STUDY GUIDE (Questions to think about while you read):

I. Knowing

A. Knowledge as power/knowledge as virtue

1] Schleirmacher wrote: "…it is here in Plato's philosophy that form and content are inseparable." The dialogue, he argued, forms a whole, a complete pattern, and the parts cannot be properly understood out of context, and without understanding of the pattern of the whole. The choice of Plato to present his philosophy in the form of a story or play has something to do with the Greek conception of knowledge. The truth about our relationship to this kosmos cannot be told adequately as a series of proofs, or in the form of a treatise [Aristotle, as well, presented all of his polished work in the form of dialogues. What we have of Aristotle is only his lecture notes]. What does it say about Plato's understanding of knowledge that he chooses this style of presentation [note also that today no 'philosopher' could get a dialogue published in any journal addressed to other philosophers- what does this say about our change of attitude?].

2] What is knowledge in the dialogue *Meno?* You can think about this both from the perspective of what is said in the dialogue, and what happens in the dialogue. There may be competing answers: can you reconcile them?

3] Why is a mathematical example chosen to illustrate the *supposed* theory of knowledge Socrates is expounding? Why this *particular* mathematical example?

4] Why is this particular mathematical example chosen to illustrate our pursuit of knowledge of *arête* [virtue, human excellence]

5] Socrates asks if we can know whether Meno is handsome or well born if we are entirely ignorant of who he is. Is this a good analogy for our position with respect to human excellence?

6] Consider these quotes from some of the founders of modernity, and put them together to form a theory [or, if more than one is implied, theories] of knowledge. From what you've seen in *Meno,* how do you think Plato would respond to these quotes, and the conception[s] of knowledge that lie behind them?

a] Knowledge and human power are synonymous. Francis Bacon
b] The Great End of life is not Knowledge but Action. Francis Bacon
c] Science is the knowledge of consequences, and dependence of one fact upon another. Thomas Hobbes
d] No man's knowledge here can go beyond his experience. John Locke
e] The only fence against the world is a thorough knowledge of it. John Locke

f] Man is the helper and interpreter of nature. He can only act and understand in so far as by working upon her or observing her he has come to perceive her order. Beyond this he has neither knowledge nor power. For there is no strength that can break the causal chain: Nature cannot be conquered but by obeying her. Accordingly those twin goals, human science and human power, come in the end to one. To be ignorant of causes is to be frustrated in action. Francis Bacon.

Lastly, I would address one general admonition to all; that they consider what are the true ends of knowledge, and that they seek it not either for pleasure of the mind, or for contention, or for superiority to others, or for profit, or fame, or power, or any of these inferior things; but for the benefit and use of Life; and that they perfect and govern it in charity. – Francis Bacon *The Great Instauration* [General Preface]

g] Human knowledge and human power meet in one; for where the cause is not known the effect cannot be produced. Nature to be commanded must be obeyed; and that which in contemplation is as the cause is in operation as the rule. Francis Bacon *Novum Organon*

h] Knowledge is the image of existence. Francis Bacon *Novum Organon*

i] On a given body, to generate and superinduce a new nature or new natures is the work and aim of human power. Of a given nature to discover the form, or true specific difference, or nature-engendering nature, or source of emanation (for these are the terms which come nearest to a description of the thing), is the work and aim of human knowledge. Francis Bacon *Novum Organon*

j] True knowledge is knowledge by causes." Francis Bacon *Novum Organon*

k] Roads to human power and to human knowledge lie close together and are nearly the same. Francis Bacon *Novum Organon*

l] For it was not that pure and uncorrupted natural knowledge whereby Adam gave names to the creatures according to their propriety, which, gave occasion to the fall. It was the ambitious and proud desire of moral knowledge to judge of good and evil….which was the form and manner of the temptation… For it was from lust of power that the angels fell, from lust of knowledge that man fell" Francis Bacon. *The Great Instauration*

m] For man is but the servant and interpreter of nature: what he does and what he knows is only what he has observed of nature's order in fact or in thought; beyond this he knows nothing and can do nothing. For the chain of causes cannot by any force be loosed or broken, nor can nature be commanded except by being obeyed. And so those twin objects, human knowledge and human power, do really meet in one; and it is from ignorance of causes that operation fails. Francis Bacon *The Great Instauration*

n] The end of our foundation is the knowledge of causes, and secret motions of things; and the enlarging of the bounds of human empire, to the effecting of all things possible. Francis Bacon, *The New Atlantis*

B. Logos as Definition: What is "What is?"

1] When you give a definition of 'straight line', what are you trying to define? [i.e. how can you state what you are trying to define without in fact defining it?]

