Philosophy 244: #14— Existence and Identity

Existence Predicates

The problem we've been having is that (a) we want to allow models that invalidate the CBF ($\Box\forall x\alpha\supset\forall x\Box\alpha$), (b) these will have to be models in which w can see w' although D_w has members not in $D_{w'}$, (c) models like that invalidate one of our theorems, namely $\Box(\forall x\varphi(x)\supset\varphi(y))$ (= $\Box\forall$1).

So, for instance, it does hold necessarily that if everyone is happy, Kripke is happy, because the antecedent holds in worlds where Kripke does not exist.

Remember how the problem arises. Validity of a wff α is defined as $V_\mu(\alpha$,w)=1 for all w and all μ assigning members of D_w to α's free variables. The reason μ is limited to members of D_w is that otherwise you could make $\forall x\varphi(x)\supset\varphi(y)$ (without the box) false, just by assigning *y* a member of D-D_w.

OK, but there's a simple solution to this. You can stipulate in $\forall$1 itself that *y* has to be one of the existing things: $(\forall x\varphi(x)\&Ey)\supset\varphi(y)$. This requires an existence predicate, which is governed by the obvious rule:

[VE] <*x*,w>ϵV(E) iff $x\epsilon D_w$.

The availability of an existence predicate E takes some of the pressure off of μ as the guarantor of appropriate instantiation, thus allowing us to redefine validity so that $V_\mu(\alpha$,w) has to be 1 for all w *and all* μ *as well*. $\forall$1 will not be valid on this approach but we can replace it with

$\forall$1E $(\forall x\alpha\&Ey)\supset\alpha[y/x]$.

An axiom of this sort is the standard replacement for $\forall$1 in so-called free logic: logic free of existential assumptions.

Existence Systems

LPC*E*+S is defined as follows.

Axioms

S' $\vdash\alpha$ for each α an LPC substitution instance of an S-theorem,
$\forall$1E $\vdash(\forall x\alpha\&Ey)\supset\alpha[y/x]$
$\forall^\supset$ $\vdash\forall x(\alpha\supset\beta)\supset(\forall x\alpha\supset\forall x\beta)$
VQ $\vdash\alpha\equiv\forall x\alpha$ where *x* is not free in α
UE $\vdash\forall xEx$

Rules

NE $\vdash\alpha\Rightarrow\vdash\Box\alpha$
MP $\vdash\alpha$, $\vdash(\alpha\supset\beta)\Rightarrow\vdash\beta$
UG $\vdash\alpha\Rightarrow\vdash\forall x\alpha$
UG$\Box^{\forall n}$ $\vdash\alpha_1\supset\Box(\alpha_2\supset...\Box(\alpha_n\supset\Box\beta)...)\Rightarrow\vdash\alpha_1\supset\Box(\alpha_2\supset...\Box(\alpha_n\supset\Box\forall x\beta)...)$ — *x* not free in α_i

A number of standard results follow, including

$\forall$1' $\vdash\forall y(\forall x\alpha\supset\alpha[y/x])$

RBV $\vdash\forall x\alpha\equiv\forall y\beta$ — α and β differ only in that α has free *x* where β has free *y*
QR $\vdash\exists y(\alpha[y/x]\supset\forall x\alpha)$
Etc.

This axiomatization is sound w.r.t. the new definition of validity. Take ∀1E. Assume for contradiction that $V_\mu(\forall x\alpha(x),w)=1$ and $V_\mu(Ey,w)=1$ but $V(\alpha[y/x],w)=0$. Choose ρ so that $\rho(x)=\mu(y)$. By the Principle of Replacement (241), $V_\rho(\alpha(x),w)=0$. $V_\mu(Ey,w)=1$, so $\rho(x)\epsilon D_w$. But then $V_\mu(\forall x\alpha(x),w)=0$ after all, since ρ is an *x*-variant of μ.

PR basically says that V doesn't care whether an object is picked out as $\rho(x)$ or $\mu(y)$.

