1. Symmetry.

a. Complete the combination theorem $T \bullet A_\alpha = B_\alpha$ and illustrate the result with a sketch.

b. If a 4-fold axis is to be combined with a two-dimensional lattice, that lattice must be square (that was a result of the equation derived in an earlier construction, cos α = (1-**p**)/2, where **p** is an integer used to derive the rotation axes that are compatible with translation). Use the theorem in part (a) to derive the rotation axes that arise when a 4-fold axis is combined with a square lattice. Note that (a) is an equation in *individual operations*, not symmetry elements.

c. The three-dimensional space lattices may be derived by "stacking" the two dimensional plane groups. (These give the set of symmetries that require all possible types of two-dimensional lattices. Symmetry and the "special" geometry of a lattice are two inseparable aspects of a pattern. For example, a 2-D lattice is square only if there is a 4-fold axis present that **demands** that this be so, and if a 4-fold axis is present it requires that, if it is combined with translational periodicity, that the lattice is exactly square.)

Combining a third translation that is not coplanar with plane group P4 will, therefore, "stack" the square nets above one another. There is however a severe constraint on how the third translation may be chosen: It must translate the symmetry elements in the base to locations directly above themselves – That is, be normal to the plane group (This will work for every plane group) or alternatively, slide the plane group normal to the base and laterally such that symmetry elements are mapped into other, independent symmetry elements of the same sort.

With this as background, please show the way or ways plane group P4 may be stacked to conform to the above requirement. Also, please sketch the unit cell of the space lattice or lattices that result. These are called the tetragonal space lattices ("tetra"=four; "gonal"= angled)

3. **Tensor Quantities**. A particular oxide crystal is orthorhombic, a = 8.7 Å , b=12.4 Å, c=4.5 Å, $\alpha = \beta = \gamma = 90^0$. The conductivity tensor σ_{ij}, which relates the current density vector J_i to an applied electric field ε_j according to $J_i = \sigma_{ij}\,\varepsilon_j$ the conductivity tensor for this crystal is:

$$\sigma_{ij} = \begin{vmatrix} 5.6 & 0 & 0 \\ 0 & 2.4 & 0 \\ 0 & 0 & 10.60 \end{vmatrix} 10^{-6} \text{ ohm}^{-1} \text{ cm}^{-1}$$

An electric field of 10^6 volts/m is applied along the [111] direction of the crystal.

a. What are the components of ε , ε_i ?
b. Compute the components J_i and the magnitude of J , $|J|$.
c. What is the angle between J and ε ?
d. What is the value of σ along the direction [111] ?

(Beware! Crystallographic coordinates are used to specify the indices of planes and directions in a lattice, but Cartesian coordinates are used to specify components of vectors and elements of tensors!!)

3. Formation of an ideal solution is always thermodynamically favorable. Consider the mixing process described schematically below for an isolated two-component system.

d. Using the ideal solution model of chemical potentials, show that at constant temperature and pressure, the Gibbs free energy change for taking an initially pure quantity of a component **A** and mixing it with a quantity of a second pure component **B** is always favorable (mixing will occur spontaneously) if the two components form an ideal solution.

e. Show that the entropy change on mixing two components to form an ideal solution is always positive or zero. (Hint: How can I directly get the entropy change in the process from the Gibbs free energy change calculated in part (a)? Or in other words, how is S related to G?)