Harvard-MIT Division of Health Sciences and Technology
HST.584J: Magnetic Resonance Analytic, Biochemical, and Imaging Techniques, Spring 2006
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HST.584 / 22.561 Problem Set #1 Solutions

Marking Scheme: Question 1 – 3.5 points, Question 2 – 3 points, Question 3 – 3.5 points
General Comments: Watch out for radians! Angles should be expressed in radians and γ adjusted between linear and angular units accordingly ($^{i.e.}$ Hz vs rad / s)

1-a) $B_1 = \dfrac{\theta}{T\gamma}$ for $\theta = \pi/2$, T = 1 ms, $\gamma = 2\pi * 43 \times 10^6$ rad/s/T

$B_1 = 5.81x10^{-6} T$

This is 6 orders of magnitude smaller than a typical B_O.

1-b) $B_1 = \dfrac{\mu_O NI}{2r}$ for a short solenoid.
Rearranging, we can calculate I = 0.462 A to produce our desired field. To estimate power, we need to estimate resistance (only a resistive element causes power loss – can't dissipate power through an inductance). Let's pick copper ($\rho = 1.56 \times 10^{-8}\ \Omega$m) with a circular cross-section of 2 mm (maybe a little small for the current we're pushing, but good enough for an order of magnitude estimate).

$R = \dfrac{\rho L}{A}$ where L is the total length of wire (4 turns * circumference = 5.03m)

$R = 0.025\Omega$

$P = I^2 R = 5.33 mW$ for an estimate

1-c) B_{eff} is the vector sum of B_1 and our off-resonance contribution.
Off resonance $= B_O - \omega_{rot}/\gamma = 0.1163 mT$.
So $B_{eff} = 0.1163\,\hat{z} + 5.81x10\text{-}3\,\hat{x}$ mT = 0.116 mT at 2.9° tilt off the z-axis.
$\theta = \gamma B_{eff} T = 31.3 rad$ = 5 rotations around B_{eff} = NOT A 90° pulse!

2-a) $\gamma_e = 2.8 \times 10^{10}$ Hz / T. At 1.5 T, ν = 42.0 GHz. Is this practical? On chemical samples yes – ESR is a common technique. However, this frequency is in the microwave range, so this is not practical for humans → will potentially have a great deal of energy deposition in your subject (this is BAD!)

2-b) In 1.5 T NMR experiment, our Larmor frequency is 64.5 MHz. To generate 64.5 MHz in an ESR experiment, we need a 2.3 mT field ($B = \dfrac{\upsilon}{\gamma}$).

2-c) Using our expression from 1-a, we can calculate that we would a $B_1 = 8.93 \times 10^{-7}$ T to generate the desired RF pulse.

3-a) $\frac{n_{\downarrow}}{n_{\uparrow}} = \exp(-\frac{\gamma Bh}{K_B T})$ is the expression for the difference in spin population levels based on a Boltzmann distribution (again, watch your units for γ and h – if you use one in angular form, they both must in angular form).

Field Strength (T)	**Ratio for ^{1}H**	**Ratio for ^{13}C**
7.0	0.99995	0.99999
3.0	0.99998	0.999995
1.5	0.99999	0.999997

So net magnetization increases with field strength, but we are still dealing with incredibly small signals!

3-b) If we re-arrange our initial expression, to get 1:2 ratio of spins, we need temperatures of 20.6 mK for ^{1}H and 5.2 mK for ^{13}C.

3-c) For ^{1}H, we'd need a field of 9.84×10^{4} T to attain a 1:2 ratio of spins. For ^{13}C, we'd need a field of 3.95×10^{5} T. Clearly, these are not attainable fields in a laboratory setting.