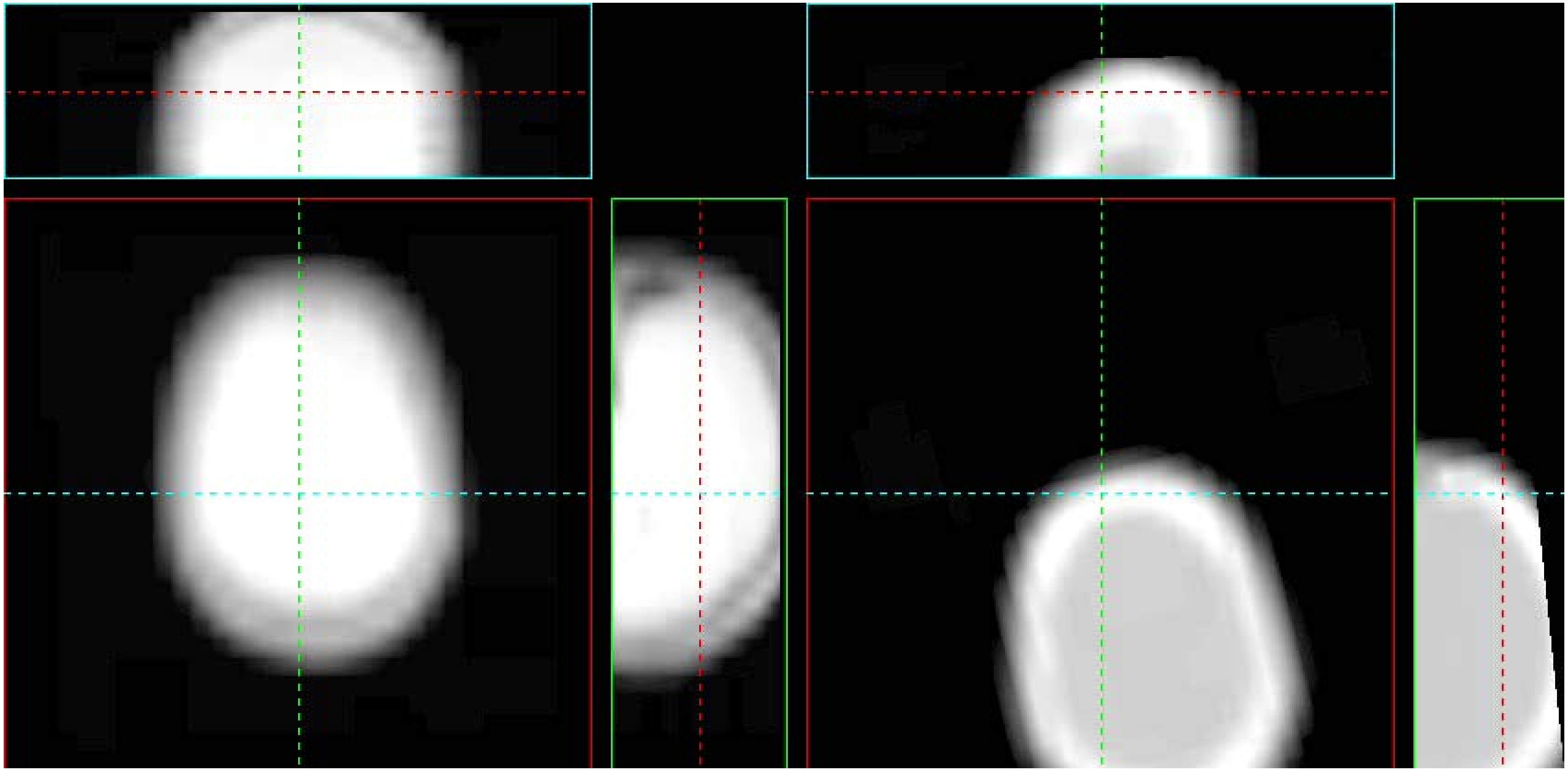
HST.582J / 6.555J / 16.456J Biomedical Signal and Image Processing
Spring 2007

Medical Image Registration II

HST 6.555

Lilla Zöllei and William Wells

CT-MR registration movie



From: Wells, W. M., et al. "Multi-modal Volume Registration by Maximization of Mutual Information." *Medical Image Analysis* 1, no. 1 (March 1996): 35-51.
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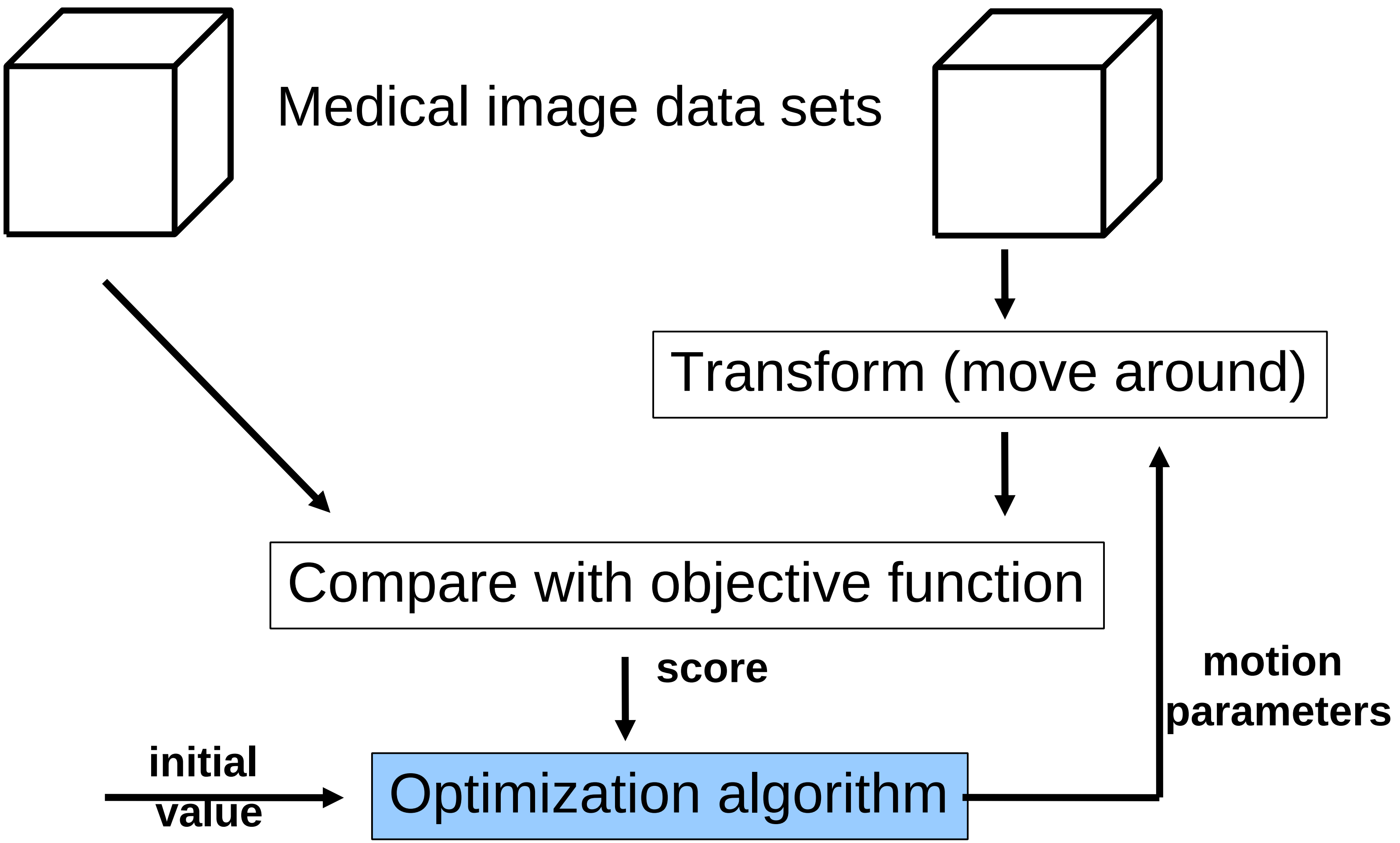
Roadmap

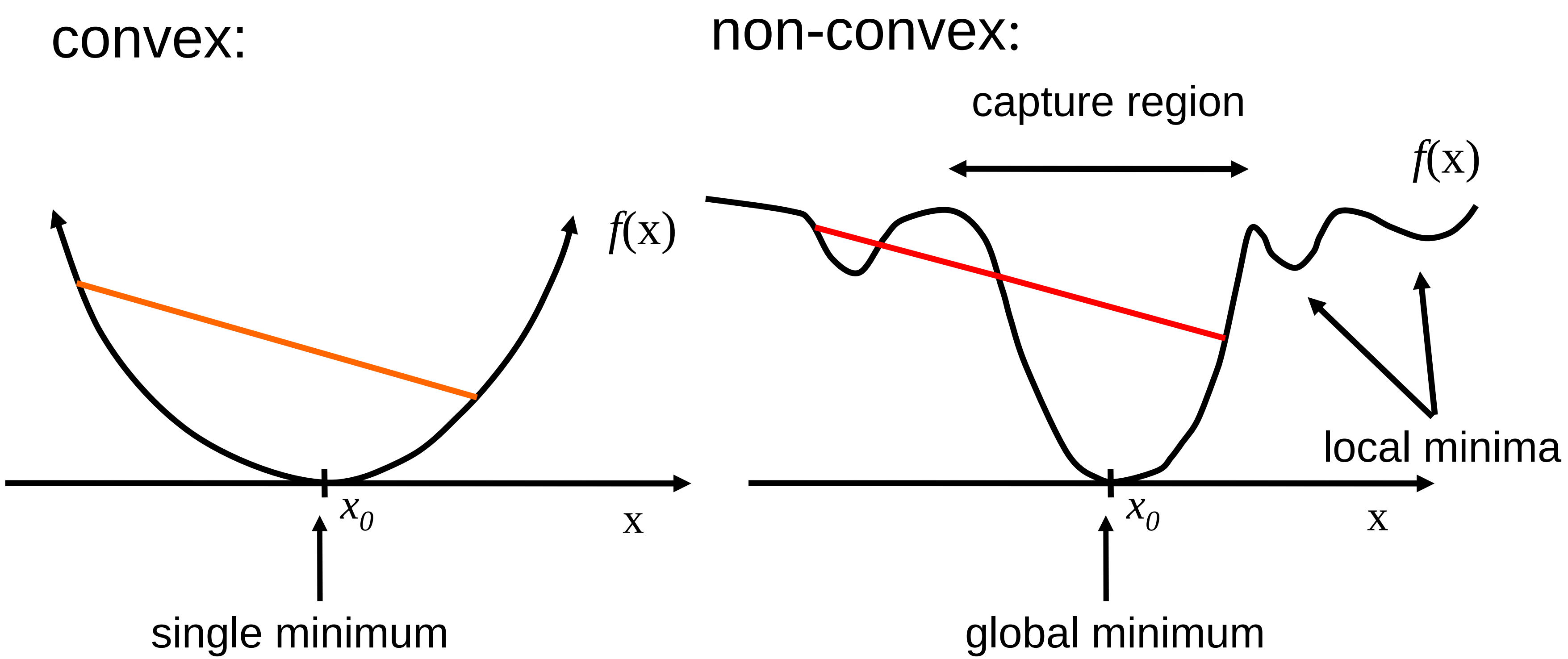
- ✓ Data representation
- ✓ Transformation types
- ✓ Objective functions
  - ✓ Feature/surface-based
  - ✓ Intensity-based
- ➡ Optimization methods
- Current research topics

Medical Image Registration



Optimization -- terminology

- find *x* that minimizes *f*(*x*)



Optimization (for registration) (1)

- Goal: find **x** that optimizes *f*(**x**)
  - do it quickly, cheaply, in small memory; (or evaluate *f* as few times as possible)
- Parameter recovery: “search” for solution
  - Standard mathematical function (with T dependency) to be optimized
    - use only function evaluations
    - use gradient calculations (more guidance, but costly)
- Based upon prior information:
  - *constrained,* e.g.: $x_1 \leq x \leq x_2$
  - *unconstrained*

Optimization (for registration) (2)

- No guarantees about global extremum
  - Local extrema:
    - sometimes sufficient**
    - find local extrema from a wide variety of starting points; choose the best
    - perturb local extremum and see whether we return
  - Ambitious algorithms:
    - simulated annealing methods
    - genetic algorithms

