# Engineering, Economics & Regulation of the Electric Power Sector

ESD.934, 6.974

Recitation
Module E.2

# Electricity transmission expansion models

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## Regulation of transmission services

INVESTMENT

# Annex
# A model for transmission network planning

3

## Nature of the transmission Expansion problem

Determine the technical characteristics and installation time of new network facilities, so that:

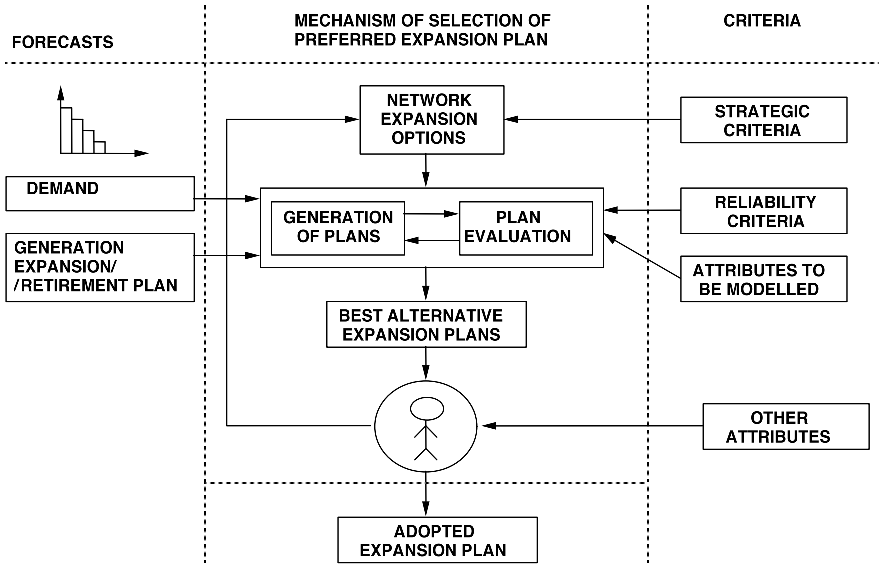
- Total expected cost of supply (including consumer outage costs) is minimized)

subject to acceptability criteria

- Technical
- Reliability
- Financial
- Environmental
- other

4

# Transmission expansion planning




# Nature of the transmission expansion problem

**TIME PERSPECTIVE**

- **LONG-TERM (15-30 YEARS)**
  - **Guidelines for network development**
  - **Simplified models are acceptable**
  - **Synthesis of plans is main priority**
- **MID TERM (6-10 YEARS)**
  - **Decisions for network development**
  - **Detailed models are required**
  - **Analysis of proposed plans is main priority**

6

# Mono-attribute optimization of expansion plans

**MINIMIZE M(p)**
**$p \in P$**

## Subject to

$$G_{k, min} \leq G_k(p) \leq G_{k, max}, \; k = 1,\ldots, K$$

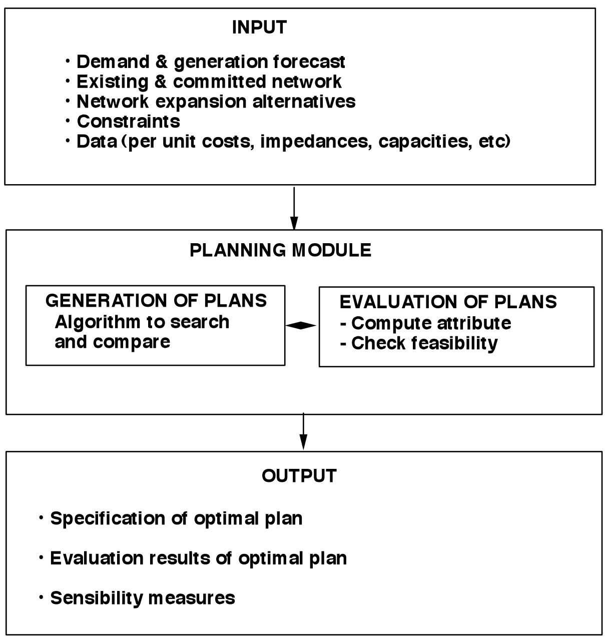
## Where

p: individual plan
P: set of all possible plans
M: attribute to be minimized (e.g., total cost of supply
G: result of each one of the k=1,...K technical/or reliability constraints that the plan has to meet

