Department of Materials Science and Engineering
Massachusetts Institute of Technology
3.044 Materials Processing – Spring 2012

Exam II

Wednesday, May 2, 2012

The Rules:

1) No books allowed; no computers allowed; etc.

2) A simple calculator is allowed

3) Two hand written 3x5 index cards may be prepared as a crutch.

4) Complete 5 out of the 6 problems. If you do more than 5 problems, I will grade the first 5 that are not crossed out.

5) Make sure that you READ THE QUESTIONS CAREFULLY

6) Supplementary materials are attached to the end of the test (eqns., etc.)

7) WRITE YOUR NAME HERE:

SOLUTIONS

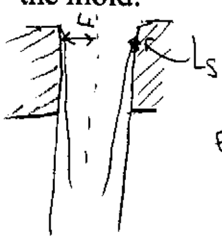
Problem #1: A Continuous Casting is Subjected to Scrutiny Following Some Concerns About the Stability of the Solidification Front

In this problem, you will explain the following statement:

If we continuously cast a thick section of a binary alloy, the solidification begins in a stable plane front mode, but later transitions into an unstable, dendritic growth mode.

1.5 **Part A:**

Even though the solid and liquid have about the same 'k' (thermal conductivity), the resistances, or conductances, of these two components are different. Compare the thermal conduction in the liquid and solid of a continuously-cast alloy for the early stages of casting, when we are still in the mold.



THERMAL CONDUCTANCE = $\frac{k}{L}$

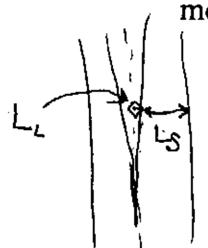
EQUAL

$$\frac{k_L / L_L}{k_S / L_S} = \frac{L_S}{L_L} \to 0$$ AT SHORT TIMES (MOSTLY STILL LIQUID)

∴ THE THERMAL GRADIENTS ARE MOSTLY IN THE LIQUID → $\left.\frac{dT}{dx}\right|_{LIQ}$ IS LARGE.

1.5 **Part B:**

Compare the conductances of the solid and liquid at a much later time, when we are out of the mold and solidification is almost complete.



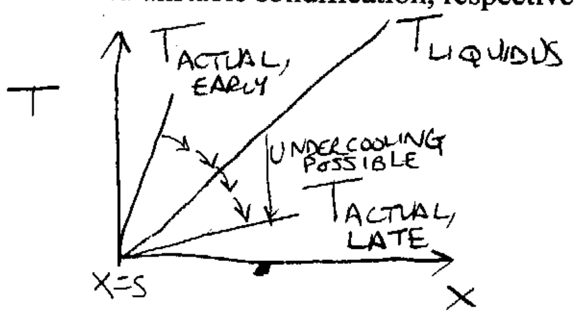
THERMAL CONDUCTANCE RATIO $= \frac{L_S}{L_L} \to \infty$ AT LONG TIMES (MOSTLY SOLIDIFIED)

∴ THE THERMAL GRADIENTS IN THE LIQUID, $\frac{dT}{dx}$, ARE SMALL.

Part C:

2

Explain how the conditions in the early (Part A) and late (Part B) stages of casting lead to stable and unstable solidification, respectively.



