HST.583 Functional Magnetic Resonance Imaging: Data Acquisition and Analysis
Fall 2008

HST.583
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Problem Set #2

Problem 1: Understanding K-space

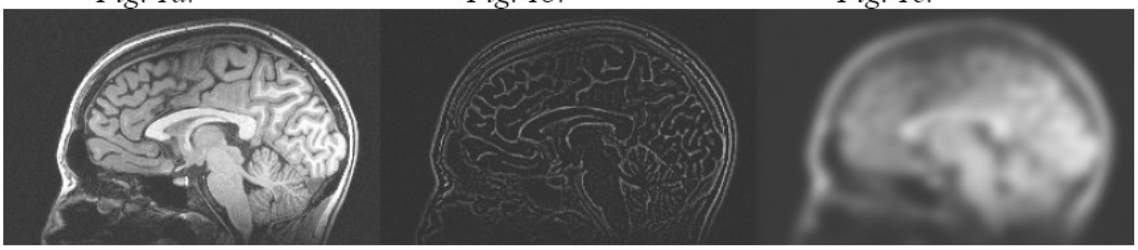
Describe what you would see AND explain why when you look at a T1 image of the mid-sagittal slice through the brain after:

a) discarding the LOW frequency k-space information
b) discarding the HIGH frequency k-space information

You may include images, but a verbal description is required.

a) After discarding the low frequency k-space information, the resulting image would have the same detail of the original image, but would have a lower signal level. This is because the high frequency region which is retained encodes the finer details of the signal, but the low frequency region which is discarded is responsible for the higher signal-to-noise ratio of the image.
b) After discarding the high frequency k-space information, the resulting image will have a similar signal to the original image, but will have a lower spatial resolution due to the loss of the high frequency information which encodes the finer details of the image.

Fig. 1a. *Fig. 1b.* *Fig. 1c.*



Problem 2: Factors that affect the spin level populations

The difference in level population in a two level system is denoted by $\Delta n = N_\alpha - N_\beta$, where N is the number of spins and the levels are denoted by α or β. The time rate of change of Δn is given by $d(\Delta n)/dt = -(\Delta n - \Delta n_0)/T1$, where T1 is the spin-lattice relaxation time and Δn_0 is the population difference at equilibrium.

a) Find the equation for the population difference at some time, t, after placing the sample in the magnet?
b) How long does it take for the distribution to get to 90% of the Boltzmann distribution if the T1 is 800 ms?

We want $\Delta n(t)$ and we have $d(\Delta n)/dt = -(\Delta n - \Delta n_0)/T1$.

Rearranging,

$d(\Delta n) / (\Delta n - \Delta n_0) = -dt/T1$

Integrating both sides and recalling that integral(dx/x) = ln(x),

$\ln(\Delta n - \Delta n_0) = -t/T1 + C$, where C is a constant arising from indefinite integration limits. Since C is constant, we can set t=0 and arrive at

$C = \ln(\Delta n(0) - \Delta n_0)$

$\ln(\Delta n - \Delta n_0) - \ln(\Delta n(0) - \Delta n_0) = -t/T1$

Recall that $\ln(x) - \ln(y) = \ln(x/y)$

$\ln[(\Delta n - \Delta n_0) / (\Delta n(0) - \Delta n_0)] = -t/T1$

Raise both sides by e and recall that $e^{\ln(x)} = x$

$(\Delta n - {}_{\Delta} n0) / (\Delta n(0) - \Delta n_0) = e^{-t/T1}$

$\Delta n(0)$ is the spin level difference of the sample immediately after being placed in the magnet, and cannot change instantaneously. Therefore, $\Delta n(0) = 0$, the value it had before being placed in the magnet.