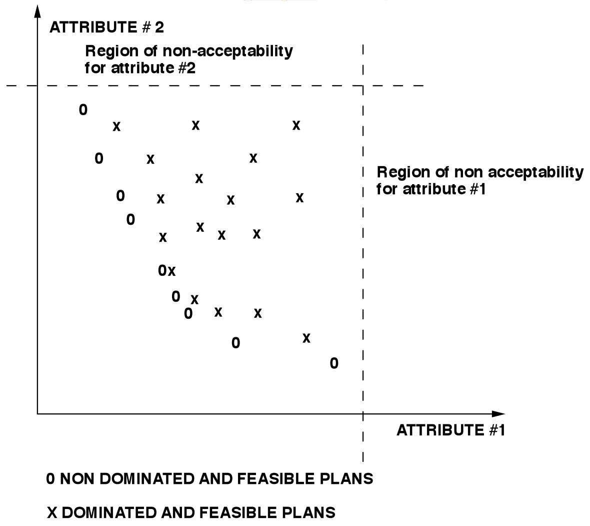
## Alternative: Heuristic search model

- Same as above, but algorithm (typically computationally efficient) does not guarantee that the optimal plan is obtained

7



## Multi-attribute optimization models




9

## Mono-Attribute Optimization Models

- **STATIC MODELS**
  - Only the final year of the considered time horizon is analyzed
  - Only models that seem to have been actually used in practical applications
- **DYNAMIC MODELS**
  - The entire time horizon is simultaneously considered

10

## Methodology. Modeling Aspects *(1 of 2)*

- **Main issues**
  - Demand
    - Generation of scenarios
  - Expansion alternatives/investment model
  - Discrete or continuous variables
  - Financial/economic constraints
- **Attributes (objectives function)**
  - Reliability: constraint, cost or both
  - Other attributes (e.g. environmental impact)
- **Network representation**
  - Transportation, DC, AC, hybrid model
  - Ohmic losses
  - Security limits

11

## Methodology. Modeling Aspects *(2 of 2)*

- **Production cost model**
  - Thermal generation units representation
  - Hydro units
  - Security constraints (preventive vs. corrective)
  - Uncertainty: hydro, load availability
- **Reliability model**
  - Contingency list vs. Probabilistic approach
  - Uncertainty: hydro, load availability

12

## Mono-Atribute static & strictly optimization models

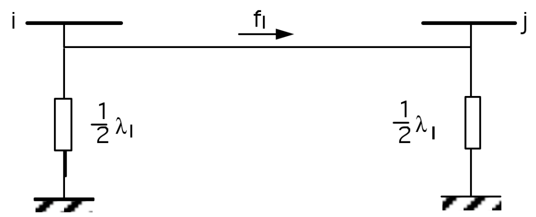
### Main Features

- Single attribute: Total supply cost (network investment cost + system operation cost + consumers outage cost)
- Optional constraints of the investment subproblem
  - Maximum number of lines per corridor
  - Maximum number of lines of a type per corridor
  - Maximum investment per corridor
  - Maximum total investment
  - Maximum non served energy
- Several options of network representation (DC has been chosen in the example shown here)
- Investment variables
- type of line & volume of investment at each corridor