2] What does Euclid's definition of a straight line mean?

3] I have in my possession a high school geometry text that I found at the town dump. On the inside cover it has the name Charles P. Drew written, and then, right under the warning not to write in the book is written, in a woman's handwriting, 'Hi Charlie!' with the dots over the i's in the shape of hearts. The textbook says the following about definitions: "A **definition** uses known words to describe a new word. In geometry, some words, such as point, line, and plane, are undefined terms. Although these words are not formally defined, it is important to have general agreement about what each word means. A **point** has no dimension. It is usually represented by a small dot. A **line** extends in one dimension. It is usually represented by a straight line with two arrowheads…A **plane** extends in two dimensions. It is usually represented by a shape that looks like a tabletop…" Compare these definitions to Euclid's, and this 'theory of definition' to Aristotle's. Is there a 'modern/ancient' distinction here?

4] Go back through *Meno*, list all the definitions of anything, and create categories of types of definitions. Use Aristotle to categorize and/or make up your own.

5] Thinking of Plato and Bacon, if the ancient and modern account of knowledge is quite different, what is it that they are disagreeing about? That is what is this 'knowledge' the nature of which is in dispute?

6.] <u>For those familiar with Non-Euclidean geometry</u>: For those who deny the postulates that create Euclidean space, there is an argument about the nature of a straight line. If we accept the postulates of spherical geometry (that is, not merely conceiving of geometry on a sphere, within Euclidean space, but imagining that geometry on what we perceive to be a plane has the characteristics of geometry on a sphere) then we will assign different properties to straight lines and their relationships (e.g. the possibility of parallel lines). If we are arguing about what the properties of a straight line are, what is the object about which we both are disagreeing? If we can come up with a definition of this object (it's not too hard) that applies in both realms, is that, then, necessarily the most 'essential' definition?

7] Why does Postulate 4 need to be stated? How is it other than the principle of identity ("All angles of the same measure are of the same measure")?

8] Aristotle distinguishes between definitions based on what is better known to us, and definitions which are based on what is better known (i.e. more *knowable*) absolutely. Of which sort are Euclid's definitions?

9] Aristotle claims [*Posterior Analytics* 90a31]: "to know what a thing is [ti estin] is the same as knowing *why* it is [dia to estin]". How could this apply to Euclid's definitions of mathematical objects?

C. The Idea of a Proof

1] Smith reports "The idea of demonstrating the truth of a proposition which had been discovered intuitively appears first in the teachings of Thales (c. 600 B.C.) It is probable that this pioneer knew and proved about six theorems, each of which would have been perfectly obvious to anyone without any demonstration whatever." Proofs are not found anywhere in antiquity, except in Greek intellectual life. Speculate: what would cause someone to prove something that is perfectly obvious, and thereby inaugurate a system in which the non-obvious could be proven, especially given that the entire rest of the world, for the 1000 years following Thales, were perfectly content to operate with empirically reliable, unproven theorems?

2] Euclid's Proposition 6, proposition 14, and proposition 19 [Bk. I] and others are negative or 'reductio ad absurdam' proofs. There is currently a school of mathematical thought [constructivism] that rejects the validity of such proofs [can you guess why?] At very least, it is generally considered more enlightening to prove something directly. What forces Euclid to resort to a 'reductio' proof? [Think about what the reduction proofs in Euclid have in common]. [Is I.48 a reductio proof in disguise?]

3] What are the conditions of a reasonable axiom? What constitutes a reasonable 'common notion'?