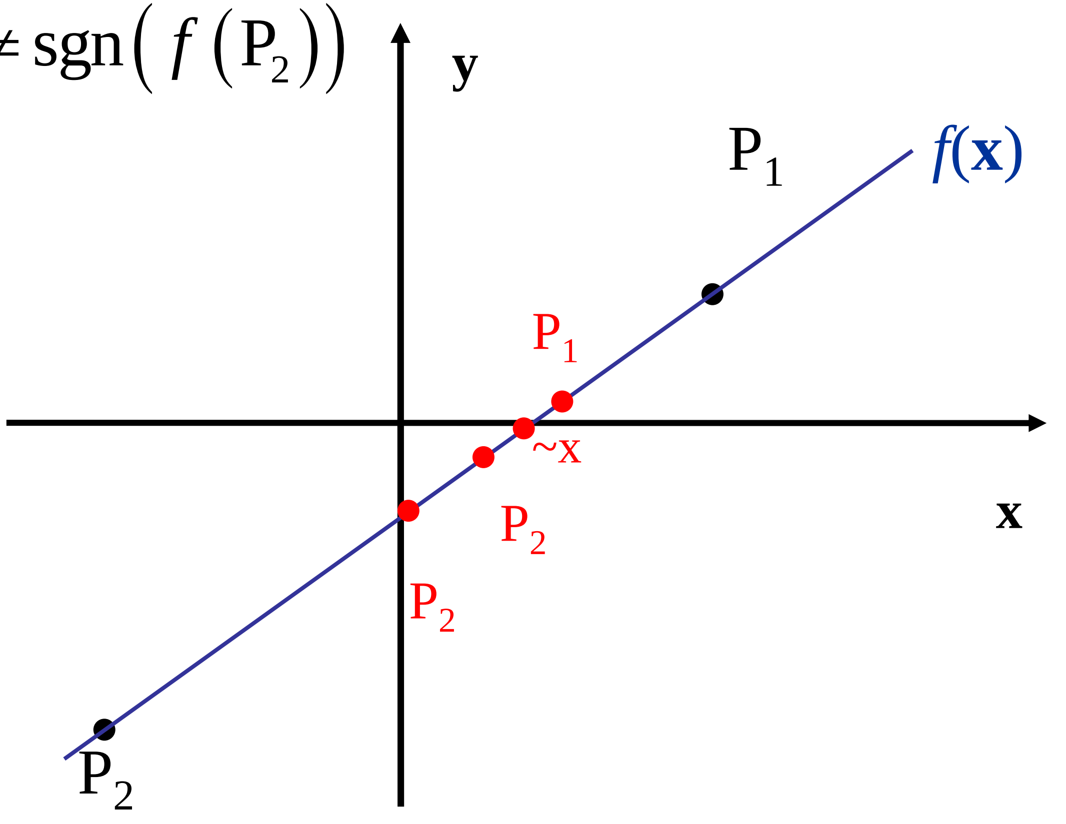
Search Algorithms

- **1D solutions** – minimum bracketing is possible
  - Golden Section Search
  - Brent's Method
  - Steepest Descent
- **Multi-dimension** – initial guess is important!
  - Downhill Simplex (Nelder & Mead)
  - Direction Set Methods
    - Coordinate Descent
    - Powell's Method
  - Gradient Methods
    - Conjugate gradient methods

Root finding by bisection

- Root-bracketing by two points: (P_1, P_2)

$$\operatorname{sgn}\left(f\left(P_1\right)\right) \neq \operatorname{sgn}\left(f\left(P_2\right)\right)$$



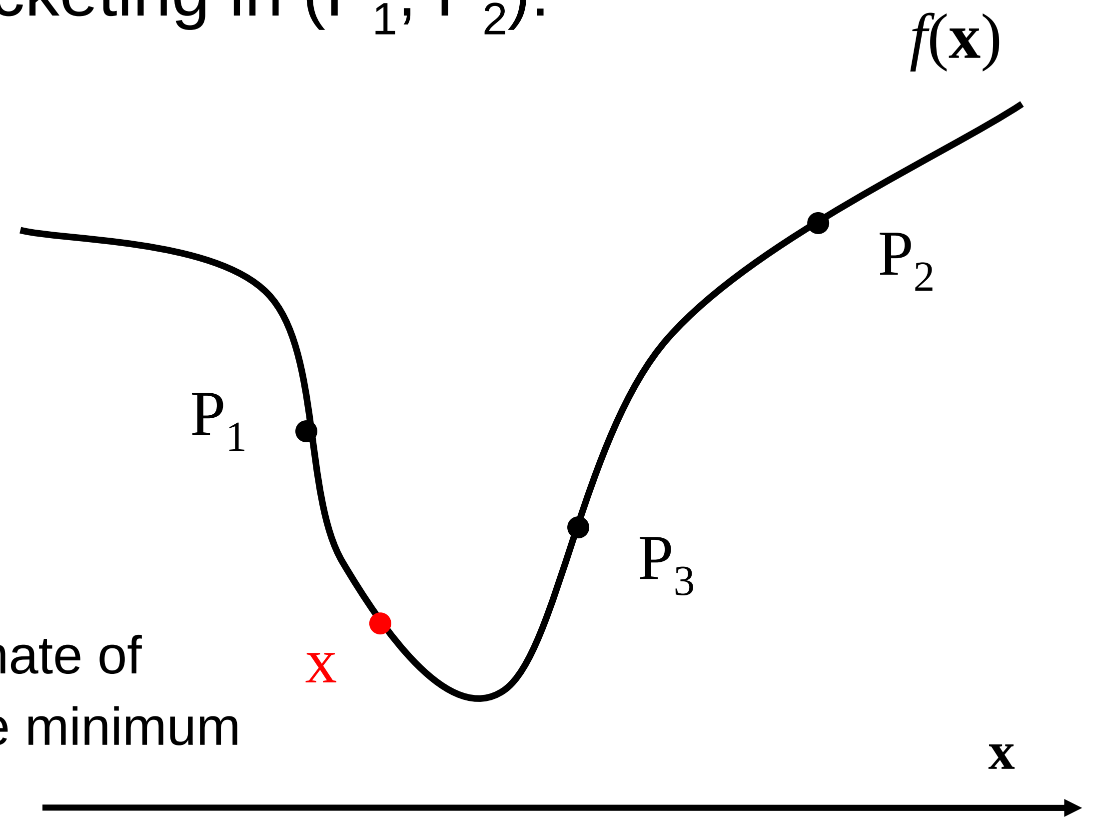
Minimization of convex function

- Minimum bracketing in (P_1, P_2):

$$P_1 < P_3 < P_2$$

$$f(P_3) < f(P_1)$$

$$f(P_3) < f(P_2)$$



P_3: best current estimate of the location of the minimum

Golden Section Search

- Minimization strategy:
  - *(i) select the larger of* $\overline{P_1P_3}, \overline{P_3P_2}$
    → assign this interval to be L
  - *(ii) position x in L s.t.*

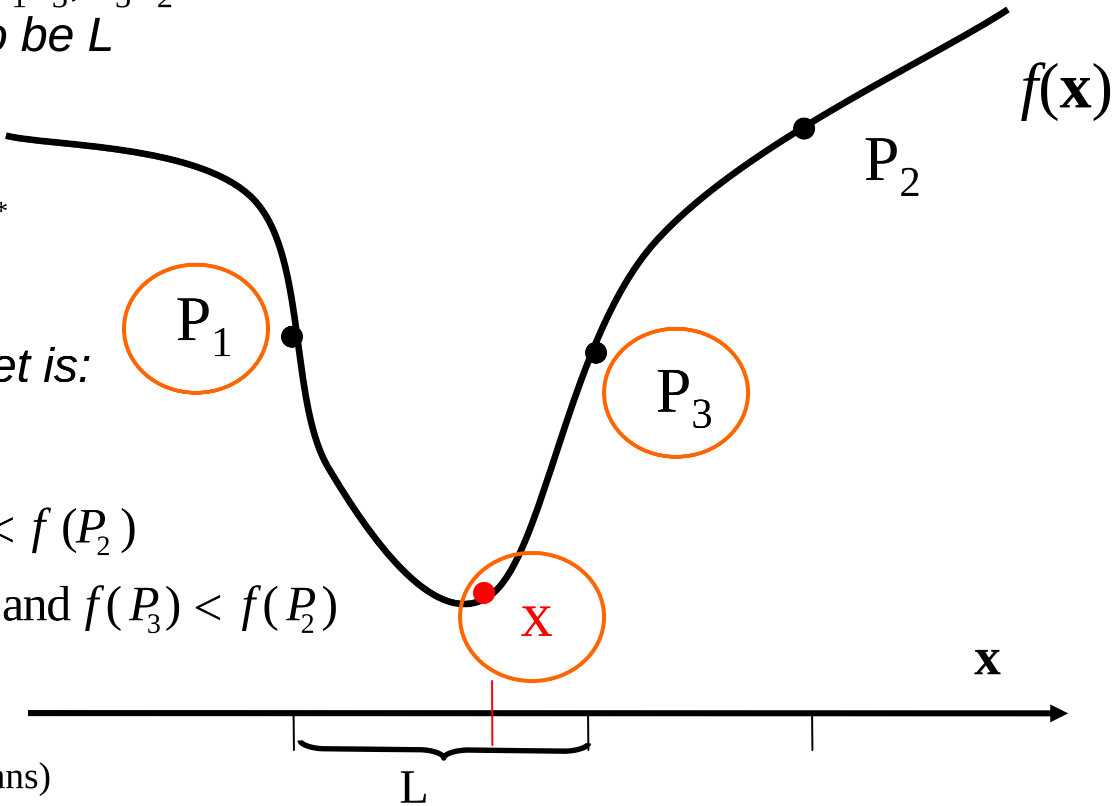
    $$\frac{\left\|\overline{P_3x}\right\|}{\|L\|} = \frac{\left(3-\sqrt{5}\right)}{2} \approx .38197^{*}$$

  - *(iii) new bracketing triplet is:*

    $\left(P_1, x, P_3\right)$ if

    $f(x) < f(P_1)$ and $f(x) < f(P_2)$

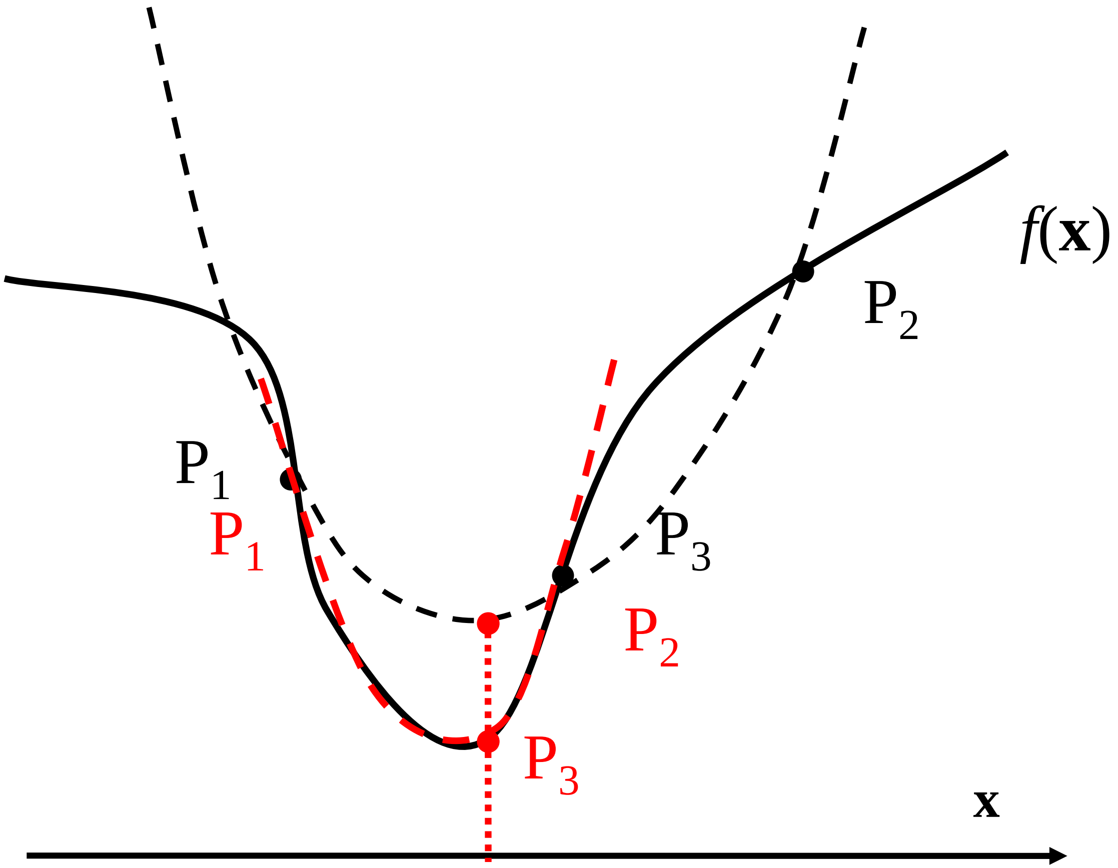
    $\left(x, P_3, P_2\right)$ if $f(P_3) < f(x)$ and $f(P_3) < f(P_2)$



