Harvard-MIT Division of Health Sciences and Technology
HST.750: Modeling Issues in Speech and Hearing, Spring 2006
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Problem/Discussion Set for "Forward vs Inverse"

1. **Dimensional Analysis.** The simple passive model whose transfer function is show in Zweig's Fig. 2 is defined by the variables/parameters

$$x,\ \omega,\ \bar{M}(x),\ M(x),\ R(x),\ K(x),\ \text{and}\ l\ .$$

   (a) What are the dimensions of each? How many independent dimensionless combinations (groups) can be formed?

   (b) Identify and interpret the dimensionless groups used by Zweig. For each group, identify its role in determining/shaping the model transfer function shown in Fig. 2 (e.g., what feature(s) of the transfer function or its variation with position does the dimensionless variable/parameter affect)?

   (c) What additional dimensionless variables/parameters does Zweig introduce to model the empirical wavelength? What are their physical interpretations?

   (d) The function $N(x) \equiv (l/4)\sqrt{\bar{M}(x)/M(x)}$ can be interpreted as the approximate number of wavelengths of the traveling wave in the cochlea in response to sinusoidal stimulation at a frequency $f = f_c(x)$. In the Zweig model, $N(x)$ is constant. What empirical evidence supports this? What is the general condition for constant N? Figure 2 shows the predictions of a model with $N = 5$. Draw the approximate phase response of the model with $N = 4$ and with $N = 6$?

   (e) Wobbles or oscillations in the "tails" of Neely's IHC tuning curves are apparent in Fig. 8. Similar wobbles are also seen in Figs. 6, 7, and 9. Speculate on the physical origin of these wobbles. Do the wobbles in these four figures suggest anything suspicious about Neely's model calculations?

2. **Wavelengths.** Plot (e.g., using Matlab) the real and imaginary parts of $(4N\lambda\!\!\!^{-})^2$ using Zweig's Eq. (135). Compare with the "empirical" wavelength shown in Fig. 11. Overlay a plot of the wavelength for the passive model shown in Fig. 2 (using the same value of N and $\delta = 0.02$). Explain how the forms of the passive/active wavelengths create the features of the passive/active transfer functions?

3. **Sensitivity Analysis.** In this problem you'll perform a simple sensitivity analysis on Zweig's model. In particular, you will estimate the sensitivity of the model's stability to changes in the parameter $\mu \equiv \psi/2\pi \approx 1.75$ controlling the time delay of the slower feedback force. The locations of the zeros, s_n, of the impedance of the organ of Corti (see Fig. 14) depend on model parameter values (via Eqs. 149 and 150). For the parameter values given in Eq. 136, $\mathrm{Re}\{s_n\} \le 0$ for all n so that all the zeroes in Fig. 14 are to the *left* of the solid line representing the $\mathrm{Im}\{s\}$ axis; the model is therefore stable (i.e., impulse responses approach 0 as $t \to \infty$). For example, the two zeroes of λ^2 closest to the imaginary s axis in Fig. 14 occur at

$$s_1 \approx -0.0185454 + 0.958904i \qquad s_2 \approx -0.0327025 + 1.03244i\ .$$

   (a) Using Eq. 150, which says that

$$\angle\{(s_n - S)(s_n - S^*)\} = \pi - 2\pi\mu\,\mathrm{Im}\{s_n\}\ ,$$

   compute the value of the parameter μ at which $\mathrm{Re}\{s_1\} = 0$ (the zero then lies *on* the imaginary s axis, and the model hovers on the edge of instability). Keep all other parameter values fixed and assume that $\mathrm{Im}\{s_1\}$ is approximately equal to the value given above. (Extra credit: Justify this assumption.) Hint: Remember that the inverse tangent is multi-valued, so that that $\tan^{-1}(x) = \theta + m\pi$ with $m = 0, \pm 1, \pm 2, \ldots$; choose m so that μ comes closest to 1.75.

   (b) Now compute the value of μ at which $\mathrm{Re}\{s_2\} = 0$. Again, keep all other parameters and $\mathrm{Im}\{s_2\}$ fixed.

What do your results tell you about the sensitivity of the model to changes in the parameter μ?

4. **Modeling Strategies.** This week's papers take three very different approaches to modeling the cochlea.
   (a) In his Introduction, Zweig writes "In conventional theoretical treatments of cochlear mechanics, models are proposed and their consequences explored. This paper adopts another approach." Characterize Zweig's approach. How does it differ from the approaches taken by the other two papers? What are the strengths and weaknesses of each? Illustrate your remarks with examples from the three papers?
   (b) In the first sentence of their abstract, Kolston and Ashmore write "A new cochlear modeling technique has been developed in which the number of assumptions required in model formulation is significantly less than in previous modeling studies." Discuss whether you agree or disagree with this summary of their approach.
   (c) In his Discussion, Neely writes that "There is always a trade-off in models between complexity and completeness." Is Neely right? If so, what are the elements of the trade-off? How do the various authors frame the issue and justify their individual approaches? What are the relative strengths and weaknesses of so-called lumped vs finite-element models?
   (d) To what extent do the papers reach similar conclusions? Where do they disagree? How do you resolve any discrepancies?
5. **Motivation and Justification.** How do the different papers motivate and justify their work to the reader? How are the different models and interpretations of the data influenced by these goals?
6. **Assumptions.** List what you consider to be the most important assumptions for each model. How do the assumptions differ from paper to paper? Do the authors adequately justify their assumptions? Which assumptions are critical to the conclusions reached? How might the assumptions be tested?
7. **Predictions.** One measure of the power of a model is its ability to suggest experiments that test the model. Describe predictions made by each model that you find particularly interesting or telling. Can you suggest new (and/or identify existing) experiments that might test these predictions?
8. **Sanity Checks.** What "sanity checks" do the authors use to determine whether their results are at all reasonable (e.g., whether their numerical solutions are accurate, their parameter values reasonable, their models stable, etc)?