

Quick Introduction to the Semantics of Modals¹

- Syntactic assumptions (following von Stechow & Heim 2005): modals are raising predicates (see Wurmbrand 1999, Bhatt 1998)

- 1) John must go home.
- 2) [John [λ_1 [must [t_1 go home]]]]

For simplicity, we will work with the reconstructed structures:

- 3) must [John go home]

- In a possible world semantics, modal expressions are treated as quantifiers over worlds.

Necessity modals (e.g., **must**, **have to**) are universal quantifiers.
Possibility modals (e.g., **might**, **can**) are existential quantifiers.

First try:

- 4) [[**must**]] = $\lambda p_{\langle st \rangle} \lambda w \forall w': p(w') = 1$

- 5) [[**can**]] = $\lambda p_{\langle st \rangle} \lambda w \exists w': p(w') = 1$

- 6) John must be home
[must [John be home]]

true iff John is home in all possible worlds

- 7) John may be home
[may [John be home]]

true iff there's some possible world where John is home.

- BUT: this doesn't account for

- Contingency of modal statements:

- 8) John must be home now.

- 9) It must be raining outside

In order to evaluate (8) and (9), we only consider worlds that are compatible with the evidence available to us.

¹ Some parts of this handout are borrowed / adapted from von Stechow and Heim's notes on Intensional Semantics. I have also made use of Valentine Hacquard's handouts on modality, available at <http://people.umass.edu/hacquard/ling720/calendar.htm> Of course, all errors are mine.

- Different flavors of modality

- | | | |
|-----|----------------------------------|------------------|
| 10) | John must have been the murderer | [epistemic] |
| 11) | John must go to jail | [deontic] |
| 12) | John must sneeze | [circumstantial] |

(Kratzer 1991)

Maybe modals are ambiguous? Kratzer 1977, 1981, 1981: NO.

The kind of modality involved can be specified overtly:

- 13) In view of the available evidence, John must have been the murderer
- 14) In view of what the law provides, John must go to jail
- 15) In view of the present state of his nose, John must sneeze.

The paraphrases above in 13), 14), 15) are not redundant. The **must** in these examples is a 'neutral' modal. "The existence of neutral modals suggests that non-neutral modals are not truly ambiguous. They just need a piece of information provided by the context of use" (Kratzer 1991: 640)

Second try:

- What we want:

- 16) (In view of what the law provides) John must go to jail

true in a world w iff John goes to jail in all the worlds that are compatible with what the law provides in w .

- 17) In view of the available evidence, John must have been the murderer

true in a world w iff John is the murderer in all the worlds that are compatible with the available evidence in w .

- 18) In view of the available evidence, John might have been the murderer

true in a world w iff there's at least a world compatible with the evidence in w where John is the murderer.

- Kratzer's conversational backgrounds:

Restrictions on the domain of quantification of modals are provided via conversational backgrounds.

A conversational background is the sort of thing that is identified by phrases like **what the law provides**, **what we know**, etc.

what we know

What we know varies from one possible world to another.

What we know in a particular world is a set of propositions.

The denotation of **what we know** is a function which assigns to every possible world the set of propositions that we know in that world.

- 19) $[[\text{what we know}]] = \lambda w. \lambda p. p$ is one of the propositions that we know in w .

what the law provides

The denotation of **what the law provides** is a function which assigns to every possible world the set of propositions that the law provides in that world.

- 20) $[[\text{what the law provides}]] = \lambda w. \lambda p. p$ is one of the propositions that the law provides in w .

More generally, a conversational background is a function that takes maps any world w to (the characteristic function of) a set of propositions.

Note: For any conversational background f of type $\langle s \langle s, t \rangle, t \rangle$, we can define the corresponding accessibility relation R_f of type $\langle s \langle s, t \rangle$ as follows:

- 21) $R_f := \lambda w. \lambda w'. \forall p [f(w)(p) = 1 \rightarrow [p(w') = 1]]$

((21) is taken from von Stechow and Heim 2005; see definition 2 in Kratzer 1991)

- Denotation of modals.