**golden mean* / *golden section* (Pythagoreans)

Brent's Method

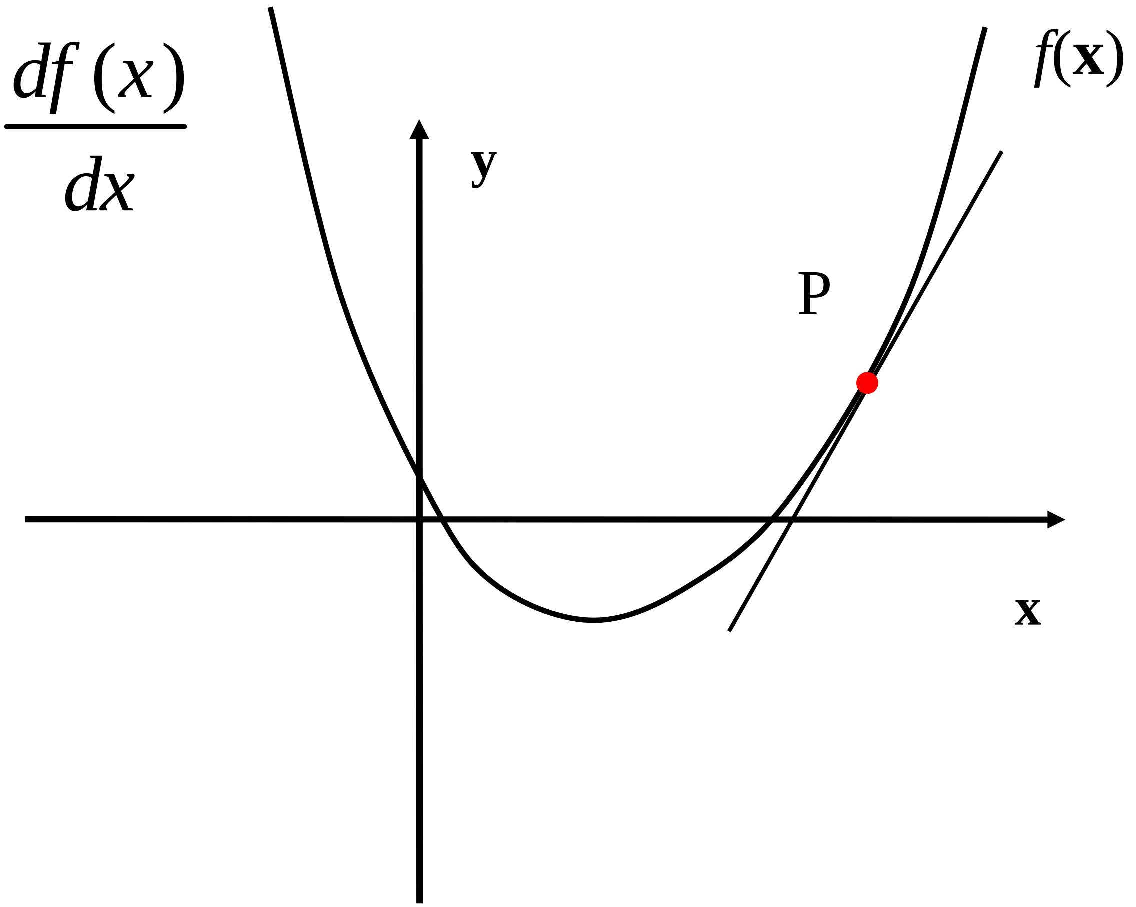
- Minimum bracketing
- Parabolic interpolation



Gradient Descent

$$m = \text{slope} = \frac{df(x)}{dx}$$

$$P' = P + \alpha m$$



Downhill Simplex Method (1)

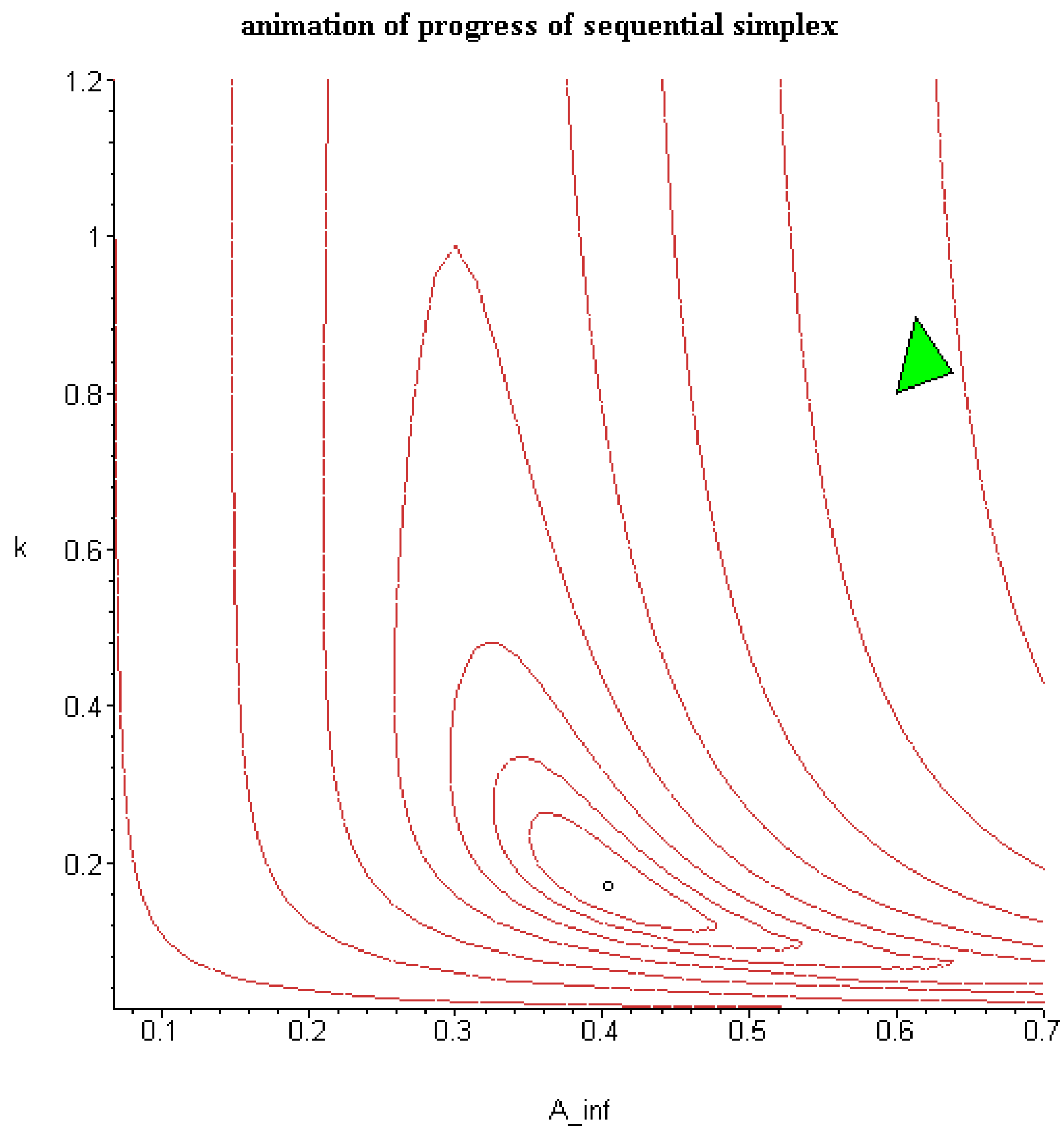
- due to Nelder and Mead*
- self-contained; no 1D line minimization
- only function evaluations, no derivatives
- not efficient in terms of number of function evaluations, but easy-to-implement
- geometrical naturalness
- useful: when f is non-smooth or when derivatives are impossible to find

Nelder, J.A, and Mead, R. 1965, Computer Journal, vol. 7, pp. 308-13.

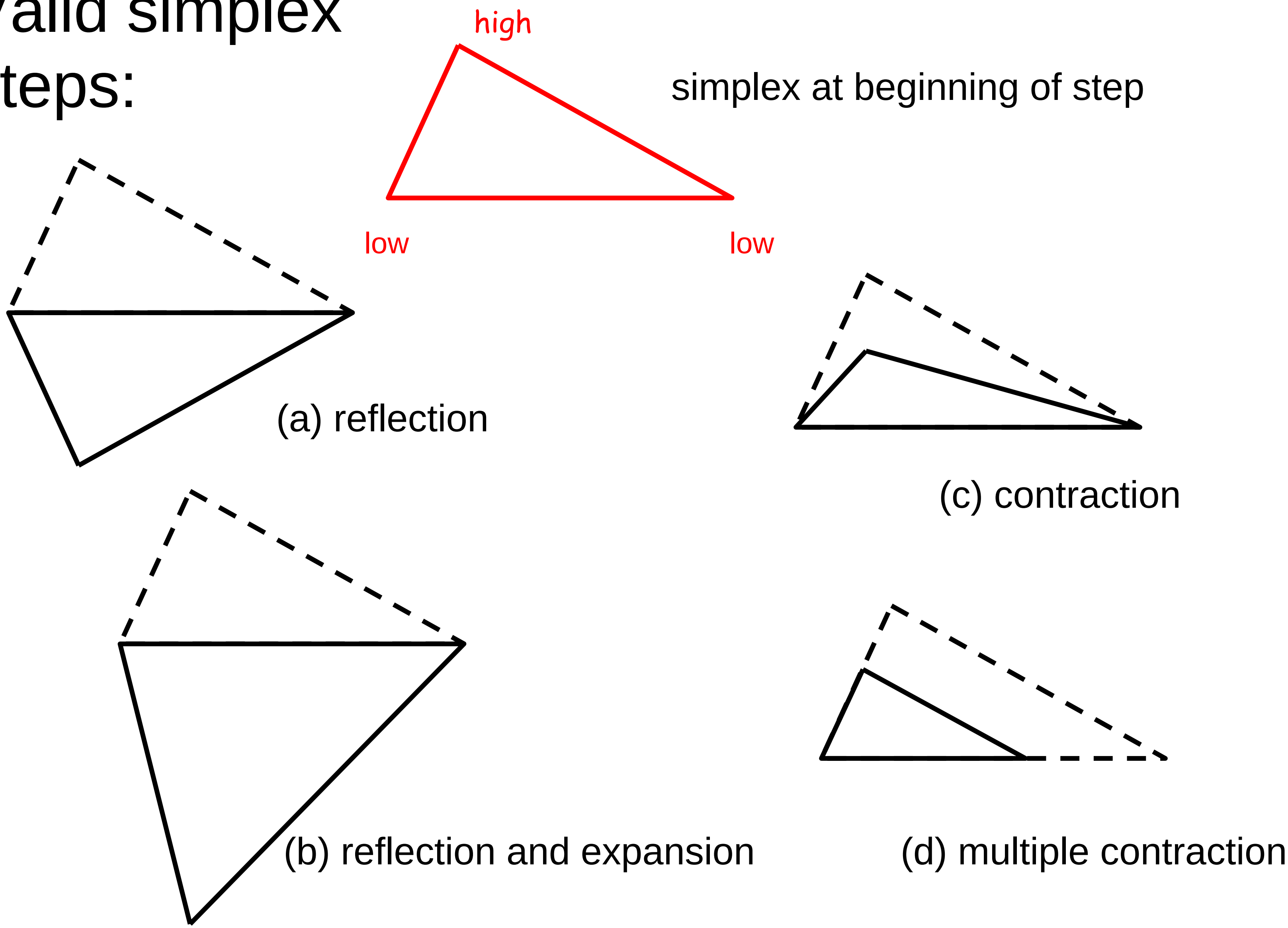
Downhill Simplex Method (2)

- *simplex*: geometrical figure; in N dimensions, (N+1) points/vertices
  - e.g.: in 2D: triangle, in 3D: tetrahedron
  - non-degenerate! (encloses a finite N-dimensional volume)
- starting guess ((N+1) points)
  - $\mathbf{P_0}$ and $\mathbf{P_i} = \mathbf{P_0} + \lambda \mathbf{e_i}$ $\mathbf{e_i}$: unit vectors;

    λ: constant, guess of characteristic length scale


animation of progress of sequential simplex
k
1.2
1
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
A_inf

Valid simplex steps:



Downhill Simplex Method (3)

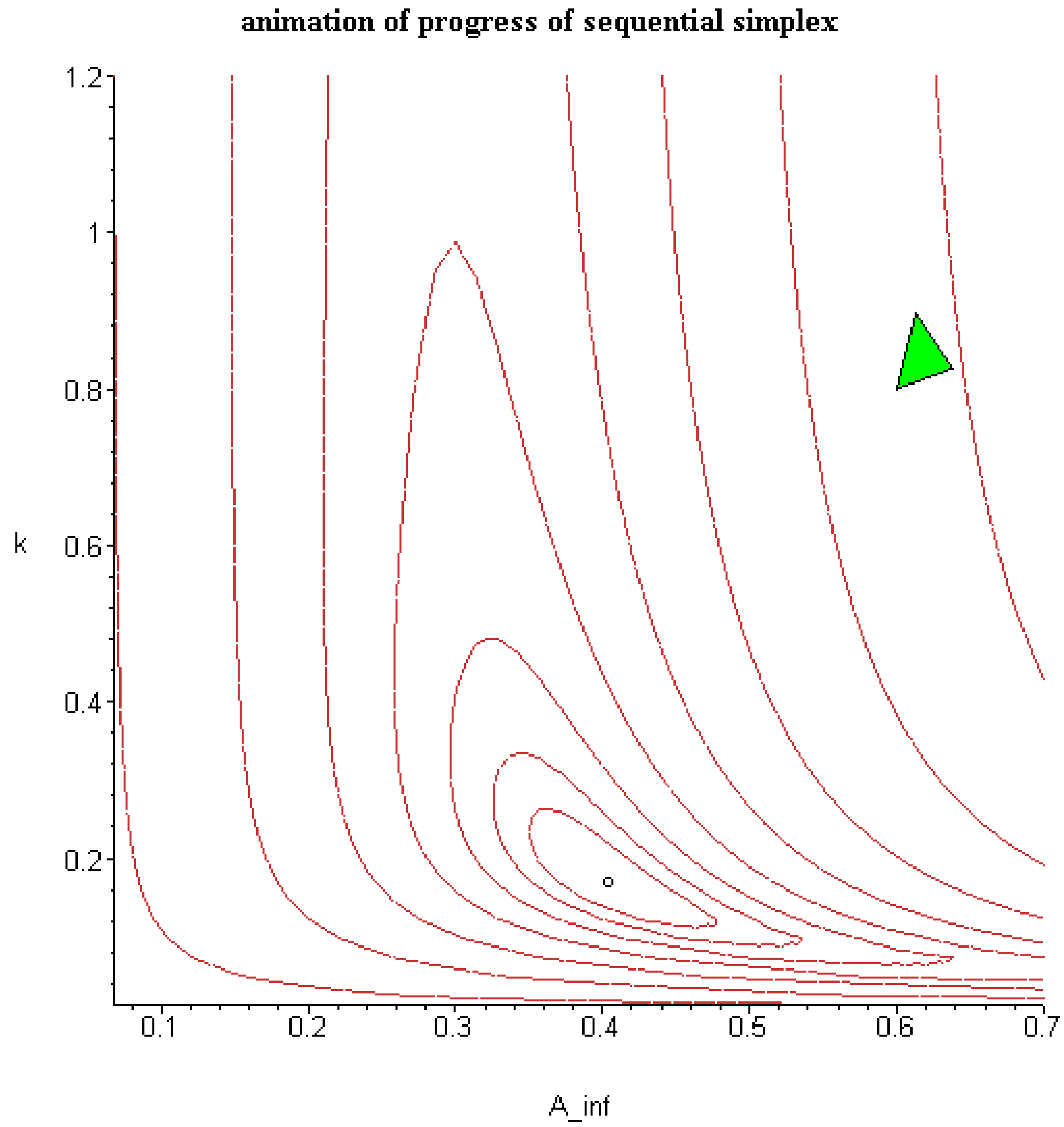
- possible moves (from previous figure):
  - reflection (conserving volume of the simplex)
  - reflection and expansion
  - contraction
  - multiple contraction
- termination criterion
  - use threshold on moved vector distance
  - or threshold on function value change
- *restart* strategy
  - needed as even a single anomalous step can fool the search algorithm

