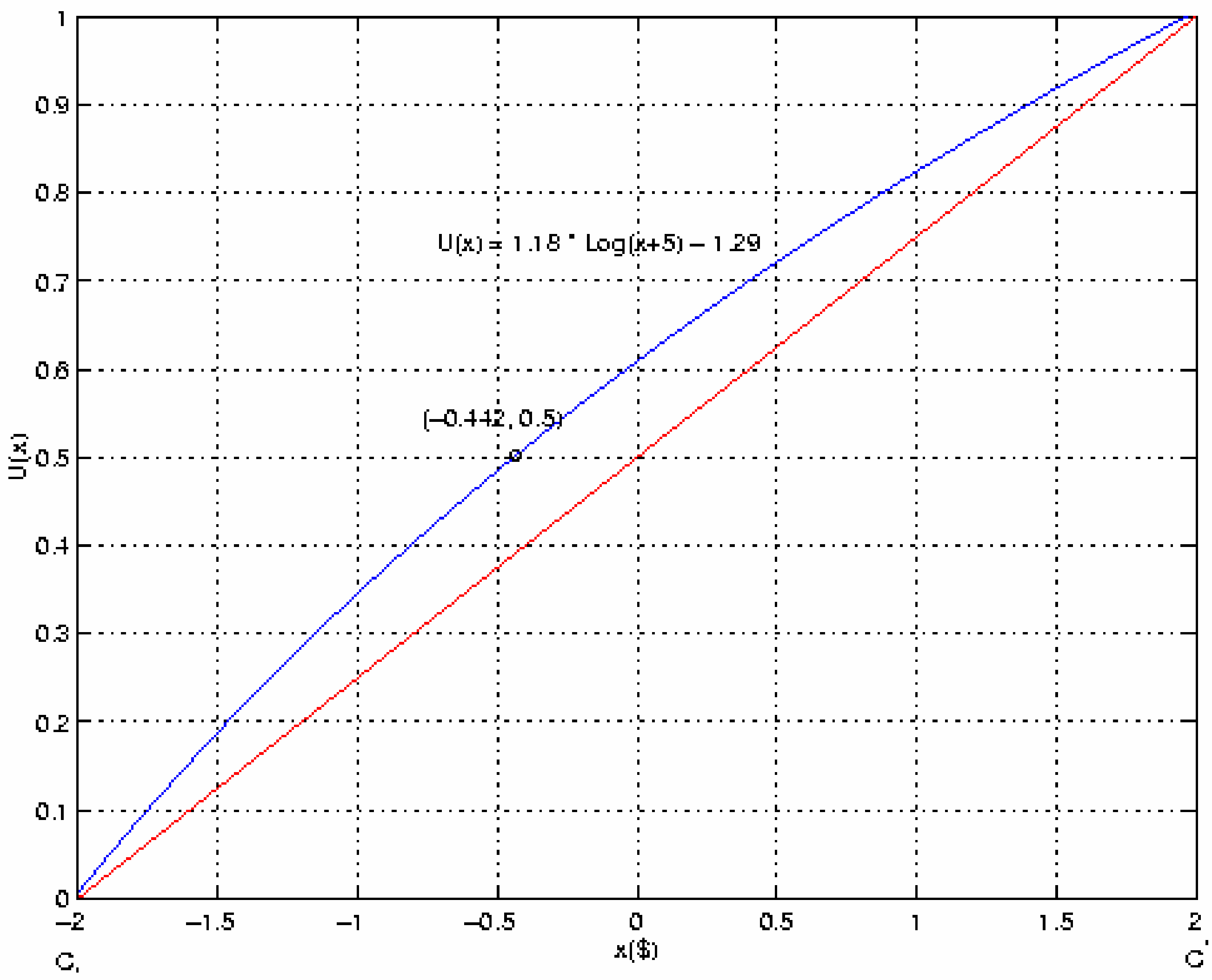
$-2 \leq x \leq 2$ (x in \$M)