By (V∀'), which says that $V_\mu(\forall x\alpha,w)=1$ iff $V_\rho(\alpha,w)=1$ for each *x*-variant ρ of μ such that $\rho(x)\ \epsilon\ D_w$.

Completeness

Again the procedure is to construct a canonical model. Assume Λ is a consistent set of $\mathcal{L}$-wffs in LPCE+S, and that $\mathcal{L}$ is an infinitely proper sublanguage of $\mathcal{L}$+. A set Δ of $\mathcal{L}$+-wffs has the □∀-property iff

(i) for every α of $\mathcal{L}$+ and variable *x*, there is a variable *y* s.t. $Ey\&(\alpha[y/x]\supset\forall x\alpha)\epsilon\Delta$

(ii) for $\beta_1...\beta_n$ and α in $\mathcal{L}$+, and every *x* not free in β_i, there's a variable *z* such that $\Box(\beta_1\supset...\Box(\beta_n\supset\Box(Ez\supset\alpha[z/x])...)\supset\Box(\beta_1\supset...\Box(\beta_n\supset\Box\forall x\alpha)...)$

Prop. 16.1 Any consistent set Λ of $\mathcal{L}$-wffs can be extended to a consistent set Γ of $\mathcal{L}$+-wffs with the □∀-property.

Prop. 16.2 If Γ is a maxiset with the □∀-property, and $\Box\alpha\notin\Gamma$, then there's a consistent set Δ with the □∀-property such that $\Box^-(\Gamma)\cup\{\neg\alpha\}\subseteq\Delta$.

The canonical model is defined as before except that D_w is the set of variables *x* such that $Ex\epsilon w$.

Prop. 16.3 $V_\sigma(\alpha,w)=1$ in the canonical model iff $\alpha\epsilon w$.

From this it follows that the canonical model of LPCE+S validates exactly the theorems of LPCE+S. Completeness follows as before for any S such that the frame of the canonical model of LPCE+S is an S-frame. That includes all of the main systems we have been working with.

Possibilist Quantification

The way we've been interpreting the Barcan Formula makes it look as though it presupposes that the same things exist in every world, or at least that you never get new things as you move from w to a world u that w can see. If you do get new things then the fact that everything in w is necessarily φ leaves it wide open that something in u isn't φ even accidentally. And that appears to go directly against BF. Likewise BFC assumes, apparently, that things never disappear. If they do then existence becomes a counterexample to $\Box\forall x\varphi\supset\forall x\Box\varphi x$.

Does CBF really assume this, though? It does if you interpret $\forall x$, uttered in connection with a world w, as ranging over just the things that exist in w. Another option would be to interpret it as ranging, whenever it is used, over *all possible* things. An example of this from English might be "there are things which could have existed but don't actually exist, e.g., the 1998 Moose Jaw Winter Olympics." The first interpretation is called "actualist," the second "possibilist." Now that we have an existence predicate, the "actualist" interpretation isn't forced on us; an actualist quantifier if is wantedn can be defined in terms of a possibilist one and the existence predicate. Using "$\forall$" for the possibilist quantifier and "Π" for the actualist one, we can simply say that

DefΠ $\quad \Pi x\alpha =_{df} \forall x(\text{E}x\supset\alpha)$.

The rule for Π is what above we called [V$\forall$']:

(VΠ) $V_\mu(\Pi x\alpha)$=1 iff $V_\rho(\alpha)$ = 1 for every x-alternative ρ of μ such that $\rho(x)\epsilon D_w$.

The rule for $\forall$ can now go back to something very like a constant domain rule:

(V$\forall$) $V_\mu(\forall x\alpha)$=1 iff $V_\rho(\alpha)$ = 1 for every x-alternative ρ of μ such that $\rho(x)\epsilon D=\cup_w D_w$

Now it seems we can have our cake and eat it too. We can stick to our original constant domain rule for the quantifiers without losing access to the expressive possibilities opened up by allowing domains to vary. BF and CBF come out valid—but only formulated in terms of the possibilist quantifier $\forall$. Formulated in terms of Π they are *not* valid because domains are in fact changing from world to world; this affects the interpretation of predicate E and hence that of Π.