Implementation details

- *fminsearch* in MATLAB

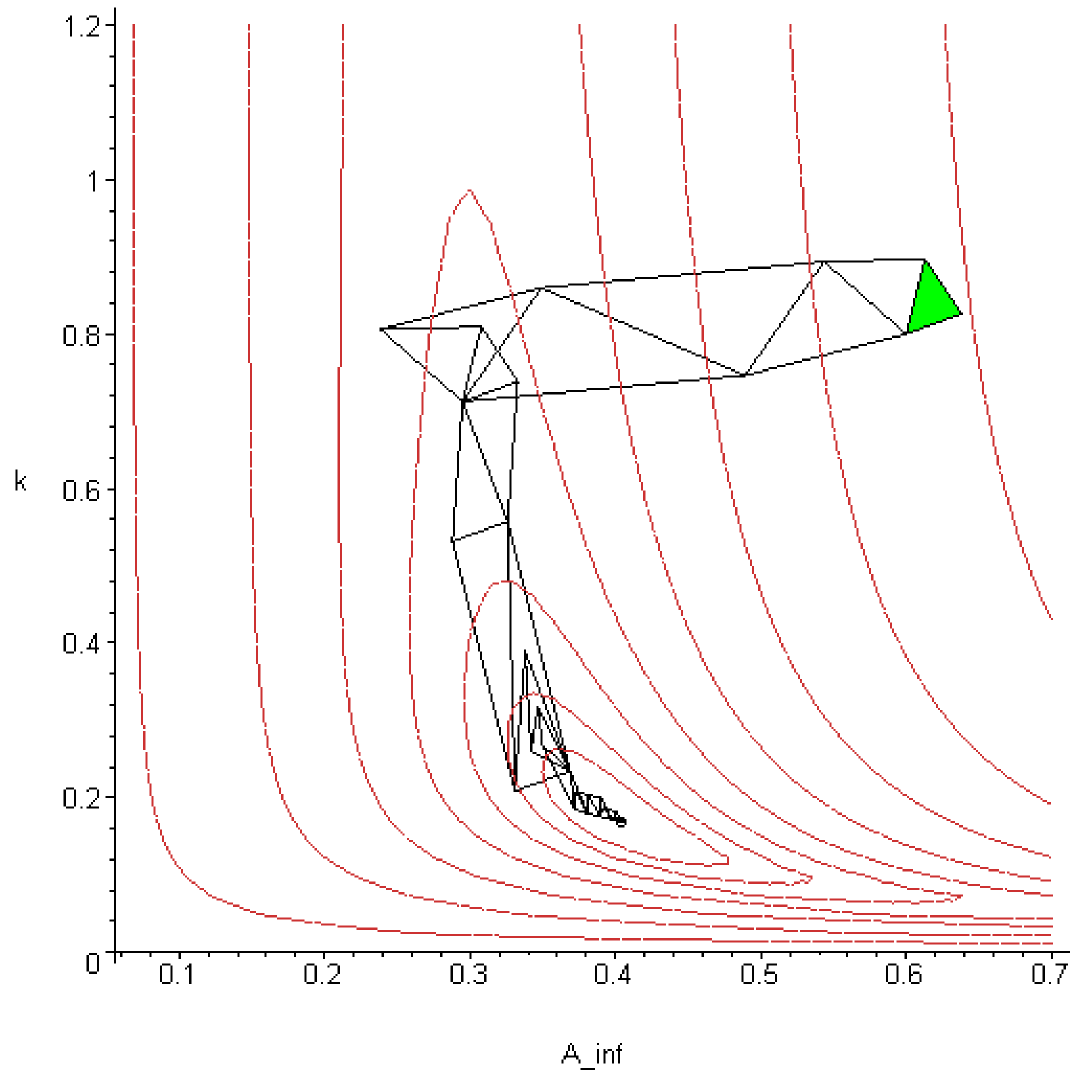
1. build initial simplex
2. do reflections, expand if appropriate
3. in "'valley floor" contract transverse
   - ooze down valley

- works well in some medical registration methods
- has implicit *coarse-to-fine* behavior



Courtesy of E. G. Romero-Blanco and J. F. Ogilvie. Used with permission.

sequential simplex track

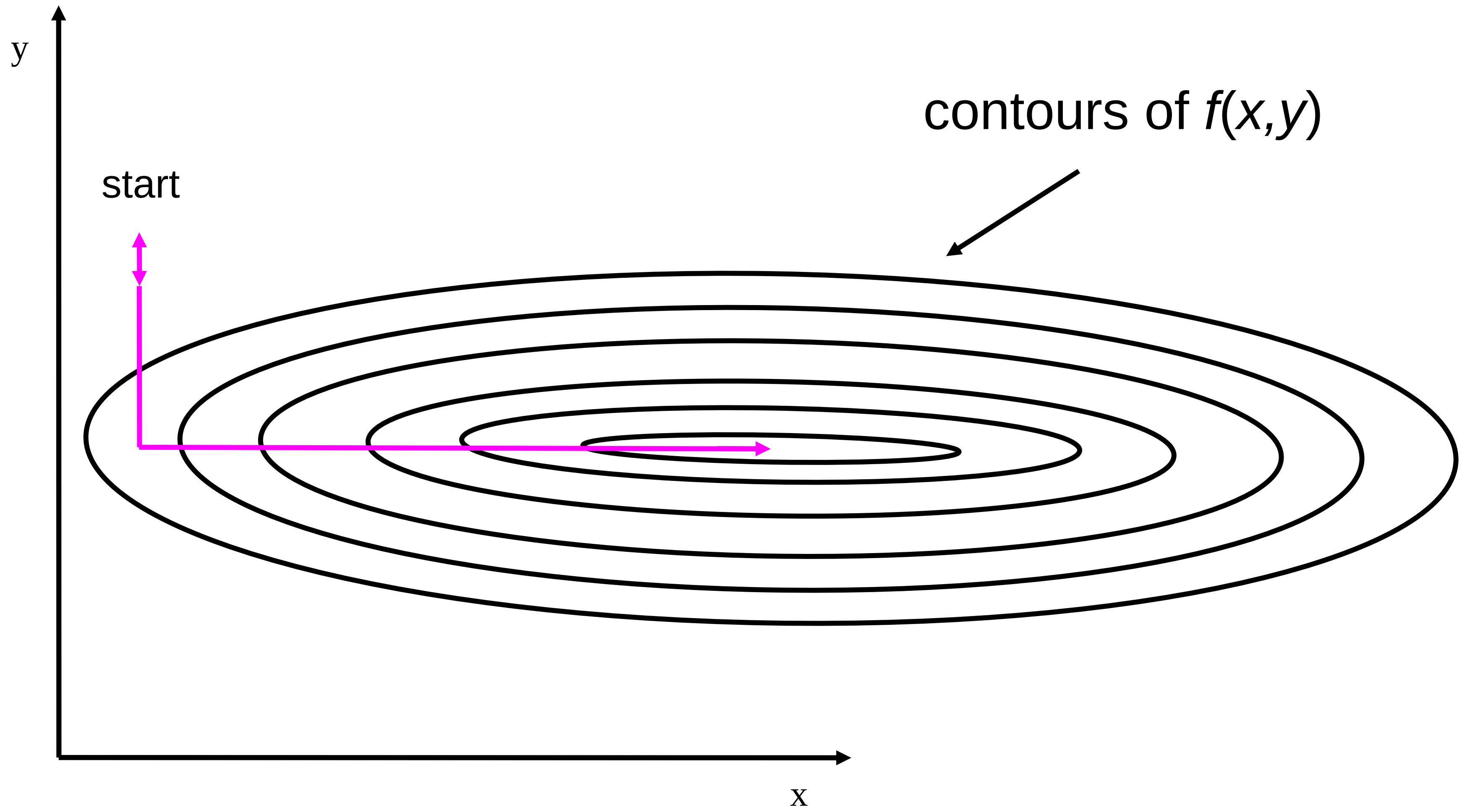


Courtesy of E. G. Romero-Blanco and J.F. Ogilvie. Used with permission.

Direction Set Methods

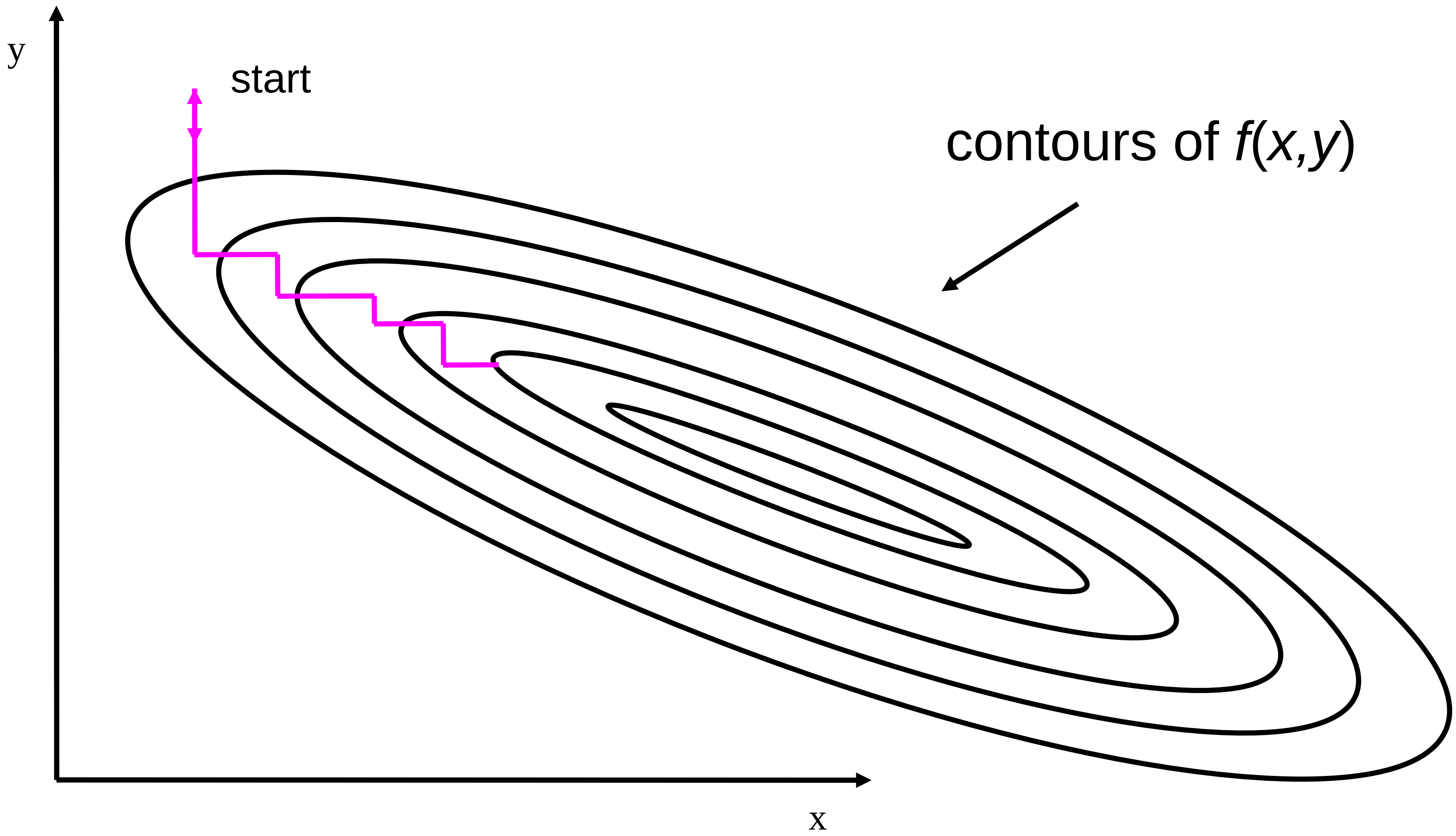
- Successive line minimizations
- No explicit gradient calculation
- How to select the best set of directions to follow?
  - simple example: follow the coordinate directions
  - direction set methods: compute “good” or non-interfering (conjugate) directions

Follow the coordinate directions



Ideal situation: only two steps are enough to locate the minimum

Follow the coordinate directions



In general: can be very inefficient; large number of steps can be required to find the minimum.