PART A/EARLY: $\frac{dT}{dx} > \frac{dT_L}{dx}$

⇒ STABLE

PART B/LATE: $\frac{dT}{dx} < \frac{dT_L}{dx}$

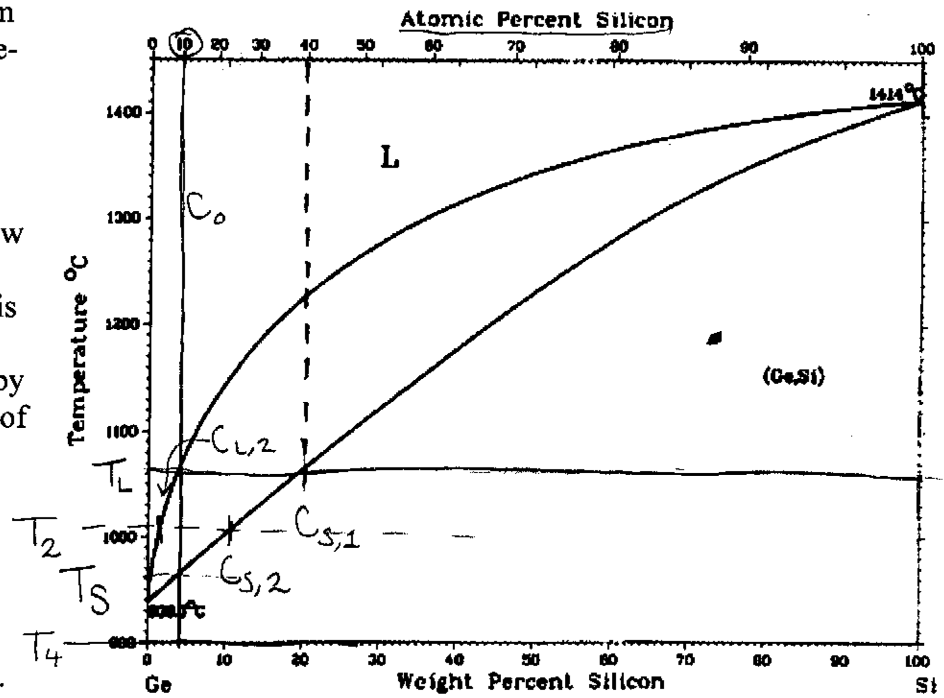
⇒ UNDERCOOLING, WHICH LEADS TO AN UNSTABLE LIQUID ⇒ DENDRITES.

Problem #2: Solidification: Inside, Outside, Upside Down

Here is a phase diagram for a binary alloy of Ge-Si. We are going to solidify a dilute alloy, say Ge-10 at% Si.

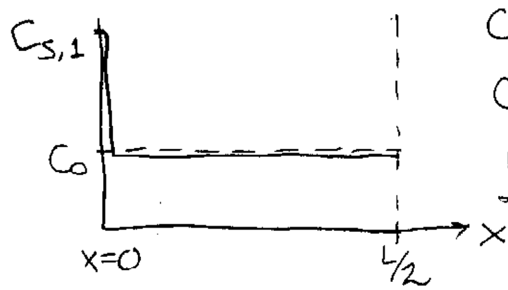
I would like you to show me how this solidification problem is different from all those we have seen in class, by drawing some pictures of the composition profile as the alloy solidifies.

Assume that mixing/diffusion in the liquid is rapid, but nonexistent in the solid.



Please draw three pictures of the composition profile, at at least three times/temperatures. One near the beginning of the solidification ($T = T_1$ near the liquidus), one at an intermediate time ($T = T_2$), and one near the end of solidification ($T = T_3$ near the solidus). Please make your charts as QUANTITATIVE as possible; label every quantity you can.

AT $T = T_L$:



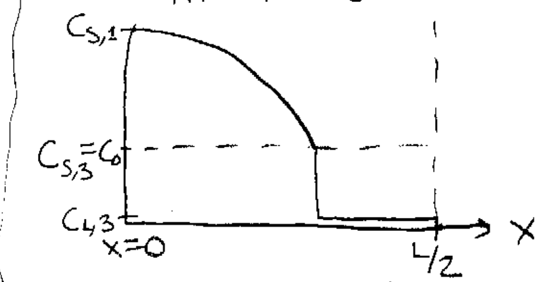
$C_{S,1} = 40$ AT % Si

$C_o = 10$ AT % Si

$$K_1 = \frac{C_{S,1}}{C_{L,1}} = \frac{40}{10} = 4$$

$C_{L,1} = C_o$

AT $T = T_S$:

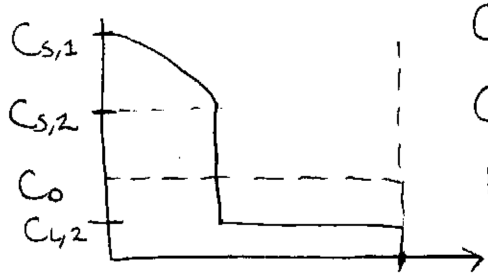


$C_{S,3} = C_o = 10$ AT % Si

$C_{L,3} \cong 1$ AT % Si

$K_3 \cong 10$

AT $T = T_2$:

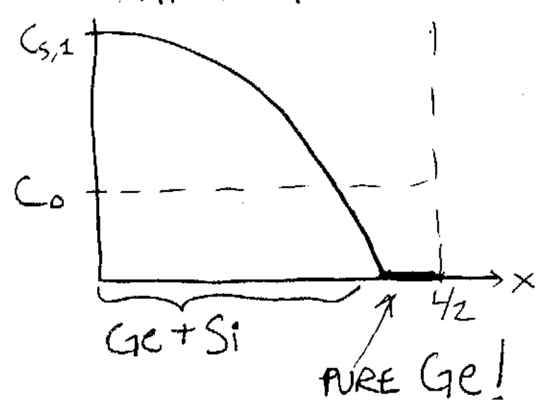


$C_{S,2} \cong 25$ AT % Si

$C_{L,2} \cong 5$ AT % Si

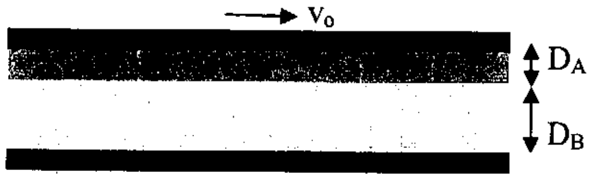
$$K_2 = \frac{C_{S,2}}{C_{L,2}} = \frac{25}{5} = 5$$

AT $T = T_4$:



Problem #3: Go With the Co-Flow

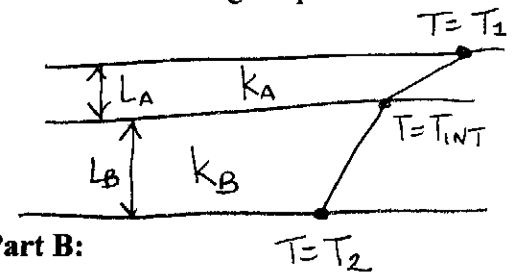
Imagine our classical two-parallel-plates-in-shear situation, but now there is not a single fluid between the plates, but two immiscible fluids…



The fluids, call them A and B, in general have different properties, and different dimensions, D. The bottom plate is fixed and the top one is moving at a fixed velocity v_o.

1 **Part A:**

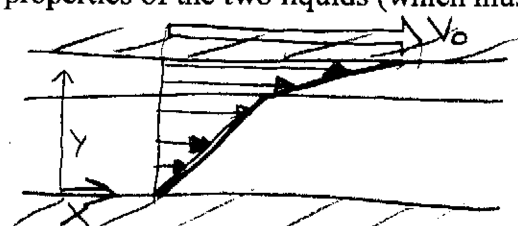
Describe the analogous problem in heat transfer.



THE TOP SURFACE IS FIXED AT T_1. THE BOTTOM SURFACE IS FIXED AT T_2. THE 2 MATERIALS HAVE DIFFERENT THICKNESSES (L_A, L_B) AND THERMAL CONDUCTIVITIES (k_A, k_B) AND WE ASSUME NO INTERFACE RESISTANCE.

1.5 **Part B:**

Draw the steady-state flow profile for this system; state any assumptions you make about the properties of the two liquids (which must NOT be the same).



HERE, I ASSUMED:

① NO INTERFACE RESISTANCE, SO $V_x|_B(Y = D_B) = V_x|_A(Y = D_B)$

② $\rho_A < \rho_B$ (GRAVITATIONALLY STABLE)

③ $\mu_B > \mu_A$ (STEEPER GRADIENT IN A)

④ NO SLIP AT TOP & BOTTOM ($V_x(Y = D_A + D_B) = V_o$ & $V_x(Y=0) = 0$)

1 **Part C:**

Identify any sets of conditions in which you can neglect one of the two fluids.

IF $\frac{\mu_A}{D_A} >> \frac{\mu_B}{D_B}$, NEGLECT A (NO GRADIENTS IN A)

IF $\frac{\mu_B}{D_B} >> \frac{\mu_A}{D_A}$, NEGLECT B (NO GRADIENTS IN B)

(ANALOGOUS TO THERMAL CONDUCTANCE, $\frac{k}{L}$)

1.5 **Part D:**

Describe what you could do to guarantee mixing of these two liquids; what is the specific mathematical condition you would need to achieve?

TO GET MIXING, BOTH FLUIDS MUST BECOME TURBULANT, SO $Re_A = \frac{\rho_A V_{A,min} D_A}{\mu_A} > 1000$ & $Re_B = \frac{\rho_B V_{B,max} D_B}{\mu_B} > 1000$

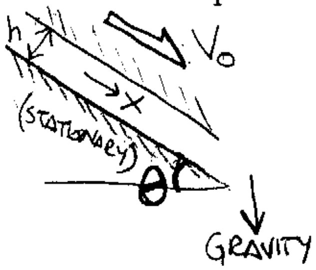
USE $V_{A,MIN}$ B/C MUST BE TURBULENT AT THE A/B INTERFACE. LIKEWISE, USE $V_{B,MAX}$ TO GET TURBULANCE AT THE INTERFACE.