Suppose we want to get, say LPC+T without BF or CBF, but using, not quite a constant domain semantics, but a constant domain *rule* for the universal quantifier, which is what made the constant domain semantics so convenient in the first place.

First, let your language be LPCE, which has the possibilist quantifier $\forall$ and E.

Second, let your models be variable domain models with reflexive frames.

Third, use the "constant domain" rule (V$\forall$) for your evaluations.

Fourth, replace each occurrence of $\forall x(\text{E}x\supset\alpha)$ in the resulting validities with $\Pi x\alpha$.

Fifth, erase all formulas still containing $\forall$.

Sixth, the formulas remaining are the desired LPCE+T, bearing in mind that your universal quantifier is now written Π.

Identity

Ordinary, non-modal predicate calculus often makes special provision for a binary predicate intended to express identity. Strictly speaking the predicate should be a capital letter P and it should appear before its two arguments, as in Pxy. But the practice has long been to write it "=" and allow it to appear between its arguments, which yields the more familiar "x=y".

The semantics of identity what you'd expect. If <DV> is a model for LPC then V(=) is $\{<o,o>|o\epsilon D\}$. It follows that $V_\mu(x=y)$=1 iff $\mu(x)=\mu(y)$.

A complete axiomatic basis for LPC with identity is given by adding two axioms, the Laurie Anderson axiom.

After a song on *Big Science* called "Let x = x."

I1 x=x

and the Leibniz axiom

I2 $x{=}y\supset(\alpha\supset\beta)$ – β has y free in some or all of the places where α has x free

Now let's look at adding identity to modal LPC. For simplicity we limit ourselves to systems which satisfy BF. S+BF+I1+I2 will be written S+I; BF will be taken for granted. A surprising fact that makes the whole enterprise interesting:

□I $x{=}y\supset\Box x{=}y$

Proof:
(1) $x{=}y \supset (\Box x{=}x \supset \Box x{=}y)$ I2
(2) $\Box x{=}x \supset (x{=}y \supset \Box x{=}y)$ (1)×PC
(3) $\Box x{=}x$ I1×N
(4) $x{=}y \supset \Box x{=}y$ (2)(3)×□P

It may seem easy to think of counterexamples, e.g.,

the person who lives next door is the mayor
the number of planets is 8
Michelle's husband is the president
my favorite color is red

The thing to notice for now is that these statements link not variables but definite descriptions. So the theorem as stated doesn't apply. It doesn't imply that if my favorite color is red, then necessarily it is red. Why the nature of the referring term should make so much of a difference is something we'll have to come back to, after looking at definite descriptions.

Compare a temporal example. Sparky's age = 8. Dogs are always their-age years old. So, Sparky is always eight years old.

Diversity

Necessity of identity is one thing, necessity of diversity is something else. You'd think they went together but it depends. Some versions of S+I. but not all (all where S extends B) have in addition

□NI $x{\neq}y \supset \Box x{\neq}y$

(1) $\neg\Box x{=}y \supset x{\neq}y$ □I×PC
(2) $\Diamond x{\neq}y \supset x{\neq}y$ (1)×□◇I (□◇ Interchangeability)
(3) $x{\neq}y \supset \Box x{\neq}y$ (2)×DR4

DR4 = $\vdash(\Diamond\alpha\supset\beta) \Rightarrow \vdash(\alpha\supset\Box\beta)$. DR4 depends on the Brouwer axiom.

How can you have contingent distinctness without contingent identity? The answer is that maybe a world where x and y are distinct can see a world where they're identical, but not the other way around. If accessibility is symmetrical, as in system B or above, this can't happen; it should come as no surprise then that it's B you need to deduce □NI from □I. We'll use S+□NI for the result of adding □NI to S+I.

Next time, identity and descriptions.