Conjugate Directions

- non-interfering directions: subsequent minimizations should not spoil previous optimization results
- *goal*: come up with a set of N linearly independent, mutually conjugate directions

  $\Rightarrow$ N line minimizations will achieve the minimum of a quadratic form

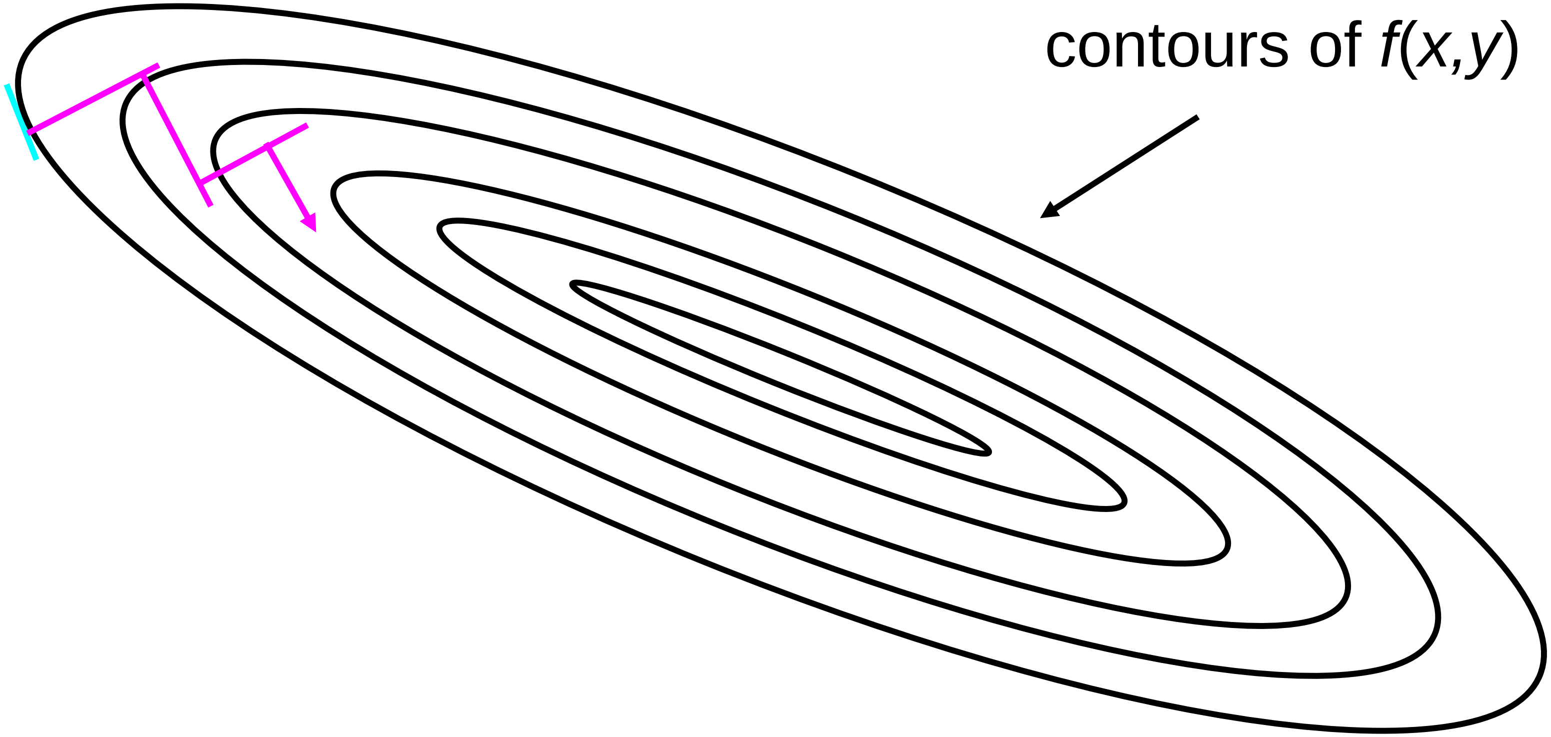
Powell's Method

- N(N+1) line minimizations to achieve the minimum
- possible problem with linear dependence after update
  - fix
    - re-initialize the set of directions to the basis vectors
    - few good directions (instead of N conjugate ones)

Conjugate Gradient Methods

- gradient calculation is needed
- order N separate line minimizations
- computational speed improvement
  - Steepest Descent method
    - right angle turns at all times
  - Conjugate Direction methods

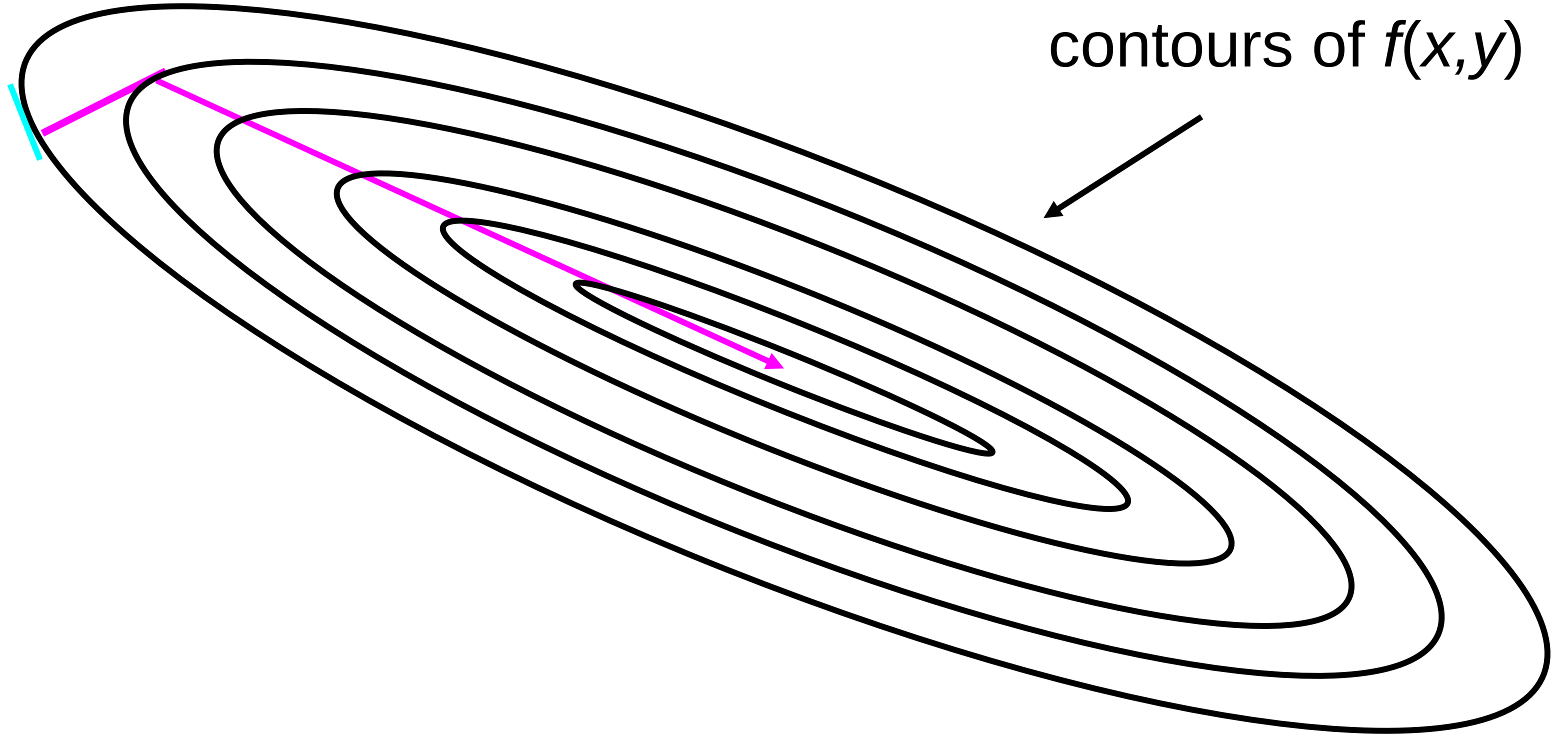
Steepest Descent method



Steepest descent method – still a large number of steps is required to find the minimum.

See http://www.tcm.phy.cam.ac.uk/~pdh1001/thesis/node57.html

Conjugate Gradients method



Conjugate gradients method - only two steps are required to find the minimum.

See http://www.tcm.phy.cam.ac.uk/~pdh1001/thesis/node57.html

Simulated Annealing

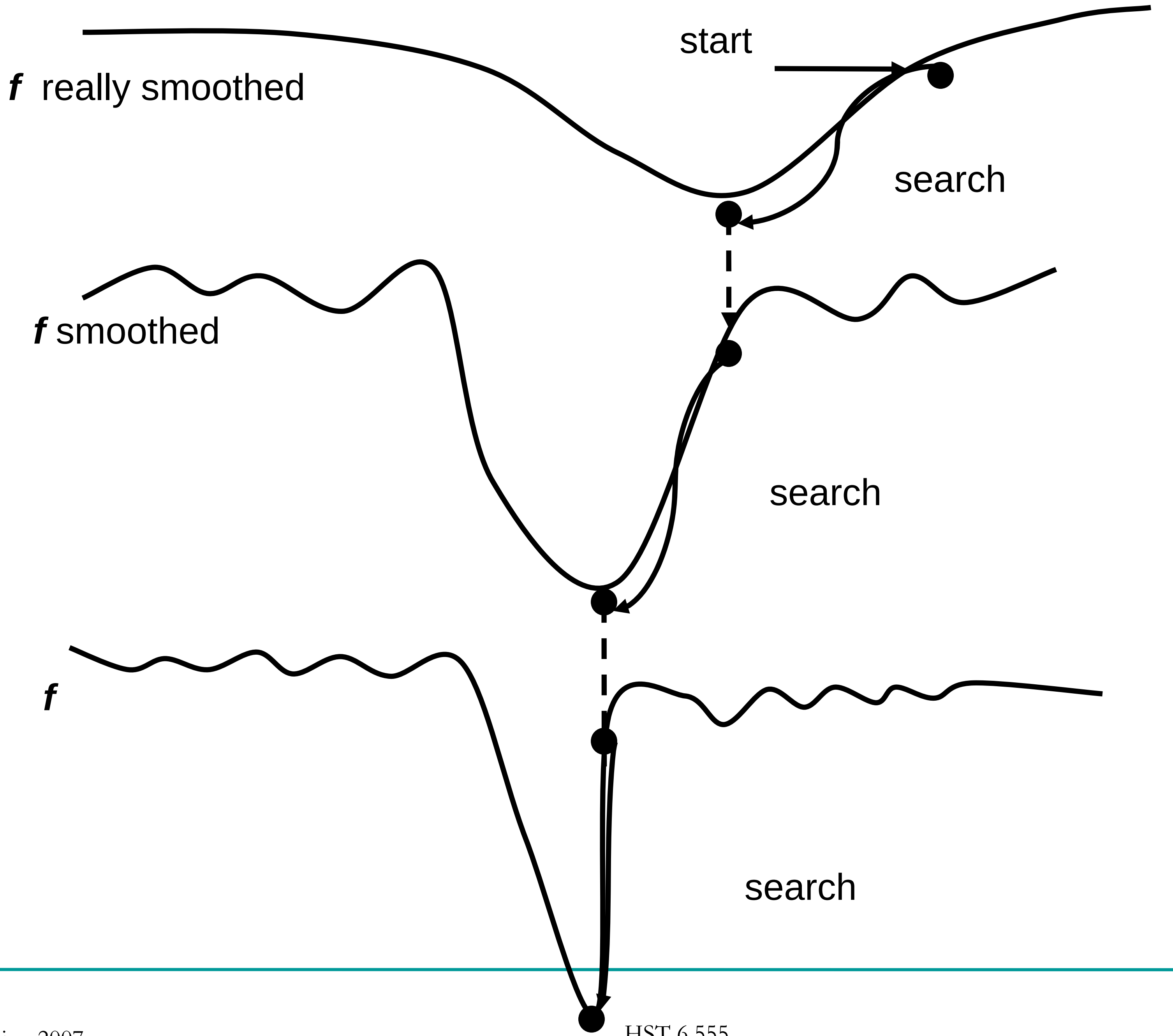
- exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the *annealing* process) and the search for a minimum in a more general system
- employs a random search accepting (with a given probability) both changes that decrease and increase the objective function
- successful at finding global optima among a large numbers of undesired local extrema