4] **Analysis and Synthesis:**
A] Anonymous scholium to Euclid XIII:

"Analysis, then, is the taking of what is sought as if it were agreed to be true and following through the consequences from this to something genuinely agreed to be true, while synthesis is the taking of something admitted and going through to something we then agree to be true." [I took some liberties with translation to make this clearer to you. Apologies to Anonymous]

B] Pappus of Alexandria [neo-Platonist, 4th century A.D.]:

"... in analysis we suppose that which is sought to be already done, and inquire what it is from which this comes about, and again what is the antecedent cause of the latter, and so on until, by retracing our steps, we light upon something already known or ranking as a first principle... But in synthesis, proceeding in the opposite way, we suppose to be already done that which was last reached in analysis, and arranging in their natural order as consequents what were formerly antecedents and linking them one with another, we finally arrive at the construction of what was sought..."

"Analysis, then, is the way from what is sought, taken as admitted by means of previous synthesis…but in synthesis, going in reverse, we suppose as admitted what was the last result of the analysis, and, arranging in their natural order as consequences what were formerly the antecedents, and connecting them with one another, we arrive at the completion of the construction of what was sought, and call it synthesis."

C] Francois Viete (the true founder of Western algebra, 16th century): "In mathematics there is a certain way of seeking the truth, a way which Plato is said first to have discovered, and which was called ‘analysis’ by Theon and was defined by him as ‘taking the thing sought as granted and proceeding by means of what follows to a truth that is uncontested’; so, on the other hand, ‘synthesis’, is ‘taking the thing that is granted and proceeding by means of what follow to the conclusion and comprehension of the thing sought

D] Rene Descartes (16th century) in *Objections and Replies*:
“…the method of proof is two-fold, one being analytic, the other synthetic. Analysis shows the true way by which a thing was methodically discovered and derived, as it were effect from cause, so that, if the reader care to follow it and give sufficient attention to everything, he understands the matter no less perfectly and makes it as much his own as if he had himself discovered it. But it contains nothing to incite belief in an inattentive or hostile reader; for if the very least thing brought forward escapes his notice, the necessity of the conclusions is lost; and on many matters which, nevertheless, should be specially noted it often scarcely touches, because they are clear to anyone who gives sufficient attention to them.

“Synthesis contrariwise employs an opposite procedure, one in which the search goes as it were from effect to cause (though often here the proof itself is from cause to effect to a greater extent than in the former case). It does indeed clearly demonstrate its conclusions, and it employs a long series of definitions, postulates, axioms, theorems and problems, so that if one of the conclusions that follow is denied, it may at once be shown to be contained in what has gone before. Thus the reader, however hostile and obstinate, is compelled to render his assent. Yet this method is not as satisfactory as the other and does not equally well content the eager learner, because it does not show the way in which the matter taught was discovered.

“It was synthesis alone that the ancient Geometers employed in their writings, not because they were wholly ignorant of the analytic method, but, in my opinion, because they set so high a value on it that they wished to keep it to themselves as an important secret.”

1] Which, of any, of Euclid’s proofs are analytic?

2] Is the mathematical passage in *Meno* analytic or synthetic?

3] Rewrite Euclid II.4 as an analytic proof. [You are excused, for this exercise, from the general prohibition against using modern mathematics or notation]

5] Any valid proof demonstrates *that*, given certain premises, something is the case. Do some proofs also show *how* or even *why* something is the case? That is, do some proofs not only demonstrate, but explain? (Proclus, for instance, criticizing I.23 offers an alternative construction he claims proceeds in a ‘more instructive manner’) What are the characteristics of an explanatory proof, or what makes a proof more or less explanatory? Which proof in Book I do you think provides the most ‘insight’ into its subject matter?

6] Why is I.10, written as a construction problem rather than the proposition “The line bisecting the angle of an equilateral triangle bisects the opposite side?” In general, what makes him choose to present a conclusion as a construction or as a theorem [Q.E.D. or Q.E.F.]?

7] Work out a classification scheme for the proofs we read for this assignment.