13

## Network representation

## Ohmic losses




| | |
|---|---|
| **l** | **corridor identification index** |
| $\lambda_l$ | **ohmic losses (nonlinear function)** |
| $F_l$ | **active power flow in line l** |

$$\lambda_l = 2\, G_l\, [\, 1 - \cos\, (\theta_i - \theta_j)]$$

14

## Power System Model

- Production cost subproblem

$$\underset{g,r,f,\theta}{MINIMIZE}\ Z = c^T g + \mu u^T r$$

subject to

$$-\Delta - s.f + g + r = d \qquad (\pi_d)$$

$$f - \gamma.S^T.\theta = 0$$

$$0 \le g \le \overline{g}$$

$$0 \le r \le d$$

$$|f| \le \overline{f} \qquad (\pi_q)$$

$$\Delta_i = \frac{1}{2}\sum_j \lambda_{i,j} \qquad (losses)$$



## Power System Model

- Reliability subproblem

$$\underset{g,r,f,\theta}{MINIMIZE}\ Z = u^T r$$

subject to

$$-s.f + g + r = d \qquad (\pi_d)$$

$$f - \gamma.S^T.\theta = 0$$

$$0 \le g \le \overline{g}$$

$$0 \le r \le d$$

$$|f| \le \overline{f} \qquad (\pi_f)$$

# Glosary of terms

- g: active power generation at each bus
- $\bar{g}$: maximum active power generation at each bus
- f: active power flow at each line
- $\bar{f}$: maximum active power flow at each line
- r: non served power at each bus
- u: unit vector
- m: cost of unserved energy
- c: variable generation cost
- θ: voltage angle at each bus
- $\lambda_l$: ohmic losses in line l
- S: node-arc incidence matrix
- d: active power demand at each node
- pd, pf: dual variables of associated constraints
- $G_l$: susceptance of each line l

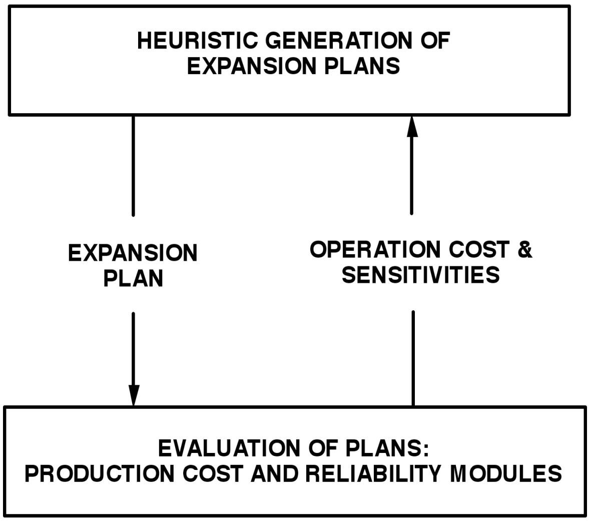
17

# Mono-Attribute static optimization model Solution by heuristic search Case example: CHOPIN

- Formulation
  - Only discrete investment variables are considered in CHOPIN
  - Production cost & reliability models with DC network formulation
- Solution method
  - The optimization of the investment subproblem is replaced by a heuristic search that consists in a truncated enumeration of the complete solution space (i.e., the set of all possible plans)
  - Investment restrictions are explicitly accounted for during the search: non feasible solutions are not accepted
  - The level of network modelling detail is not relevant for the performance of the algorithm → no restrictions to the use of DC (or even AC) models

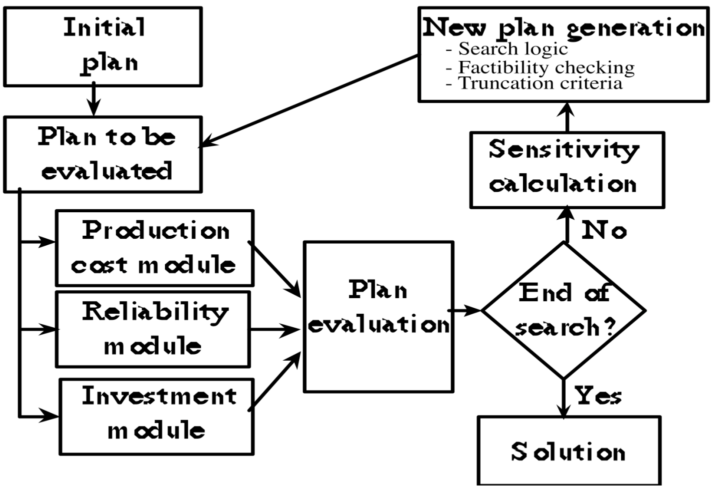
18

## CHOPIN
## Solution by heuristic search




19

## CHOPIN
## Algorithm organization




20

## CHOPIN
## Basic Philosophy of the search algorithm

- Start from a user-provided reasonable plan (*)
- Local search that is guided by
  - Sensitivities
  - heuristic rules
    - logic
    - experience from actual use of algorithm
  - Depth-first search
    - since truncation here is mostly based on extent of deviations from what locally appears to be the best decision
    - good solutions in limited time