Genetic Algorithm

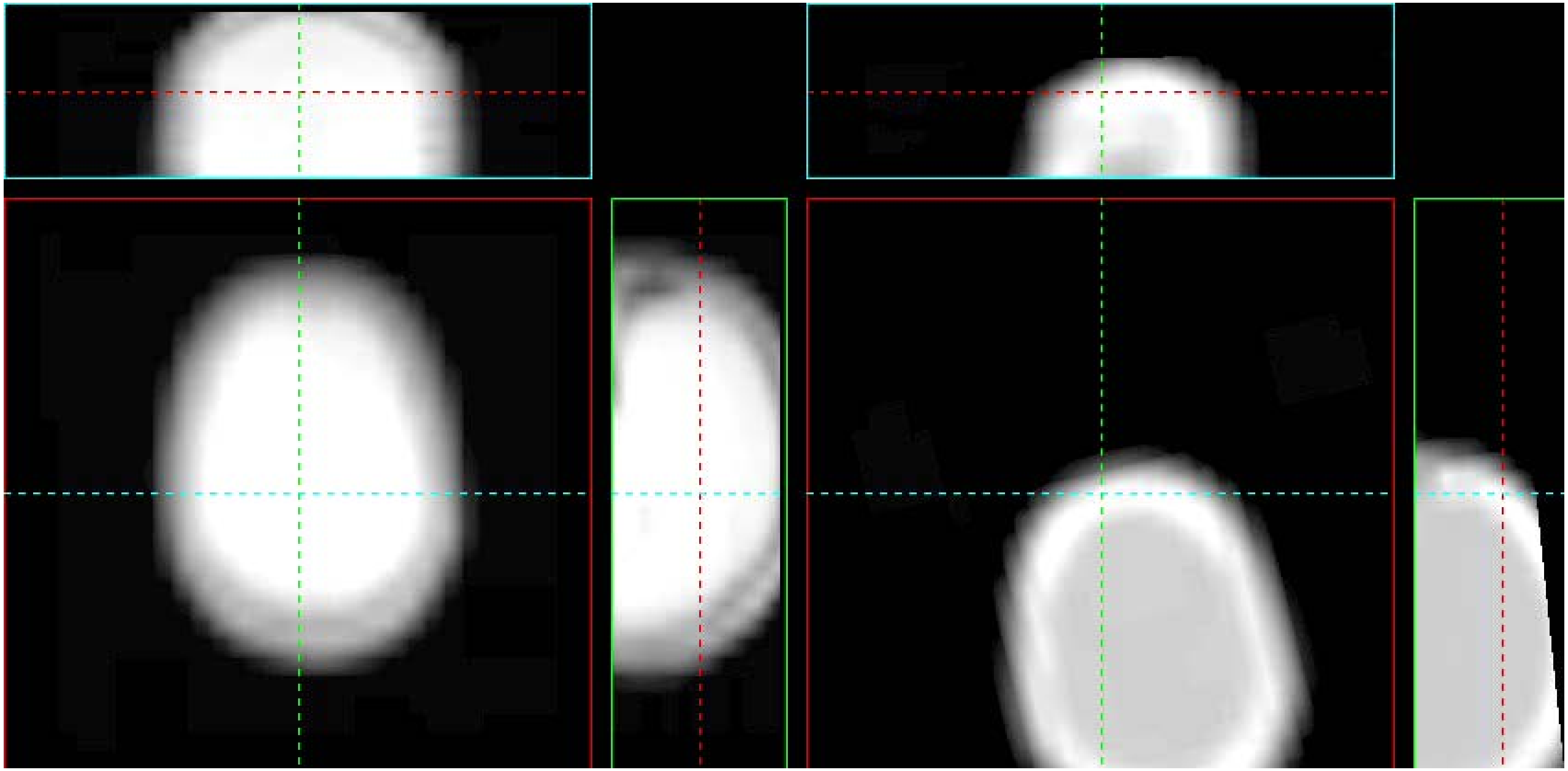
- works very well on mixed (continuous *and* discrete), combinatorial problems; less susceptible to getting 'stuck' at local optima than gradient search methods
- tends to be computationally expensive
- represents solution to the problem as a *genome* (or *chromosome*); creates a population of solutions and apply genetic operators (mutation, crossover) to evolve the solutions in order to find the best one(s).
- most important aspects of using genetic algorithms are
  - (1) definition of the objective function
  - (2) definition and implementation of the genetic representation
  - (3) definition and implementation of the genetic operators
- http://lancet.mit.edu/~mbwall/presentations/IntroToGAs/

Coarse-to-Fine Strategy

- Technique:
  - smooth objective function f_N (e.g.: blur with Gaussian)
  - optimize smoothed version (use result as start value for original objective f_N)
- Advantages:
  - avoiding local extrema
  - speed up computations


start
f really smoothed
search
f smoothed
search
f
search

CT-MR registration movie



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More examples

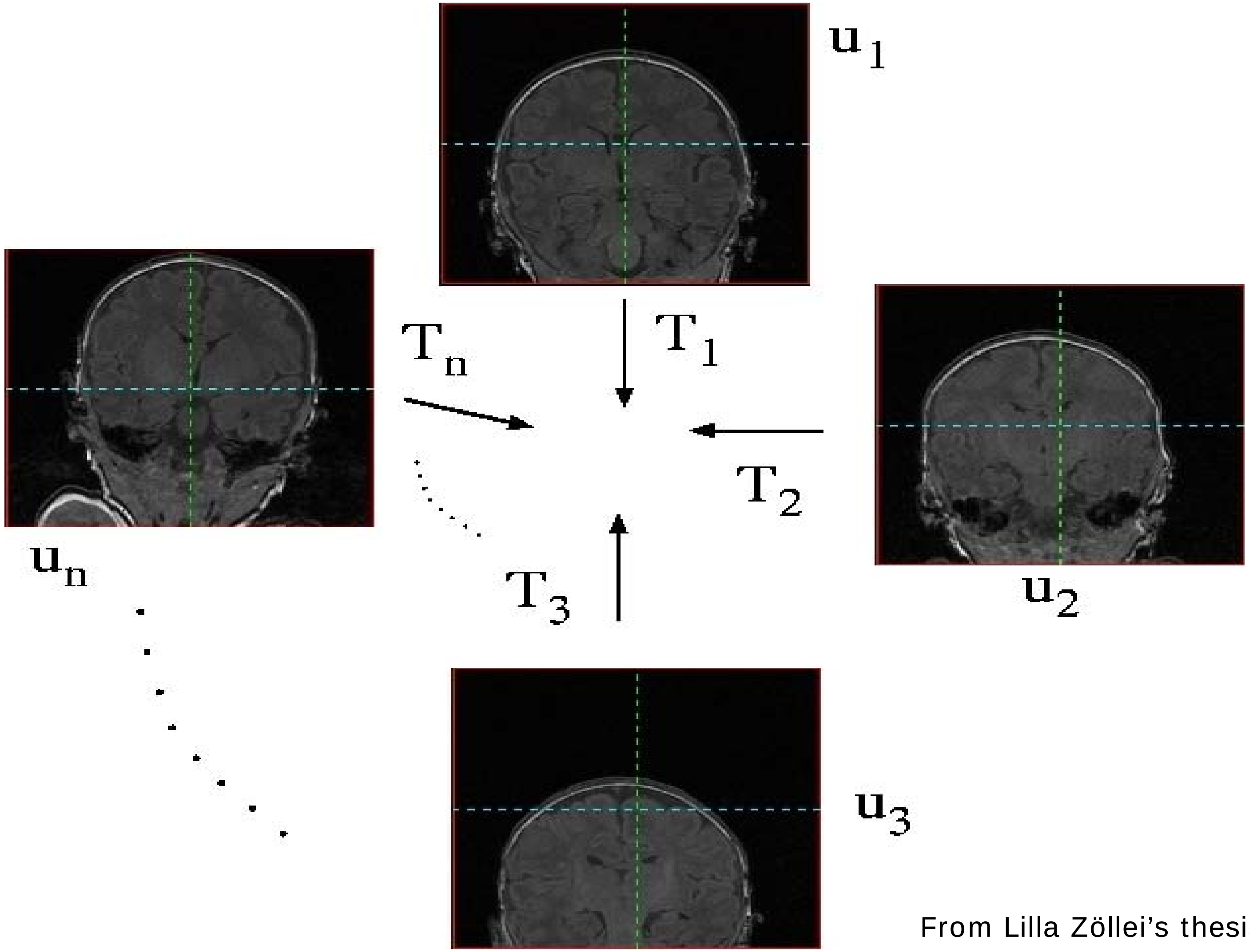
- ***Numerical Recipes in "C"***
  http://www.nrbook.com/nr3/

Current Research topics

- group-wise (vs. pair-wise) registration
- Diffusion Tensor (DT) MRI alignment
- surface-based (vs volumetric) alignment

- Open questions: tumor growth modeling, structural – functional alignment, ….
- Registration evaluation and validation

Group-wise registration

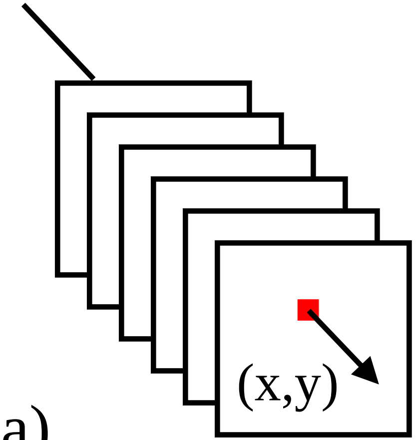


From Lilla Zöllei's thesis research.

Group-wise registration styles

- Template-dependent
  - Fixed template
    - Arbitrary member of the population
    - Pre-defined atlas
  - Online computed template
    - Sequential pair-wise alignment to evolving "mean"
- Template-free
  - Simultaneous

The Congealing method

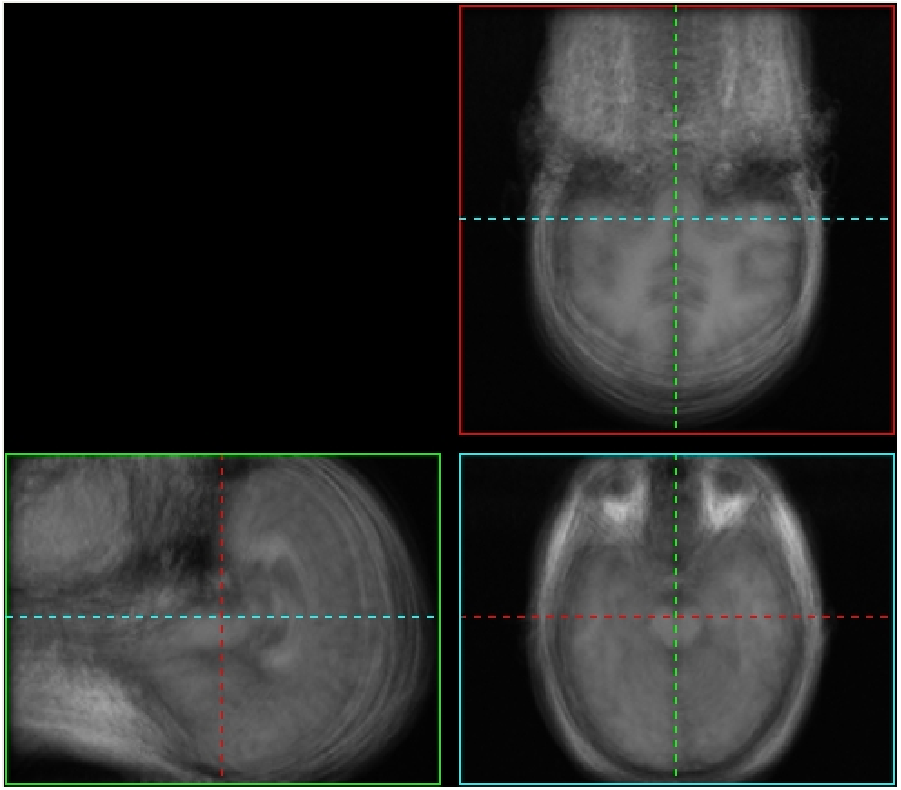


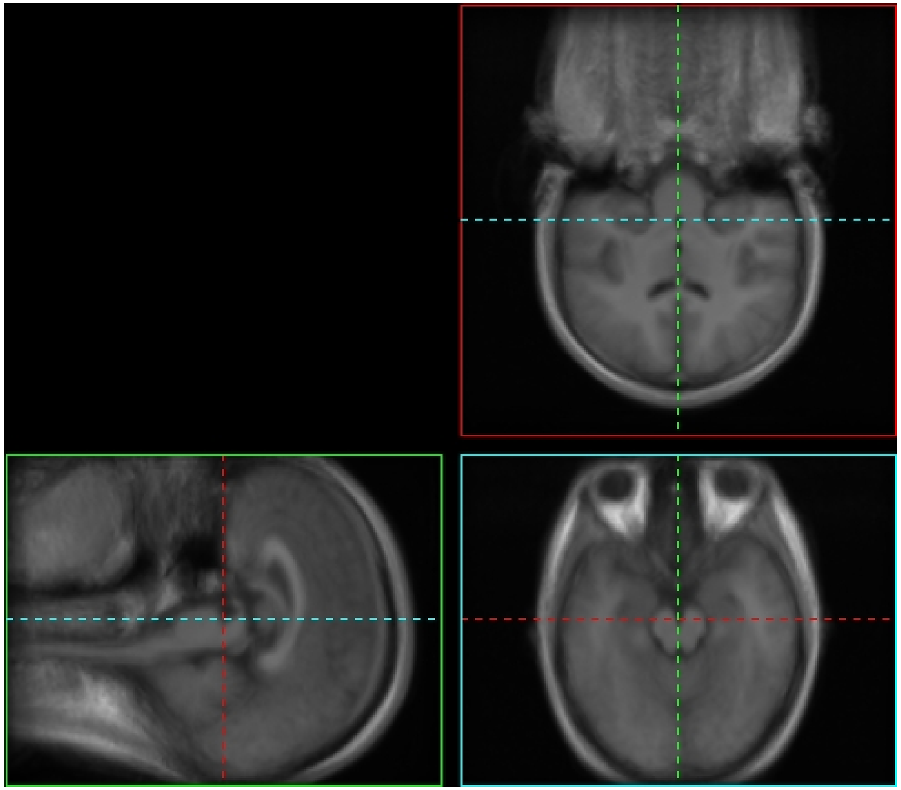
- *Def.*: simultaneous alignment of each of a set of images to each other
- Applications:
  - Handwritten digit recognition (binary data)
  - Preliminary baby brain registration (binary data)
  - Bias removal from MRI images