8] Think of the *Elements* as a work of literature. Heath says that Book I has three parts. Until Proposition 27, it is a book about triangles. Propositions 27-31 are about parallel lines. The rest of the book is about parallelograms and their relationship to triangles. What is the 'story line' of this book? **C.I. presentation: choose one proposition to present in class, and tell how it fits into the story that Book I tells.**

9] Proposition 27, which shows that two lines intersected by a transverse line, making equal alternate angles with the transverse, will never meet. This proposition does not draw in any way on Postulate 5, the famously misnomered 'parallel postulate'. We all know that alternate geometries can be formulated in which either [a] all lines meet, or in which [b] a number of lines drawn through any point are parallel to a third line ([b] is not ruled out in Euclid until Proposition 29, which *does* cite postulate 5). So what is it that has created 'Euclidean space' in the *Elements*, prior to I.27, if not the 'parallel postulate'?

II. Logos and Arithmos

A. Logos as Ratio

1] In the definitions in Book V and Book VII of Euclid's *Elements,* why is number defined but magnitude not?

2] Why do you think that for number, both part and parts are defined, in magnitude only 'part'?

3] Book V. defines a ratio between magnitudes, but Book VII has no definition of ratio between arithmoi. Why?

4] Describing what a proportion is for magnitudes is a much more complicated affair than describing proportions for arithmoi. What does the definition of proportion for magnitudes mean, and why can it not be as simple as the definition of proportion for arithmoi?

5] Why does Euclid represent numbers with lines instead of points, as his Pythagorean predecessors do? What is the difference in the 'symbolic capacity' of the two means of representation?

6] Why do you think Diophantus precedes his known numbers (rather than his variables) with the symbol 'M' (an abbreviation of 'monad')?

7] Consider Euclid VII 37. "If a number be measured by a certain number, the number measured will have a part having the same name as the measuring number." What does this mean? What does it say about how Euclid considers fractions in relationship to the concept 'arithmos'?

8) Given the Greek understanding of fractions, why do you think they were only written as strings of unit fractions?

9) One reason why 1 and 2 are not considered numbers by the Pythagoreans derives from their conception of a number as the sum of other numbers. In this conception, 1 and 2 can only be generators of number, not numbers themselves. The Pythagorean shape numbers in Nicomachus are a particular version of this conception of numbers as sums – in this case a number is a summation series. Why do you think the Pythagoreans might have thought that this quality of being a sum was definitive to the concept 'number'?

B. Magnitude and Multitude

1] It is said that the Pythagoreans were the first to consider geometry and *arithmetike* (number theory) as one subject, which they called *mathematike* or ‘learnable’. It was not until Aristotle that the term seemed to gain general usage as meaning specifically these kinds of study (including then the rest of the classic *quadrivium*, music and astronomy). It seems significant, and perhaps necessary, that in creating the axio-deductive system of theoretical geometry, the Pythagoreans convert geometry from a science of measurement to a science (perhaps) of shapes. In doing so they banish all numbering from geometry and create a disjunct between number and magnitude. Given this, why do you think the Pythagoreans thought that these subjects should be grouped together under one label?

2] Compare the definitions in Bk. V and Bk. VII of Euclid. Note the differences and explain them (most important, contrast and compare Def. V.5, 6 and Def. VII. 20).

3] Many of the propositions in Bk. V and Bk. VII are identical. Make note of anything that occurs in one book but not the other, and speculate on the cause or meaning of such distinctions.

4] What distinctions could one draw between magnitude and multitude if one considered only Euclid Bks. V. and VII?

5] Thinking about Aristotle’s division of the genus ‘quantity’, and the Greek conception of *arithmos,* why do you think that magnitude and multitude are treated as entirely distinct subjects?

6] Euclid’s Book VII takes four definitions to cover the same issues for magnitude that Book V. covers in one definition for multitude. What do you think this says about either the relative complexity of the two concepts, or their relative ‘intuitiveness’?

7] Does the fact that most of the definitions about proportion come in the book on magnitude indicate that magnitude is the genus, of which multitude is a species (given that all of these definitions apply also to arithmoi)? Consult Aristotle on this.

8] In the definitions of Book VII there is an elaborate system for classifying arithmoi. Why is there not a similarly elaborate system of classification for magnitudes? (in our section on irrationals, you might consider whether or not Book X performs this function for magnitudes).