- 22) $[[\text{must}]] = \lambda f_{\langle s \langle s, t \rangle, t \rangle} \lambda p_{\langle s, t \rangle} \lambda w \forall w' (w' \in \cap f(w) \rightarrow p(w') = 1)$

- 23) $[[\text{can}]] = \lambda f_{\langle s \langle s, t \rangle, t \rangle} \lambda p_{\langle s, t \rangle} \lambda w \exists w' (w' \in \cap f(w) \ \& \ p(w') = 1)$

[note: in Kratzer's definitions, the conversational background is a parameter of evaluation.]

Or, given (21).

- 24) $[[\text{must}]] = \lambda R_{\langle s \langle s, t \rangle, t \rangle} \lambda p_{\langle s, t \rangle} \lambda w \forall w' (R(w)(w') = 1 \rightarrow p(w') = 1)$

- 25) $[[\text{can}]] = \lambda R_{\langle s \langle s, t \rangle, t \rangle} \lambda p_{\langle s, t \rangle} \lambda w \exists w' (R(w)(w') = 1 \ \& \ p(w') = 1)$

- 26) In view of what the law provides, John must go to jail.

The proposition that is true in w iff John goes to jail in all the worlds where the laws in w are obeyed.

- What if the conversational background is not explicit?

- (i) Recovered from context.

- 27) John must be home.

- (ii) Hard-wired for some modals (see Kratzer 1981, 1991)

- This gives us:
 - Flavors of modality: different modal flavors correspond to different conversational backgrounds.
 - Contingency: modals are evaluated with respect to what the laws/the evidence/the circumstances... are in the world of evaluation.

- But this is not enough:

Consider the following example from von Stechow and Heim (2005)

28) John must pay a fine.

- The truth of 28) in the actual world depends on (i) what the facts are (what John has done), and (ii) what the law is.

For instance, 28) will be judged true if (i) the law states that nobody obstructs a driveway, (ii) the law states that anybody who obstructs a driveway pays a fine, and (iii) John has obstructed a driveway.

- What are we quantifying over in this case?

Perhaps:

29) { w' : what happened in w' up to now is the same as what happened in the actual world and w' conforms to what the law in the actual world states}

Problem: given the facts in the actual world (John has violated the law) the domain of quantification will be empty.

Thus, 28) is predicted to be trivially true.

Solution: the domain of quantification consists of all the worlds where the facts are as they are in the actual world and where the law is obeyed as well as it can be given the facts.

How: We need to rank the worlds where the facts are as they are in the actual world with respect to how well they obey the laws. To do that, we will introduce a second conversational background.

Third try: Ordering.

- Kratzer: modal operators are interpreted with respect to two conversational backgrounds.

A modal base, which determines for every world the set of worlds that are accessible from it.

An ordering source, which imposes a partial order on the set of worlds selected by the modal base.

- How this works (simplified version, from von Fintel & Heim (2005)):

Step 1: Defining a strict partial order with respect to a set of propositions [cf. with Kratzer's definition]

- 30) For any pair of worlds, w_1 and w_2 , w_1 comes closer than w_2 ($w_1 <_p w_2$) to the ideal set up by a set of propositions P iff the set of propositions belonging to P that are true in w_2 is a proper subset of the set of propositions belonging to P that are true in w_1 .

Step 2: Defining a function that selects the best worlds from any set X of worlds with respect to a partial order $<_p$

- 31) $\forall X \subseteq W: \max_p(X) = \{w \in W: \sim \exists w' \in X: w' <_p w\}$

Step 3: Formulate the semantics of **must** and **can** accordingly²

- 32) $[[\mathbf{must}]]^w = \lambda f_{\langle s, \langle s, t \rangle \rangle} \lambda g_{\langle s, \langle s, t \rangle \rangle} \lambda q_{\langle s, t \rangle} \forall w' \in \max_{g(w)} (\cap f(w)): q(w') = 1$

(von Fintel and Heim 2005: 55)

- 33) $[[\mathbf{can}]]^w = \lambda f_{\langle s, \langle s, t \rangle \rangle} \lambda g_{\langle s, \langle s, t \rangle \rangle} \lambda q_{\langle s, t \rangle} \exists w' \in \max_{g(w)} (\cap f(w)): q(w') = 1$

must p is true iff p is true in all the worlds selected by the modal base that are best with respect to the ordering source.

can p is true iff p is true in at least of of the worlds selected by the modal base that are best with respect to the ordering source.