$\Delta n(t) = -\Delta n_0 e^{-t/T1} + \Delta n_0 = \Delta n_0(1 - e^{-t/T1})$

b) We want $\Delta n(t) = 0.9\Delta n_0$

$0.9\Delta n_0 = \Delta n_0(1-e^{-t/T1})$

$0.1 = e^{-t/T1}$

Take the natural log of both sides and recall that $\ln(e^x) = x$

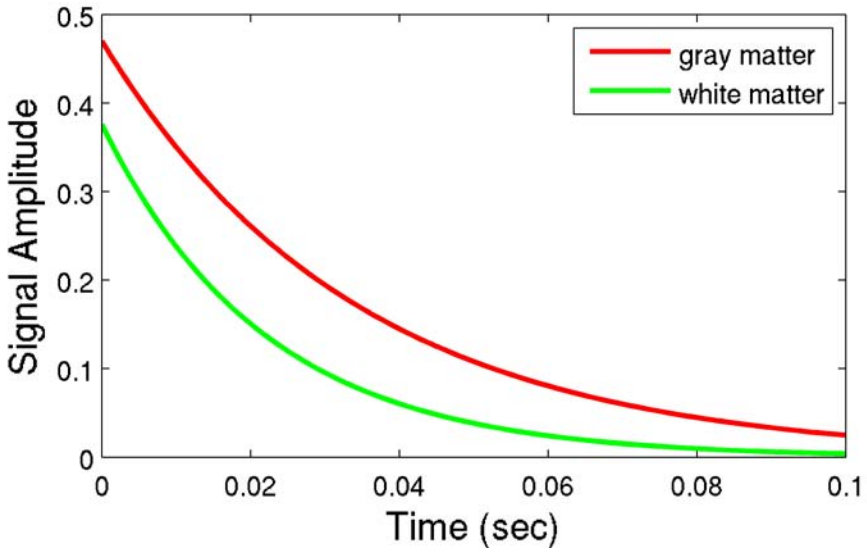
$t = -T1*\ln(0.1) = 1.84$ sec

Problem 3: Playing with T1 and T2 contrast

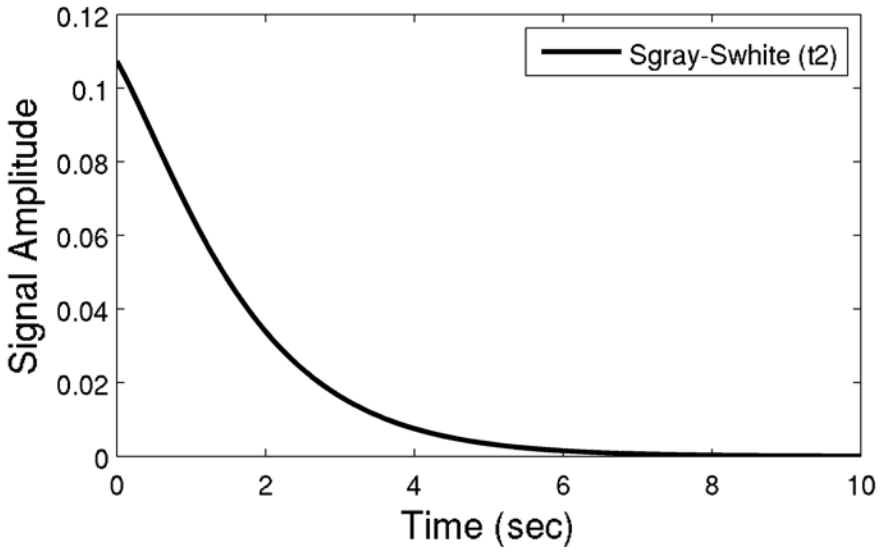
Using the table of tissue parameters below, we will investigate how the choice of pulse sequence parameters can change the contrast in the image. We will assume that the pulse sequence is the following: [$\pi/2$ – t1 - $\pi/2$ – (t1+t2) - $\pi/2$ – t1 – acquisition], a stimulated echo, and make use of the relationship: $S = (\rho/2)(\exp^{-(t1+t2)/T1})\exp^{-2t1/T2}$, where S is the MR signal strength, ρ is the proton spin density, t1=TE/2 and t1+t2=TM.

Tissue	T1	T2	ρ
Gray matter	1.2 s	70 ms	.98
White matter	800 ms	45 ms	.80

a) Plot the signal equation versus t1 for both white matter and gray matter assuming t2 = 50 ms.



b) In another figure, plot the signal difference between white matter and gray matter as t2 is changed. Assume t1 = 25 ms.



Problem 3: Calculation of Imaging Gradients

Assuming a 10 cm field of view, a 10 ms phase encoding gradient, and 8 phase encoding steps, what are the phase angles for each phase encoding step?

$\theta_y = \gamma \Delta y G_y \tau$

G_y: $FOV_y = 1/\gamma \; G_y \tau \rightarrow G_y = 1/\gamma \; FOV_y \tau$

$\theta_y = \gamma \Delta y \tau \; (1/\gamma \; FOV_y \tau)$

$\theta_y = \Delta y / FOV_y$

θ_y = 1 cm / 10 cm = .1 rad

Problem 4: Human safety in MRI

The goal of this problem is to make you aware of human safety issues when using fast brain imaging sequences. Consider the following two pulse sequences: single shot gradient-echo EPI (i.e., one 90° RF excitation pulse followed by a train of gradient recalled echoes), and a single shot spin-echo sequence (i.e., one 90° RF excitation pulse followed by a train of 180° RF refocusing pulses that form spin-echoes).

a) Find an expression for the specific absorption rate (SAR) for both sequences. **Hint:** make a simple model that considers the RF power deposited for a single RF pulse and weight it by the duty cycle for the train of RF pulses. The duty cycle is the amount of time the RF is turned on per TR, thus it contains information about each RF pulse duration, the number of slices acquired and the number of RF pulses per TR. Does the weight of the patient matters?

b) Based on the expression found in the previous point, compare the two sequences and discuss their limitations in terms of SAR. Which of the two sequences would be likely to induce more peripheral nerve stimulation?