C. Quality and Quantity

1] The first sentence in Ch. 6 of Aristotle's *Categories* divides quantity into the sub-categories 'discrete' and 'continuous'. This is the first thing that Aristotle has to say about quantity, thus one assumes that it is the most fundamental, if not most important statement one can make about quantity. If this is the beginning of a definition, what kind of definition is it?

2] These are the primary species of the genus 'quantity'. Why? Can you think of any other ways to subdivide the idea of 'quantity'?

3] The division of relative position vs. no relative position is the second way that Aristotle divides the genus quantity. The division does not coincide with the first division. Does it seem as important to you? As 'natural'?

4] Again, in Ch. 6, Aristotle says "That which is not a quantity can by no means, it would seem, be termed equal or unequal to anything else" Does this constitute a definition of quantity: if not an essential one, at least exclusive?

5] The Pythagoreans supposedly began the tradition that considered the four mathematical sciences (arithmetike, geometry, music, astronomy) to be related as parts of a greater science of 'quantity'. Does Aristotle's discussion of quantity support this contention?

6] The word 'Arithmos' does not cover the same ground as 'number.' We have a few phrases that come close to expressing what 'arithmos' expresses: counting numbers, positive integers, natural numbers. The Greeks did not have a word that covers exactly the same ground that our word number covers. Does '*poson*'[quantity, 'how much'] perform that function?

D. The Greek Conception of Number

1] Consider the following definitions of number/*arithmos* and answer these questions:

a] Which best defines a number? Defend your answer (you do not have to confine yourself to definitions where that term, rather than *arithmos*, is used)

b] Which best defines *arithmos*? Defend your answer. (you do not have to confine yourself to definitions where that term, rather than 'number', is used)

c] Based on your answer to [a] and [b], what is the *essential* difference between 'number' and 'arithmos'?

1] Thales: "*arithmos* is a collection of units" [Iamblichus *Introductio Arithmetica* p. 10]

2] Pythagoreans – "made number out of one" 985a20.

3] Chryssipus "multitude one"

4] Moderatus [neo-Pythagorean, 60 a.d.] *arithmos* "a progression of multitude beginning from a unit and a regression ending in it." Stobaeus; Eclogae i. Proem. 8

5] Nicomachus "a flow of quantity made up of units"

6]: "Number is that by which the quantity of each thing is revealed." Simon Stevin

7] "An *arithmos* is a finite multitude..." Eudoxus

8] "limited multitude". Aristotle, *Metaphysics* 1020a 30

9] "a set/system of units" μοναδων συστημα Domninus 413

10] "*arithmos* is always a multitude of indivisibles" πληθοσ αδιαιρετων. Aristotle; Metaphysics 1085b22

11] "an aggregate in the realm of quantity composed of monads" Nicomachus. 13, 8

12] Aristotle "an *arithmos* signifies a measured plurality or a plurality of measures" [*Metaphysics* 1088a6]

13] Aristotle gives a few implied definitions of *arithmos,* but here are some passages from his writings that employ *arithmos* in a way that gives a deeper sense of what he means by the term:

> **Metaphysics** *Delta* 13, 1020a8-14:
>
> Quantity [*poson*, literally "how much" or "how many"] is said to be that which is divisible into constituents, each of which is by nature one [or "a one"] and a "this" [*tode ti*, a specific indicable thing]. A plurality [*plethos*] is a kind of quantity if it [the quantity] is numerable [countable; *arithmeton*]; a magnitude is a kind if quantity if it [the quantity] is measurable. A

plurality is said to be that which is divisible potentially into parts which are not continuous; a magnitude, on the other hand, is that which is potentially divisible into parts which are continuous....Of these, a limited [*peperasmenon*] plurality is said to be a *arithmos*, a limited length a line, a limited width a surface, and a limited depth, a body.

Metaphysics *Iota* (I)6, 1057a2-6:

Plurality is as if it were [or "such as"; *hoion*] a genus of *arithmos*; for *arithmos* is a plurality measurable by the one. And in some sense [or "in a way"] the one and *arithmos* [or "a *arithmos*"] are opposed, not as contraries, but...as some relative things are; for the one in so far as it is a measure is opposed to *arithmos* in so far as *arithmos* is measurable.