Advantages of Congealing

- Computational advantages
- Can accommodate very large data sets
- Can accommodate multi-modal data
- Robust to noise and imaging artifacts
- No single central tendency assumption

Adult brain data set - mean volumes





Unaligned input data sets

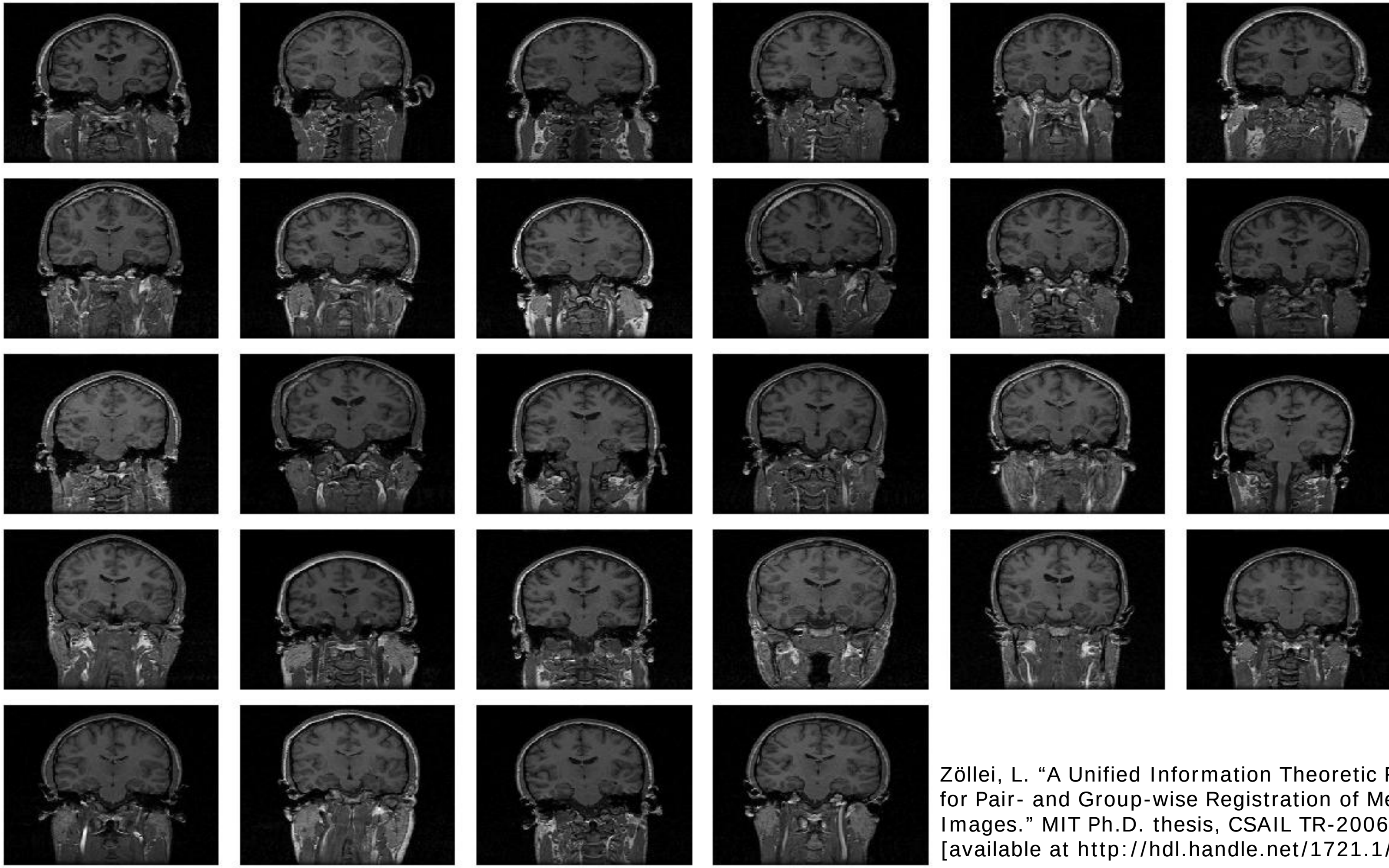
Aligned input data sets

Data set: 28 T1-weighted MRI; [256x256x124] with (.9375, .9375, 1.5) mm^3 voxels

Experiment: 2 levels; 12-param. affine; N = 2500; iter = 150; time = **1209** sec!!

Zöllei, L. "A Unified Information Theoretic Framework for Pair- and Group-wise Registration of Medical Images." MIT Ph.D. thesis, CSAIL TR-2006-005 [available at http://hdl.handle.net/1721.1/30970].

Central coronal slices



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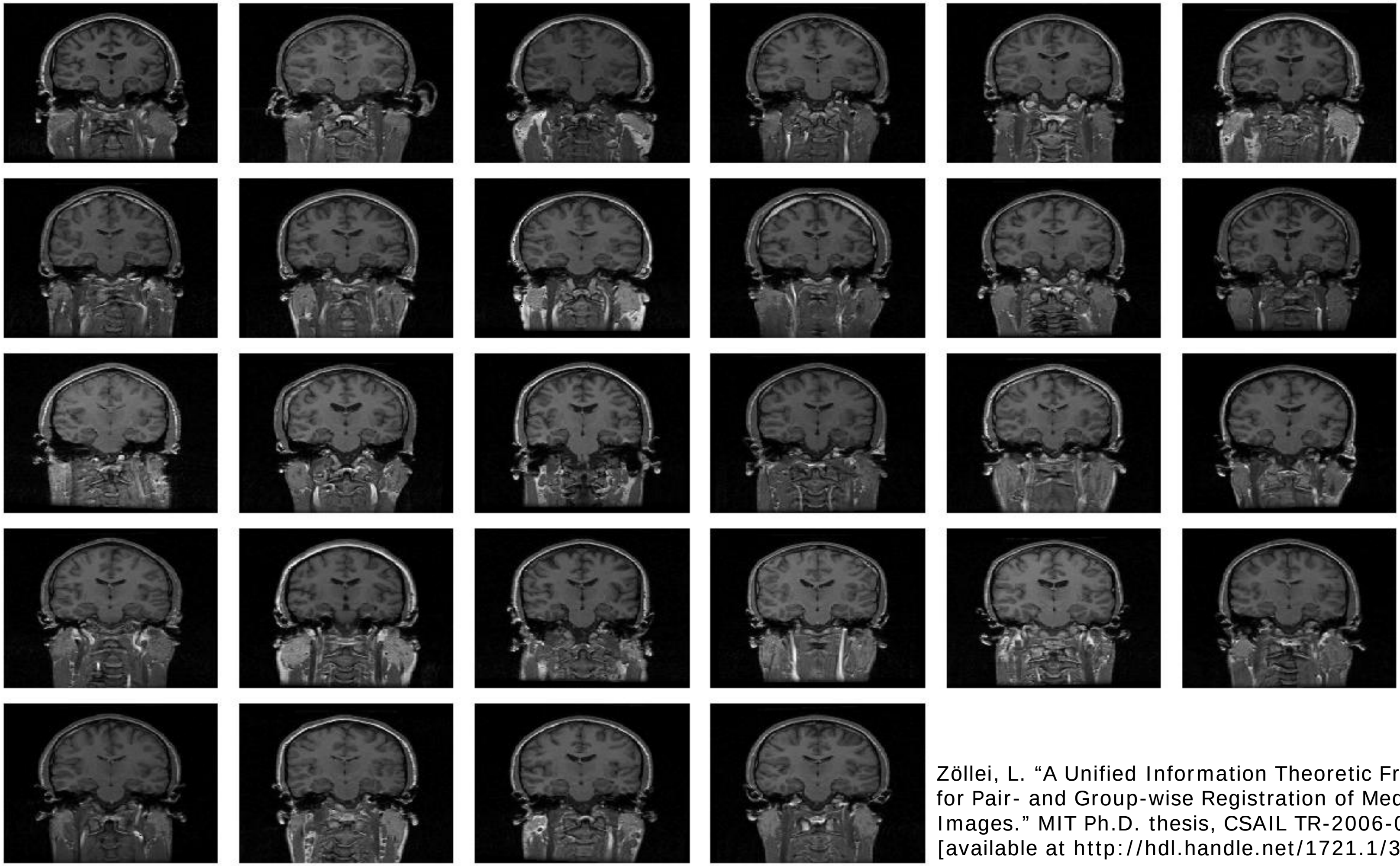
Unaligned input

Central coronal slices



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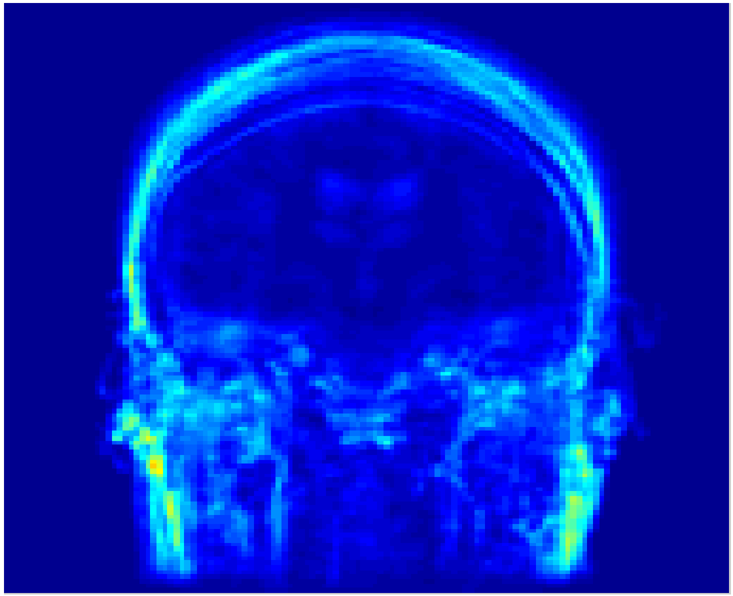
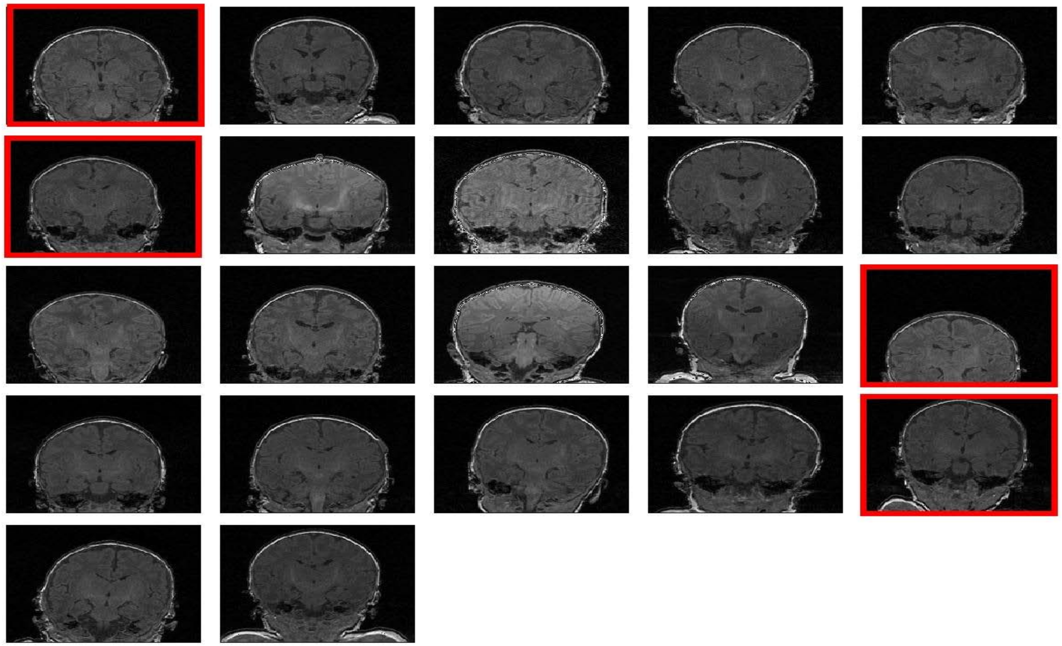
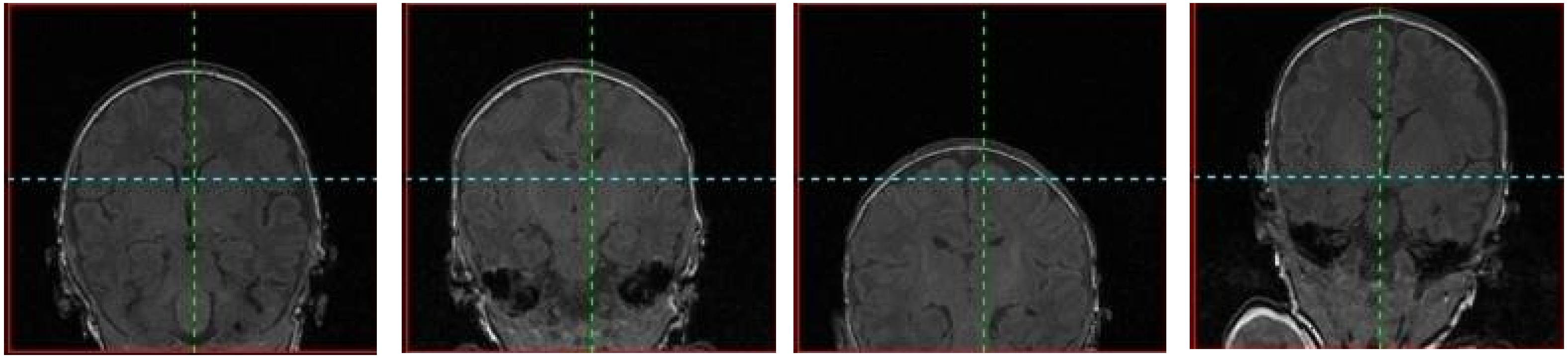
Aligned input