(*) Successful searches have been achieved in all cases even when starting from very poor initial plans

21

## CHOPIN
## Classification of the investment variables

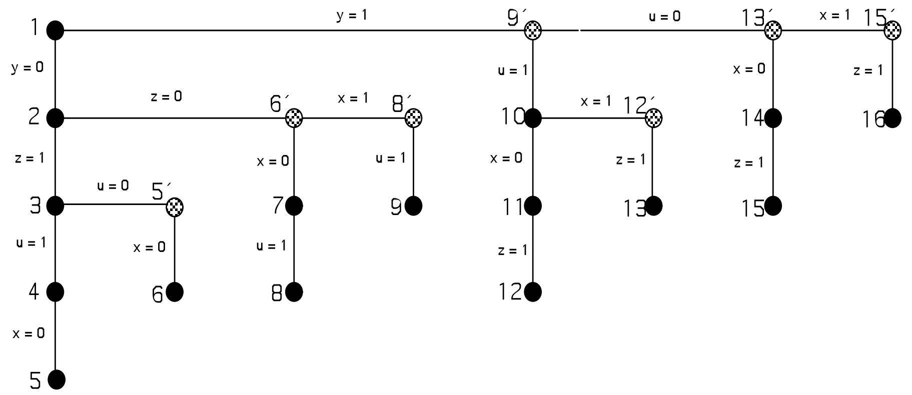
- Questioned variables
  - Lines included by user in initial plan
  - User considers they may not belong to optimal plan
  - Initial value = 1
- Attractive variables
  - Lines not included by user in initial plan
  - User considers they may belong to optimal plan
  - Initial value = 0
- Frozen variables
  - Cannot change their initial values ( 0 or 1) fixed by user
  - During the search the questioned & attractive variables become frozen variables