Metaphysics *Nu* (N)1, 1087b33-1088a15:

The one signifies a measure, evidently. And in each case there is some different underlying subject [*hupokeimenon*, thing laid down], such as in the musical scale a quarter-tone; in magnitude a finger or a foot or some other such thing; and in rhythm a beat or a syllable....And this is also according to formula [or "definition" or "account": *logos*]; for the one signifies a measure of some plurality and the *arithmos* signifies a measured plurality [a plurality that has been measured] and a plurality of measures. Therefore it is also with good reason that the one is not a *arithmos*; for neither is a measure measures, but a measure is a principle [or "source", *arche*], and so is the one.

2] Based on Nicomachus' treatment of *arithmetike*, speculate on why the Greeks did not consider [a] fractions, [b] negative numbers, or [c] irrational numbers to be *arithmoi,* or group all these with *arithmoi* under as one category, similar to our concept of 'number'.

3] Do the terms 'arithmos' and 'number' connote the same 'category of understanding', and is the argument between ancients and moderns then an argument over the proper definition and extent of that category?

4] Consider the following definitions of numerical unit. Why do you think that, from the Greeks through at least the Middle Ages, 'one' was held to not be a number?

1] Pythagoreans: "one is not a number since a measure is not the thing measured" 1088a7
2] Thymarides: [Pythagorean 4th b.c.e.] unit as a "limiting quantity" Iamblichus [11-12]
3] "an unitie is no number but the beginning and original of number." Baker *1568*
4] "..Ramus, and such that have written since his time, affirme not only that an unite or one, is a number, but also that eury fraction or parte of an vnite, is a number..." Hylles *1592*
5] "…unity alone out of all number, when it multiplies itself, produces nothing greater than itself…Unity, therefore, is non-dimensional and elementary." Nicomachus [238]
6] "[a] if from a number there is subtracted no number, the given number remains [b] if from 3 we take 1, 3 does not remain [c] therefore one is not 'no number.' Simon Stevin
7] Rabbi Ben Ezra *Sefer ha-Echad* (Book on Unity) (1140) first to entertain idea that one is a number

8] “Multiplicity is the genus of *arithmos*. Because of this *arithmos* and one are opposites.” Iamblichus [see also Aristotle Metaphysics 1056b 19ff]
9] “The one is the source of number” *Metaphysics* 1052b22ff.
10] “…unity is not an *arithmos* for neither is a measure measures, but a measure is a principle, and so is unity.” Aristotle, *Metaphysics* 1088a7

5] Is there any notable disagreement among the above definitions of unity?

6] Think back to Aristotle’s claim that a definition should aim at the ‘ti esti’ of something – the essence, or literally, the ‘what is it’. What would you, or any modern, say the ‘ti esti’ is of 3125 as opposed, say to the ‘ti esti’ of 6385)? What would Nicomachus say?

7] For Nicomachus what does it mean to ‘know’ a number***?***

8] One reason why 1 and 2 are not considered numbers by the Pythagoreans derives from their conception of a number as the sum of other numbers. In this conception, 1 and 2 can only be generators of number, not numbers themselves. The Pythagorean shape numbers in Nicomachus are a particular version of this conception of numbers as sums – in this case a number is a summation series. Why do you think the Pythagoreans might have thought that this quality of being a sum was definitive to the concept ‘number’?

ALTERNATE ARITHMOS QUESTIONS:

1] From all of the evidence of the material, come up with a definition of *arithmos* that does not refer to what in particular is included or excluded (e.g. fractions, ‘irrational numbers’, one, etc.). It might help you to think (or to frame your answer) with the question of what distinguishes *arithmos* from our concept of number (but again, we are looking for the concept – the container – not the content of that container. There is something about the ‘containers’ *arithmos* and ‘number’ that justifies what they include and exclude – what is it?)

2] Beginning by thinking about the difference in the notions of proportion and ratio in Books V and VII, speculate on what the essential difference is for the Greeks between magnitude and multitude, and why it was so important for them to consider these essentially different kinds of entities. (Does Aristotle’s distinction between quantity and quality help at all?)