[Note: what we have above only works if we can in general assume that there are always accessible worlds that come closest to the ideal set up by the ordering source. According to Lewis (1973), we shouldn't make that assumption, which he calls the Limit Assumption. Kratzer (1991) follows Lewis in this respect. Hence, her semantics for modals is more complicated than what we have above.]

Assuming conditional is material implication, and assuming a standard semantics for modals, it follows that,

It is necessary that:

- 39) If a murder occurs, the murderer will go jail.
- 40) If a murder occurs, the murderer will be knighted
- 41) If a murder occurs, the murderer will receive \$100.
- *Kratzer's solution*: the if-clause restricts the modal base of the modal
- 42) If a murder occurs, the murderer must go to jail.

In all of worlds where a murder occurs, and that are best with respect to what the law provides, the murderer goes to jail.

Two types of modal reasoning: circumstantial vs. epistemic

- 43) (a) Hydrangeas can grow here.
- (b) There might be hydrangeas growing here.

"Suppose I acquire a piece of land in a far away country and discover that soil and climate are very much like at home, where hydrangeas prosper everywhere. Since hydrangeas are my favorite plants, I wonder whether they would grow in this place and inquire about it. The answer is [43)a)] In such a situation, the proposition expressed by [43)a)] is true. It is true regardless of whether it is or isn't likely that there are already hydrangeas in the country we are considering. All that matters is climate, soil, the special properties of hydrangeas, and the like. Suppose now that the country we are in has never had any contacts whatsoever with Asia or America, and the vegetation is altogether different from ours. Given this evidence, my utterance of [43)b)] would express a false proposition. What counts here is the complete evidence available. And this evidence is not compatible with the existence of hydrangeas. [43)a)] together with our scenario illustrates the pure circumstantial reading of the modal *can*. The pure circumstantial reading of modals is characterized by a circumstantial modal base and an empty ordering source [PMB: the function which assigns the empty set to every possible world] [43)b)] together with our scenario illustrates the epistemic reading of modals (the ordering source may or may not be empty here). Circumstantial and epistemic conversational backgrounds involve different kinds of facts. In using an epistemic modal, we are interested in what else may or must be the case in our world given all the evidence available." (Kratzer 1991: 646).

- Circumstantial modal base: picks out worlds in which certain relevant facts or circumstances hold. E.g., climate, soil, special properties of hydrangeas...
- Epistemic modal base: picks out worlds compatible with all the evidence available.

If we know for a fact that hydrangeas do not grow here, (43)a) can still be true, but 43)b) is false.

Combinations of modal bases and ordering sources

- Epistemic modal base: combines with ordering source related to information: what the normal course of events is like, reports, beliefs: stereotypical ordering source (*'in view of the normal course of events'*)

Some possibilities are more far fetched than others:

- 44) Michl must be the murderer.
Michl is probably the murderer.
There is a good possibility that Michl is the murderer.
There is a slight possibility that Michl is the murderer.

Far fetched in view of the normal course of events:

- 45) Michl must be the murderer.

In all the best worlds w.r.t. the normal course of events, among those compatible with my evidence, Michl is the murderer.

- Circumstantial modal base: combines with an ordering source related to laws, aims, plans, wishes: what the law provides, what is good for you, what we aim at, what is moral...

- 46) Given your state of health, you should live in a warm climate

In view of your state or health [circumstantial modal base] and in view of what is good for you [ordering source], you should live in a warm climate.

- One of the conversational backgrounds might be empty (technically, an empty conversational background maps every possible world to the empty set).
- A possible case in point, from Kratzer 1981: in order to evaluate 47) we need only consider certain properties inherent in the cup: circumstantial modal base & empty ordering source.

- 47) Diese Tasse ist zerbrechlich
This cup is fragile