22

Initial solution

u x y z

0 1 1 0

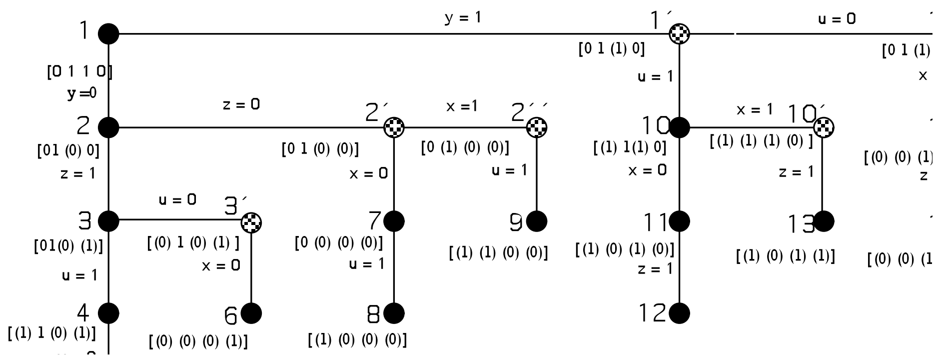



23

Initial solution

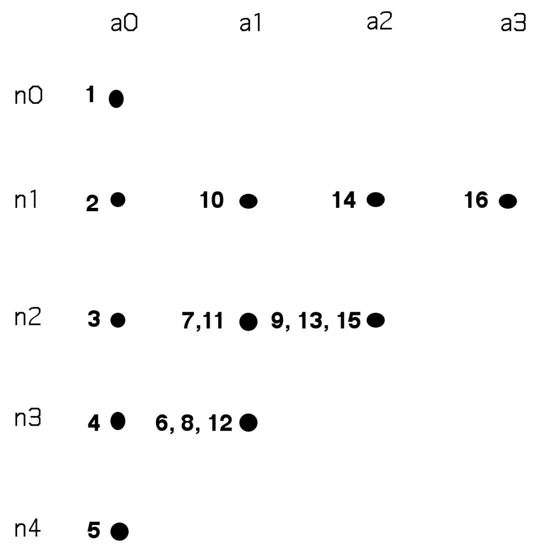
u x y z

0 1 1 0



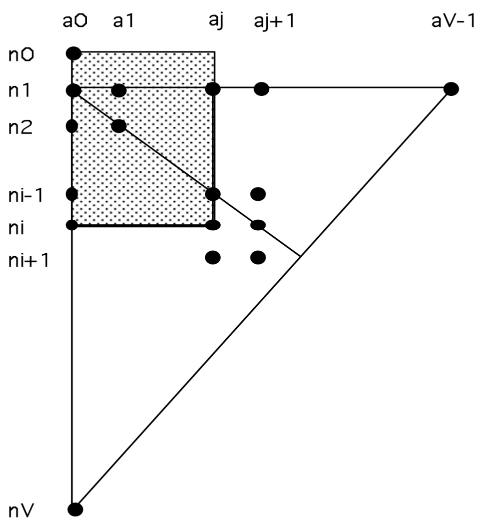

24

# CHOPIN
## Example: Solutions Space






# CHOPIN
## Solution Space in a General Case



## CHOPIN
### Use of sensitivities in guiding the local search

- **Estimate of the cost/benefit ratio of an investment option:**

$$A(X_1) = \frac{sensivity\ of\ operation\ cost\ to\ X_1}{per\ unit\ investment\ cost}$$

- **Use A(Xl) to rank investment options according to potential interest → priorities in depth-first search**
  - **A(Xl) >1 ⟶ consider for installation if attractive**
  - **A(Xl) ≤ 1 ⟶ consider for removal if questioned**



## Power system models

### Sensitivities (DC network model)

**Sensivity of the objective function Z (operation cost + outage cots) with respect to reinforcement in any corridor 1:**


$X_l$ Investment variable ( 0, 1) in corridor l

$\gamma_l^0$ Susceptance when Xl = 1

$\theta_i^*$ Voltage angle at bus i, for the optimal solution

$$\frac{\partial Z}{\partial X_1} = \underbrace{\gamma_1^0 \left(\theta_i^* - \theta_j^*\right)}_{\Delta\ flow\ when\ x_l\ changes\ 0\ to\ 1} \ . \left(\pi_{dj} - \pi_{di}\right)$$

$\pi_{dj}$ Impact on operation cost of Δ flow when injection in node j increases by one unit

$\pi_{di}$ Idem because of withdrawal from node i

## CHOPIN
### Global Truncation Criteria

- Set upper limit to
  - number of evaluations
  - size (width, depth) of search space
  - number of "wrong steps"

29

## CHOPIN
### Local Stopping Criteria

- Violation of investment constraints
- No possibility of improvement on the currently best found plan
- Exceed the allowed total volume of investment with any of the remaining open options

30

## CHOPIN
## Utilization Guidelines
### (must be taylored to each system)

- **Approach phase**
  - **No horizontal branching**
  - **No limit to number of "wrong steps"**
- **Local search phase**
  - **Sequentially allow a maximum of 1, 4 & 8 horizontal steps**
  - **No more than one "wrong step"**
- **Verification phase**
  - **Maximum of 4 horizontal steps**
  - **No more than 2 "wrong steps"**

31

## CHOPIN
## Critical Evaluation

- **Strong points**
  - **No restrictions to the level of modeling detail or nature of restrictions**
  - **Computationally very efficient (1 to 2 orders of magnitude faster than PERLA)**
  - **Successful application in large practical systems**
  - **Invariably CHOPIN has produced the optimal solution (there is no evidence against this, despite the efforts made to disprove it ) in all cases**
- **Weak(?) points:**
  - **There is no guarantee that the solution provided by CHOPIN is the actual optimal plan (this is also true for any other algorithm when applied to general non linear optimization problems)**

32

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