3] The Greeks have left us no rationale for why they only considered what we call positive integers to be *arithmoi.* To put it simply, this is because they did not have to contend with anyone arguing the case for some category called ‘number’ which includes everything that we put in there. Try to tease out of this week’s readings any clues as to why the Greeks would be so unified in maintaining such a limited concept. (the answer to this will be implicated in the answer to #1, and you can combine the two answers if you'd like).

III. Measuring the Unmeasurable: Putting the Irrational in Ratio

1] In *Topics* 158b29 Aristotle states that when a line parallel to the base cuts a triangle, it divides the sides and the areas similarly, "…for the areas and the straight lines have the same *antanairesis*, and this is the definition of 'the same ratio'." The process of *antanairesis* (called by Euclid *anthuphairesis*) is the process used to find the Greatest Common Divisor in Book X.1. How does this process 'define' a ratio? How does this definition compare with the definitions of ratio in Book V.

2] How do you think the Greeks came to suspect that two lines could have no common measure (hint: think 'Pythagorean')?

3] Thinking about the 'Pythagorean' proof of the incommensurability of the diagonal and the side of the square (or, as we might put it, of the $\sqrt{2}$), Can you use the same method to prove that $\sqrt{3}$ and $\sqrt{5}$ are irrational? Why doesn't this kind of proof demonstrate that $\sqrt{4}$ is irrational (answer is not 'because it isn't)?

4] Given that the Greeks often substituted for the word 'arithmos' the phrase 'the odd and the even', as if these mean the same thing, what might be the significance of proving that any ratio of integers expressing the relationship of the side to the diagonal would have to be both even?

5] In Euclid, Book. V: What is it about magnitudes that are commensurable in square that makes them 'expressible' in terms of each other? Why does this not hold, for instance, for magnitudes commensurable in cube?

6] Proposition X.2 is never used by Euclid to discover or prove incommensurability. In fact, it is used incidentally in Proposition X.3, but then never again employed by Euclid. What then is the function of this Proposition? Is it a kind of definition of incommensurability? If so, is it an <u>essential</u> (*ti esti*) definition, or a definition by way of identifying a necessary attribute of incommensurables (that is, a definition by exclusion)?

7] How could Proposition XIII.5 be converted into a proof of the incommensurability of the segments of the 'golden ratio.'(Hint: start performing an *anthuphairesis* between the final, larger line and its larger segment)

8] When we reverse the anthuphairetic proof of the incommensurability of the side and diagonal of the square, the ratios of the sides and diagonals seem to approach the $\sqrt{2}$. Can you prove that this progression of ratios is approaching the true value of the side/diagonal ratio (and can you do so in a "Greek", i.e. geometric manner)?

9] 'Why' does reversing the anthuphairesis give us this approximating progression?

10] The 'common measure' of the side and diagonal, if it exists in any fashion, seems to be an entity of which we can always say, with great precision, whether it is greater or less than any particular arithmos, but can never say with precision what exactly it is. Thinking back to Aristotle's categories in Chapter 2, have we grasped this entity as a 'quantity' or as a 'relation', or what?

11] The pattern generated by those incommensurable magnitudes that can be proven incommensurable through anthuphairesis is palindromic. Can you see why this is so, given that the reverse of the anthuphairesis proving them to be incommensurable, always yields a series of ratios successively approximating the ratio of the two incommensurable?

12] In Book X, are the medial, apotome, and the binonomial in any way 'essentially' different?

IV. Fitting Ideas to the World

A. Courage in *Laches*

1] What are the 'two sides' of the debate over the nature of courage? Are there more than one set of opposites contending for the definition of courage?

2] Is progress made in this dialogue? Are the two sides brought closer together? Although the dialogue ends in failure, do we have a better sense of what courage is at the end then we do at the beginning? If there is progress, exactly how does it happen (e.g., do the two sides inch closer, does one side best the other, some of both – be as 'methodical' as possible in laying this out).

B. Justice in *The Republic*

1] Are there two sides contending for the name of Justice in Book I of the *Republic?* If not, then what is the 'argument structure' of the book?

2] Do we know better at the end of Book I what justice is? What it isn't?