Without consideration of the orientation of the object to be imaged, the SAR will be proportional to: integral($B1^2(t)*dt$) * [duty cycle] / weight(kg), where B1(t) is the applied RF pulse, the duty cycle is the the number of RF pulses per TR (this takes into account the number of slices acquired per TR), and the limits of integration are 0 to TR. Since there is only 1 RF pulse per slice for the gradient echo sequence and several for the spin-echo sequence, the spin-echo sequence will distribute more energy into the subject per unit time given the same TR. For part b), the crucial factor is how fast the gradient fields change, i.e. fast gradient field switching is usually responsible for peripheral nerve stimulation. Therefore, the gradient echo sequence would be most likely to cause PNS.

Problem 5: Considerations in pulse sequence acquisition parameters

Which parameter(s) would you keep the same to ensure that the degree and direction of distortion between two different pulse sequences were the same? Why?

Assuming an EPI sequence and imaging the same volume (or slice), the main factors that affect distortions (where distortions does not include blurry images from only sampling the center of k-space) are bandwidth and echo spacing for the degree of the distortion, and the readout direction for the orientation of the distortion. For example, if we switch the frequency encoding and phase encoding directions, the ghosts you mentioned will be along the orthogonal dimension to what they were in the original sequence.

Problem 6: More on Pulse Sequence Acquisition Parameters

For each of the two pulse sequences shown below:

a) Calculate the Total Acquisition Time
b) Calculate the Voxel Size
c) State whether it is a structural or functional scan (and how you know)
d) State whether it is a 2D or 3D scan

SEQUENCE ONE

Scanner SIEMENS MAGNETOM TrioTim

Routine
Slab group: 1
Orientation: Sagittal
  Phase encode direction: A >> P
  Readout direction: R >> L
Slices per slab: 160
FoV read: 220 mm
FoV phase: 220 mm
Slice thickness: 1.20 mm
TR: 2300 ms
TE: 2.94 ms
Averages: 1
Contrast
Magnetization preparation: Non-sel. IR
TI: 1100 ms
Flip angle: 9 deg
Reconstruction: Magnitude
Measurements: 1
Resolution
Base resolution: 256
Phase resolution: 192
Slice resolution: 256
PAT mode: GRAPPA
Acceleration factor: PE 2
Geometry
Multi-slice mode: Single shot
Series: Interleaved
Sequence
Bandwidth: 240 Hz/Px
Echo spacing: 7 ms
RF spoiling: On

SEQUENCE TWO

Scanner SIEMENS MAGNETOM TrioTim

Routine
Slice group 1
  Slices: 30
  Dist. Factor: 25 %
  Orientation: Transversal
  Phase encode direction: A >> P
  Read encode direction: R >> L
FoV read: 220 mm
FoV phase encode: 220 mm
Slice thickness: 4.0 mm
TR: 2000 ms
TE: 30 ms
Averages: 1
Contrast
MTC: Off
Flip angle: 77 deg
Measurements: 142
Resolution
Base resolution: 64
Phase resolution: 64
PAT mode: None
Matrix Coil Mode: Triple
Geometry
Multi-slice mode: Interleaved
Series: Ascending
Sequence
Bandwidth: 2298 Hz/Px
Free echo spacing: Off
Echo spacing: 0.5 ms
EPI factor: 64
Dummy Scans: 3

Scan1

a). The total imaging time is the number of phase encodes multiplied by the TR time. Since we have GRAPPA (parallel imaging technique) with acceleration factor of 2, the scan time should be almost halved. So we have 192*2.3s/2 = 200.8 sec = 3.68min. Note that the GRAPPA algorithm requires acquiring several ACS (or phase-encoding) lines for proper reconstruction, which add to the scan time. The lab handout does not have this info, so I can't say what would be the precise imaging time, but in reality it should be a bit more than the calculated 3.68min. Further, all the 160 slices are acquired within the one TR, and this is possible since the TR is long and the TE is short.

b). The FOV in x and y is 220mm, and the matrix sizes are 256 and 192 respectively. Therefore, for x and y, we have (220/256)x(220/192) = $0.86 \times 1.15 mm^2$. Since the slice thickness is 1.2mm the overall voxel size is $0.86 \times 1.15 \times 1.2 mm^3 = 1.1868 mm^3$.

c). This is a structural acquisition since the number of measurements is 1.

d). We are acquiring 160 slices, so this is a 3D scan.

Scan 2

a). This is an EPI sequence used for fMRI acquisition. 30 slices with the given in-plane resolution are acquired within the rather long TR period. The total acquisition time would depend on the number of measurement, the TR time and the dummy scans. Therefore, we have (142+3)*2 = 290sec = 4.833min.

b). The slice thickness is 4mm and in plane matrix size is 64x64 over FOV is $220 mm^2$. Therefore the voxel size is $4 \times (220/64) \times (220/64) mm^3 = 47.27 mm^3 = 0.0473 cc$.

c). This is a functional acquisition since the number of measurements is 142 and the TE and TR are long, and the resolution is low.

d). This is a multislice 2D scan.