HST.582J / 6.555J / 16.456J Biomedical Signal and Image Processing
Spring 2007

**HST-582J/6.555J/16.456J-Biomedical Signal and Image Processing-Spring 2007**

# Problem Set 3

## DUE: 3/1/07

## Problem 1

Consider the continuous-time function, $\Lambda(t)$, given by

$$\Lambda(t) = \begin{cases} 1 - |t|/T_s & \text{if } -T_s < t < T_s \\ 0 & \text{otherwise} \end{cases}$$

**(a)** Show that $\Lambda(t)$ can be thought of as resulting from the convolution of a rectangular window with itself, that is,

$$\Lambda(t) = c \ \Pi_T(t) * \Pi_T(t),$$

where

$$\Pi_T(t) = \begin{cases} 1 & \text{if } -T < t < T \\ 0 & \text{otherwise} \end{cases}$$

Specify the values of $c$ and $T$ in terms of $T_s$.

**(b)** Determine $\Lambda(F)$, the CTFT of $\Lambda(t)$.

Now consider the continuous-time function, $\phi(t)$, given by

$$\phi(t) = \frac{\sin \pi F_s t}{\pi F_s t}.$$

**(c)** Sketch $\phi(t)$ and $\Lambda(t)$ on the same coordinates.

**(d)** Determine $\Phi(F)$, the CTFT of $\phi(t)$, and then sketch $\Phi(F)$ and $\Lambda(F)$ on the same coordinates.

As we saw in Chapter 1, the sampling theorem states that any bandlimited continuous-time signal can, in principle, be exactly reconstructed from its samples by means of the interpolation formula,

$$x(t) = \sum_{n=-\infty}^{\infty} x[n]\phi(t - nT_s). \tag{1}$$

In many cases this formula is not practical because it requires a summation over the entire duration of the sampled signal. Here we will consider the effect of replacing the interpolation function $\phi(t)$ by the finite-duration function $\Lambda(t)$.

**(e)** Consider using $\Lambda(t)$ in place of $\phi(t)$ to reconstruct samples of the signal $x(t) = \sin(2\pi F t)$. Sketch the original $x(t)$, along with the reconstructed signals for the two conditions corresponding to the sampling rates $T_s = \frac{1}{4F}$ and $T_s = \frac{1}{8F}$. What conditions are required to obtain a reasonably good approximation using $\Lambda(t)$?

**(f)** Discuss how the two interpolation functions differ in the frequency domain. What frequency domain features make the reconstruction based on $\Lambda(t)$ less accurate? (Note that the reconstruction described by Eq. (1) can be interpreted as the time-domain convolution of the sampled signal and the interpolation function. Also, you may find it helpful to sketch $X(F)$, the CTFT of a continuous-time sine wave, $x(t)$, and $X(f)$, the DTFT of $x[n]$ obtained by sampling $x(t)$.)

## Problem 2

**(a)** Find a closed-form expression for $X(F)$, the CTFT of the amplitude-modulated signal

$$x(t) = (1 + m \cos 2\pi F_m t) \cos 2\pi F_c t.$$

*Hint:* Show that $x(t)$ can be written as

$$x(t) = \cos 2\pi F_c t + \frac{m}{2} \cos 2\pi (F_c - F_m)t + \frac{m}{2} \cos 2\pi (F_c + F_m)t$$

**(b)** Assume that $F_c$, $F_m$, and $m$ are unknown. You propose to measure these three parameters using the following method:

1. Sample $x(t)$ at sampling frequency $F_s = 5000$ Hz for a duration of $T = 20$ ms. This gives the finite, discrete-time signal $x[n]$.
2. Compute the N=100 point DFT of of x[n]. This gives $X[k]$ for $k = 0, \ldots, 99$.

For each of the following five cases, determine whether there is sufficient information in $|X[k]|$ to estimate $F_c$, $F_m$, and $m$ unambiguously. If yes, describe your method for estimating these parameters and give numerical values for your estimates. If not, specify how you would modify the measurement parameters $T$, $F_s$, and $N$ in order to obtain unambiguous estimates. Feel free to use Matlab or any other software to compute and sketch $|X[k]|$.

i) $F_c = 1475$ Hz, $F_m = 200$ Hz, $m = 1$

ii) $F_c = 1475$ Hz, $F_m = 1200$ Hz, $m = 1$

iii) $F_c = 1475$ Hz, $F_m = 40$ Hz, $m = 1$

iv) $F_c = 1475$ Hz, $F_m = 200$ Hz, $m = 0.05$

v) $F_c = 1500$ Hz, $F_m = 200$ Hz, $m = 0.05$

## Problem 3

In this problem we will consider two $N$-point FIR filters, $h_1[n]$ and $h_2[n]$, formed from the $(\frac{N}{2}+1)$-point sequence $h[n]$ as follows:

$$h_1[n] = \begin{cases} h[n], & 0 \le n \le \frac{N}{2} \\ h[N-n], & \frac{N}{2}+1 \le n \le N-1 \end{cases}$$

and

$$h_2[n] = \begin{cases} h[\frac{N}{2}-n], & 0 \le n \le \frac{N}{2} \\ h[n-\frac{N}{2}], & \frac{N}{2}+1 \le n \le N-1 \end{cases}$$

**(a)** Sketch $h_1[n]$ and $h_2[n]$ for $N = 8$ and

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

**(b)** For arbitrary values of $N$, express the relationship between $h_1[n]$ and $h_2[n]$ in terms of cyclic convolution.

**(c)** Determine the relationship between $H_1[k]$ and $H_2[k]$, the DFTs of $h_1[n]$ and $h_2[n]$.

**(d)** We want to filter a finite duration signal, $x[n]$, with $h_2[n]$ from **(a)**. We have decided to do this by computing the $M$-point DFT of $h_2[n]$, computing the $M$-point DFT of $x[n]$, multiplying the two DFTs, and computing the $M$-point inverse DFT of the product. Given that $x[n]$ is zero outside the range $0 \le n \le L-1$, and $L = 1000$, what is the minimum value of $M$ that will produce the desired result? What value of $M$ should be used with a standard FFT algorithm that requires $M = 2^\nu$?