$[U(2) = 1, U(-2)= 0]$

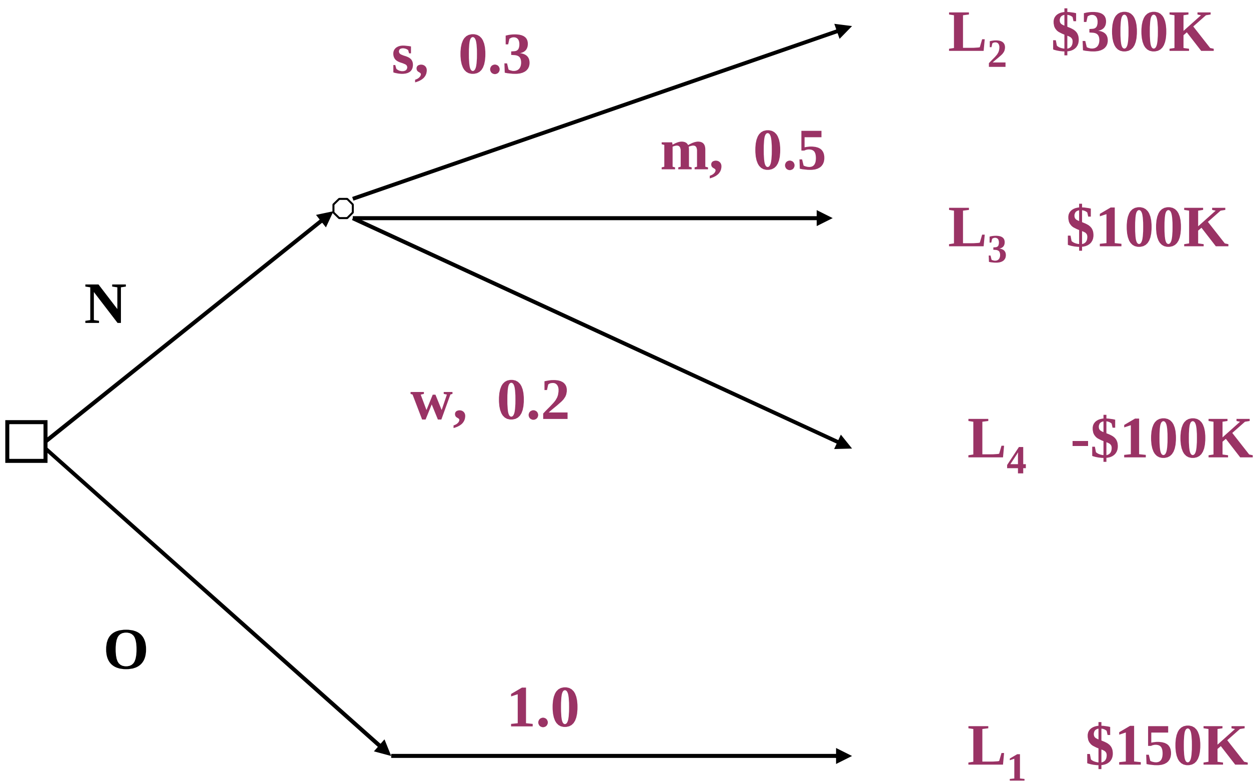
For $L(2, -2; 0.5, 0.5)$ EMV = 0

$U(L) = 0.5x1 + 0.5x0 = 0.5 = U(CE)$

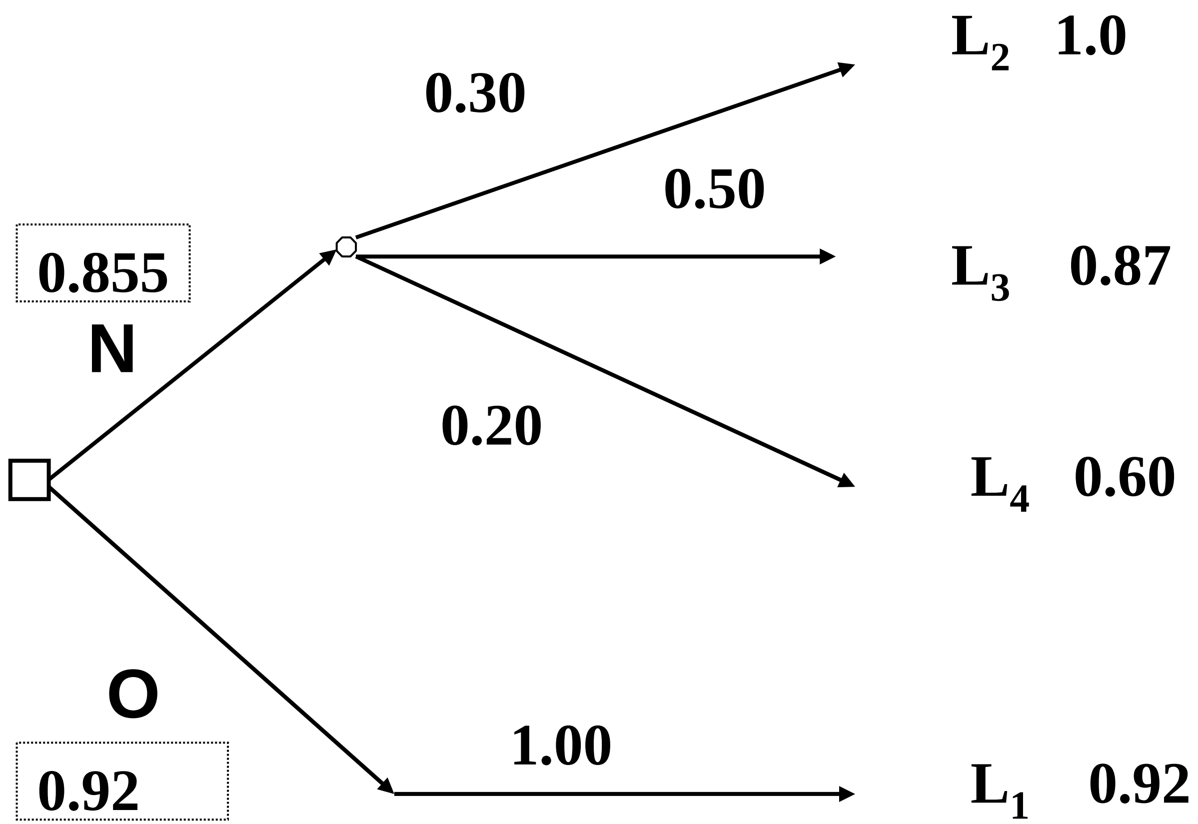
CE = -0.442M


U(x) = 1.18 * Log(x+5) − 1.29
(−0.442, 0.5)
U(x)
x($)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
C.
C*