3] If we do not have the same kind of 'two sided debate' as in *Laches,* then speculate as to why: is justice somehow inherently different from courage? Is Plato up to something different here. Any interesting guesses (with supporting argument) are welcome.

C. The Pursuit of Truth

1] The word *eidos* appears 59 times in Euclid's *Elements,* and every single occurrence of the word is found in either Book VI, which plays out the Eudoxian theory of proportions developed in Book V, or in Book X, the excruciatingly complex attempt at classifying and thus grasping the various ways in which magnitudes can be incommensurable.[1] We should be cautious about making too much of

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1 Here is a typical example of Euclid's use of the word:

"From this it is evident that, if three straight lines are proportional, then, as the first is to the third, so is the *eidos* described on the first to that which is similar and similarly described on the second." [Book VI, Proposition 19, lines 34-37]. The word *eidos* here seems to mean simply shape or figure.

this strange concentration of the word *eidos* within two sections of Euclid. Still, the question is unavoidable: Why might this notion of ‘eidos’ be most needed in these two investigations?

2] What is the relationship between the word *eidos* as used in Euclid, and as used in *The Republic?*

3] If the metaphor of the cave and the discussion of the divided line are both considerations of the nature of an ‘eidos’ and the way in which we can grasp ’eide’, what is the constant sense of *eidos* between these two passages?

4] What does Plato’s ordering of the education of the Guardians say about the nature of the *eide*?

5] Make a list of the attributes of an *eidos*, gleaned from this reading

6] Speculate, based on the following three clues, on what the relationship might be between the Platonic notion of an ‘eidos’ and the role of the idea of ‘shape’ in Greek mathematics:

> [a] There is a close connection between Plato’s conception of the *eidos* of something, and the Aristotelian demand for a definition as the statement of the *ti esti* of something.
>
> [b] Plato chooses to use for this notion which has developed into our word ‘idea’ the word that Euclid employs for ‘shape’ or ‘figure’.
>
> [c] In *Meno*, when Socrates gives an example of how to define something, he chooses the example of a ‘shape’. He notably avoids using the term *eidos*, but uses instead the word *schema*. Then, as we know, to give an example of how knowledge is acquired, he chooses the example of finding a particular length by locating it as part of a structure, or shape.

D. The Truth as a Surd

1] The saying “God is an inexpressible number” (αριθμοσ αρρητοσ θεοσ) is attributed to a thinker named Lysis, (c. 425 B.C.). Assuming that this refers to the work being done in incommensurable magnitudes at this time, what could he mean?

2] Why do you think that Theaetetus and Meno, Plato’s two dialogues most directly about knowledge, both have such a central mathematical theme, specifically concerning incommensurables?

3] If Theaetetus and Theodorus represent mathematics and Socrates represents philosophy, what does the dramatic action of this dialogue tell us about the relationship between the two disciplines?

4] If the truth of anything is only approachable by a method analogous to the dyad of the great and small, what is the relationship between humans and the truth?

5] Taking the suggestion of Theon of Smyrna that the method of approximating the ratio of the side and diagonal of the square, and the ratio we now call the ‘golden ratio’ (the method of *anthuphairesis* that we have examined) was known as the ‘ladder of the great and the small’, answer the following questions after reading the quotes I have included regarding the great and small, or the indeterminate dyad:

a] What kind of existence does the indeterminate dyad have?

b] If the truth of anything is only approachable by a method analogous to the dyad of the great and small, what is the relationship between humans and the truth?

IV. THE MODERN CONCEPTION OF NUMBER

1] In his *Universal Arithmetick* (Newton’s name for algebra), Isaac Newton defines number thus: “By number we understand not so much a multitude of unities, as the abstracted ratio of any quantity, to another quantity of the same kind, which we take for unity. And this is threeforld; integer, fracted, and surd: An integer, is what is measured by unity, a fraction, that which a submultiple part of unity measures, and a surd, to which unity is incommensurable” Is this a substantially different conception of number from that found in Nicomachus and/or Euclid?

2] Is Dedekind’s conception of number significantly different from that of Newton? If so, how is it different from that found in Nicomachus and/or Euclid**?**

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