Variance volume - during registration



Baby brain data set – central slices





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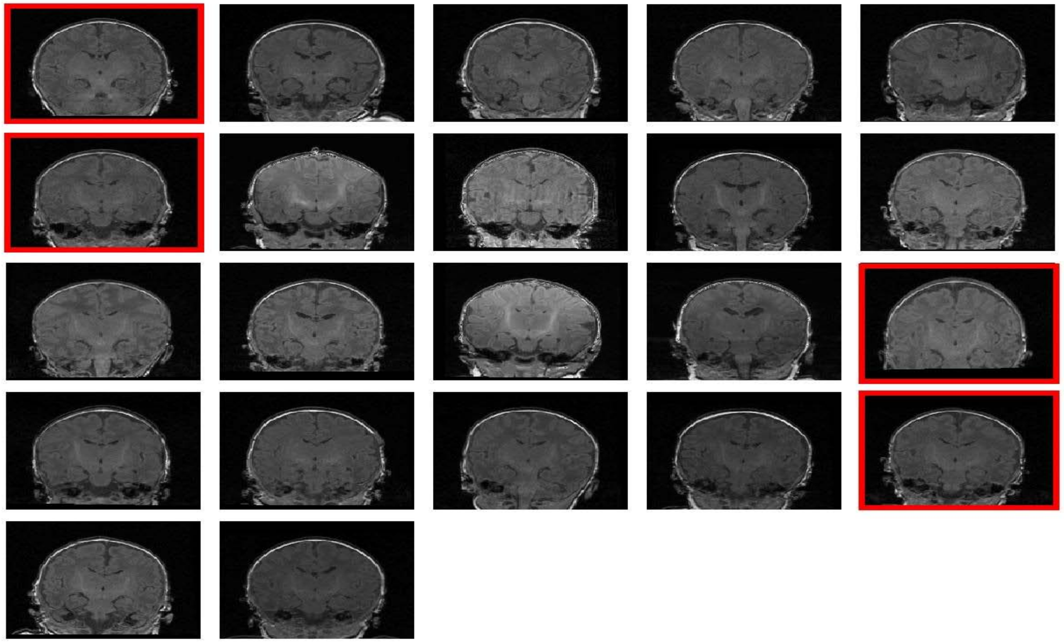

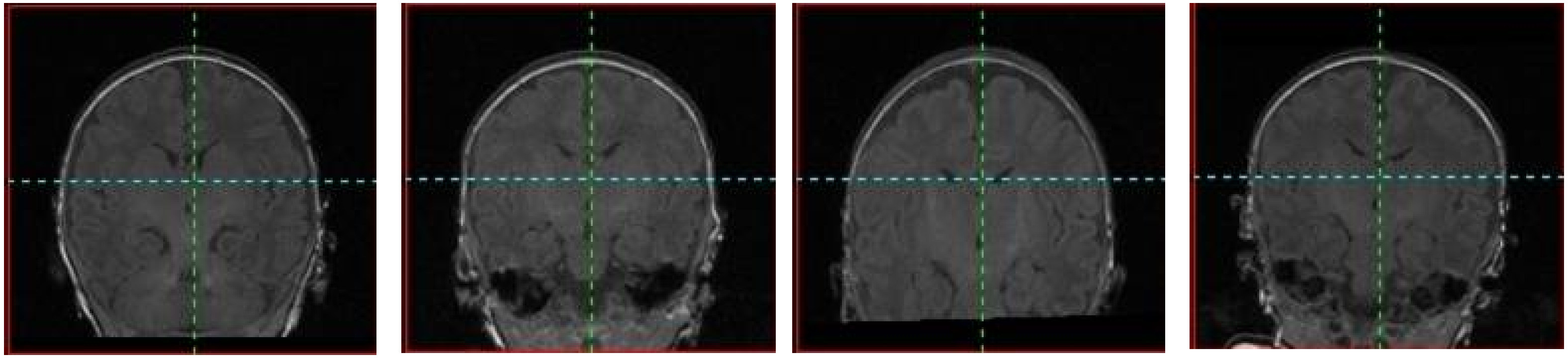

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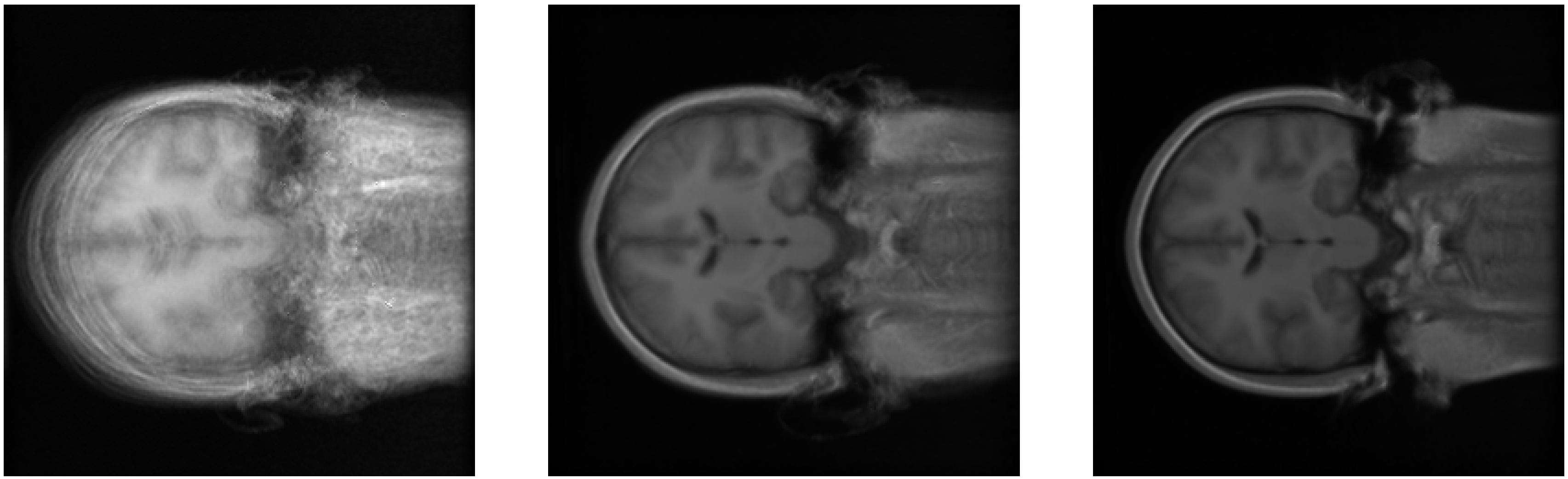
Very large data set

Zöllei, L. "A Unified Information Theoretic Framework for Pair- and Group-wise Registration of Medical Images." MIT Ph.D. thesis, CSAIL TR-2006-005 [available at http://hdl.handle.net/1721.1/30970].

Affine + B-splines Deformation



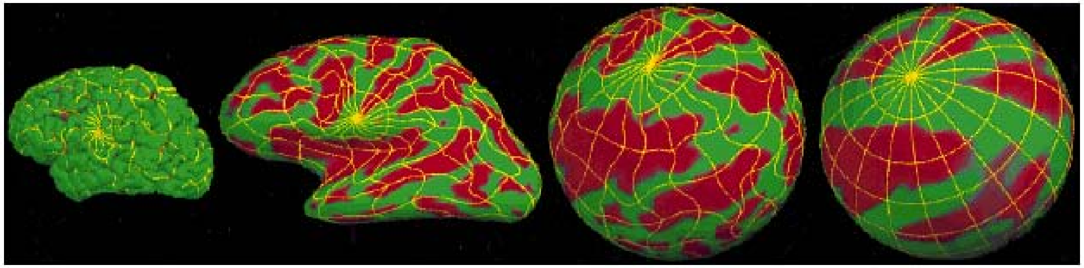
Courtesy of Serdar K Balci and Kinh Tieu. Used with permission.

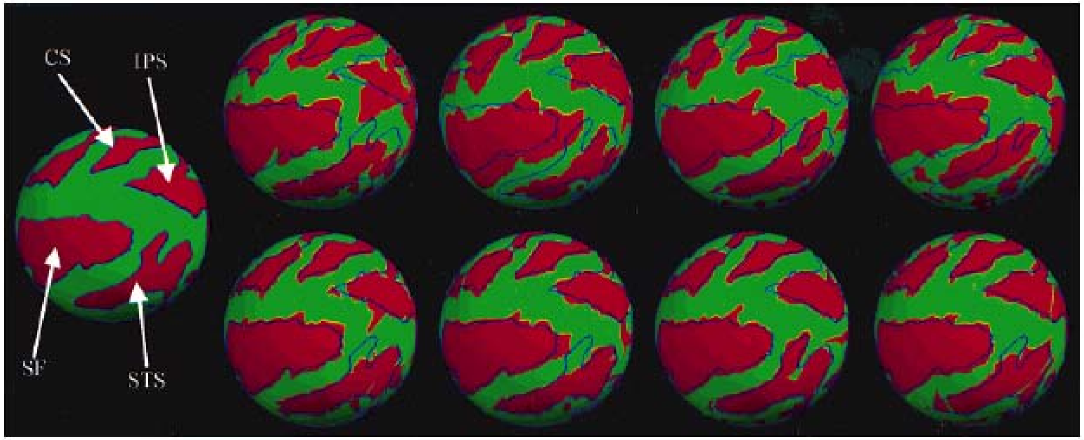
DT MRI alignment

Figure 2 from this article (sequence of eight images) removed due to copyright restrictions.

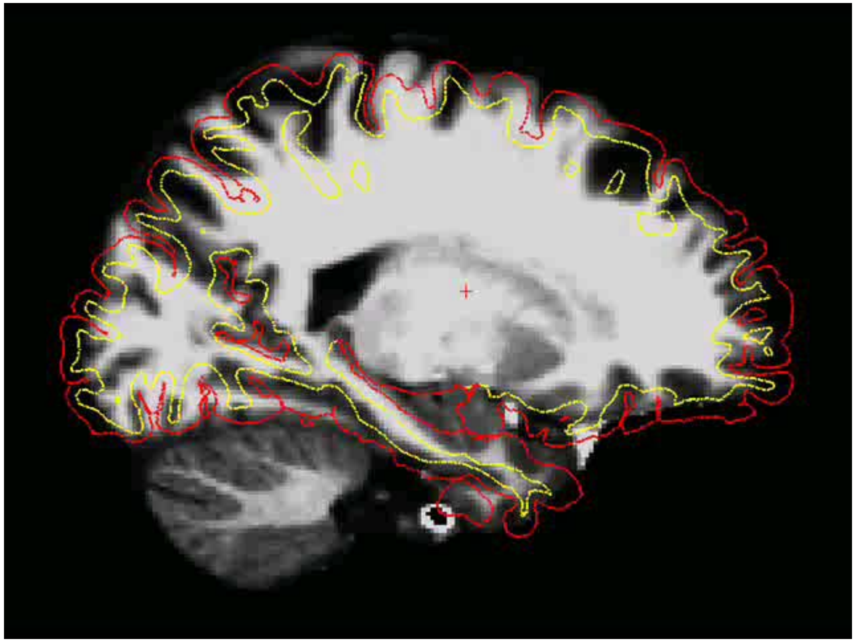
O'Donnell, L. J., et al.: *High-Dimensional White Matter Atlas Generation and Group Analysis* MICCAI, 243-251, 2006.

Surface-based alignment





Combined surface-based and volumetric alignment



Images removed due to copyright restrictions.
Two brain MRI images from Fig 3 in Postelnicu, Gheorghe, Lilla Zollei, Rahul Desikan, and Bruce Fischl. "Geometry Driven Volumetric Registration." *LNCS* 4584 (2007): 243–251.

Courtesy of Gheorghe Postelnicu. Used with permission.

Further open questions:

- tumor growth modeling
- structural – functional alignment (MRI-*f*MRI)
- population comparison
- ….

Registration evaluation and validation

- Retrospective Image Registration Evaluation Project (Vanderbilt University, Nashville, TN)
  http://www.vuse.vanderbilt.edu/~image/registration/

- Non-Rigid Image Registration Evaluation Program (NIREP); University of Iowa

  http://www.nirep.org

END