Marketing a new product



Decision tree with utilities



Choice of decision option

- **If the DM chooses to market the new product, (s)he is choosing a lottery with preference value 0.855.**

- **If the DM chooses to market the old product, (s)he is choosing a lottery with preference value 0.920.**

- **Since 0.920 > 0.855, the DM should market the old product.**

Decision Rule: Maximization of expected utility

- Each possible consequence is replaced by its utility.
- Each decision option is a lottery whose preference value is the expected utility of its outcomes.
- Choose the option with the highest preference value.
- This is the result of accepting the axioms (unlike the maximization of the EMV).

The survey problem

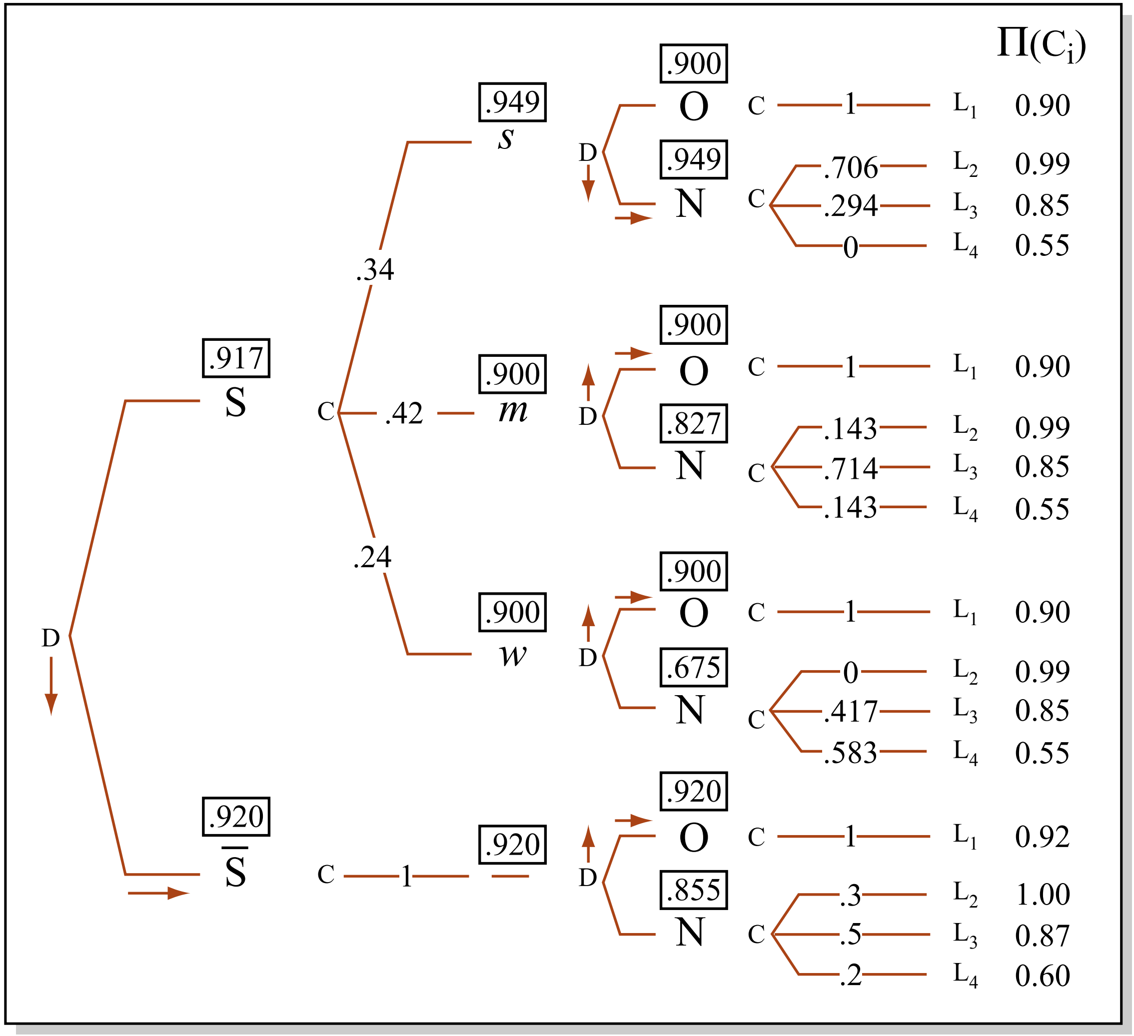


Figure by MIT OCW.

Best Decision

- **Do not buy the survey and keep marketing the old product.**
- **The best decision using EMV was to buy the survey and act according to its results, i.e., if the result is "strong," then market the new product, otherwise market the old product.**