Problem #4: How Neglectful Can We Be?

For each of the following problems, there are two driving forces for fluid flow. For each problem, do the following:

1. Write down the two driving forces,
2. Write the mathematical quantities you would need to compare the two forces (the units should match).
3. Write the conditions under which you can neglect the first, under which you can neglect the second, and under which you can neglect both

1.5 **Part A:** Parallel plates being sheared, oriented at an angle θ from horizontal



1. SHEAR FORCE & GRAVITY

2. $\frac{A\tau_{yx}}{VOL} = -\mu \frac{\partial v_x}{\partial y} \frac{1}{h}$ $\qquad \frac{F_g}{VOL} = g\rho \sin\theta$

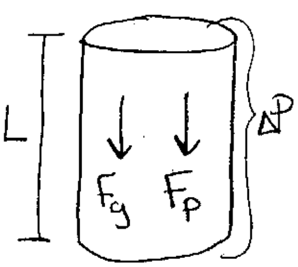
UNITS: $\frac{N}{m^2} \cdot s \, \frac{m}{s \cdot m} \, \frac{1}{m} = \frac{N}{m^3}$ ✓ $\qquad$ UNITS: $\frac{m^2}{s} \, \frac{kg}{m^3} = \frac{N}{m^3}$ ✓

3. $\dfrac{g\rho\sin\theta}{-\mu \frac{\partial v_x}{\partial y} \frac{1}{h}} = $ "C" $\qquad$ IF $C \geq 10 \rightarrow$ NEGLECT SHEAR

IF $C \leq 0.1 \rightarrow$ NEGLECT GRAVITY

CAN'T NEGLECT BOTH.

1.5 **Part B:** A vertically oriented pipe, pressurized at the top



1. PRESSURE & GRAVITY

2. $\frac{F_P}{VOL} = \frac{\Delta P}{L}$ $\qquad \frac{F_g}{VOL} = g\rho$

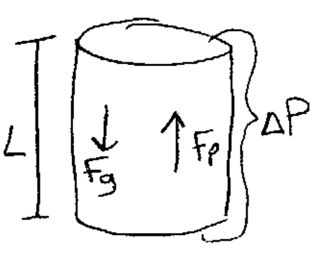
UNITS: $\frac{N}{m^3}$ ✓ $\qquad$ UNITS: $\frac{N}{m^3}$ ✓

3. LET $\frac{g\rho L}{\Delta P} = $ "D" $\qquad$ IF $D \geq 10 \rightarrow$ NEGLECT PRESSURE

IF $D \leq 0.1 \rightarrow$ NEGLECT GRAVITY

CAN'T NEGLECT BOTH.

2 **Part C:** A vertically oriented pipe, pressurized at the bottom



1. PRESSURE & GRAVITY

2. $\frac{\Delta P}{L}$ & $g\rho$ (SAME UNITS AS BEFORE)

3. $\frac{g\rho L}{\Delta P} = $ "D" $\qquad$ IF $D \geq 10 \rightarrow$ NEGLECT PRESSURE

IF $D \leq 0.1 \rightarrow$ NEGLECT GRAVITY

IF $D \cong 1$, NEGLECT BOTH B/C FORCES CANCEL

Problem #5: Laminar, Laminar, Laminar, Turbulent

Consider a transient flow problem: we have parallel plates separated by 1 cm, and we will shear a film of water between them beginning at time t = 0. The velocity of the upper plate (relative to the bottom, fixed, plate) is 1 m/s. Water properties: $\mu = 10^{-3}$ Pa-s, $\rho = 1000$ kg/m^3.

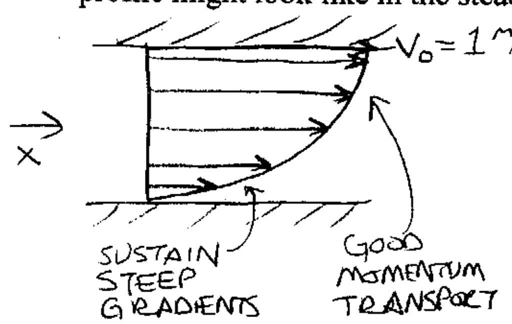
1 **Part A:** Before we think about the transient problem, let's think about the steady state, a long time after t = 0. Convince yourself (and me) that there will be turbulent flow.

$Re_{CRIT, PLATES} \sim 1000.$

$Re = \frac{\rho V_o L}{\mu} = 1000$ AT CRITICAL VELOCITY. PLUG IN VALUES FOR H_2O

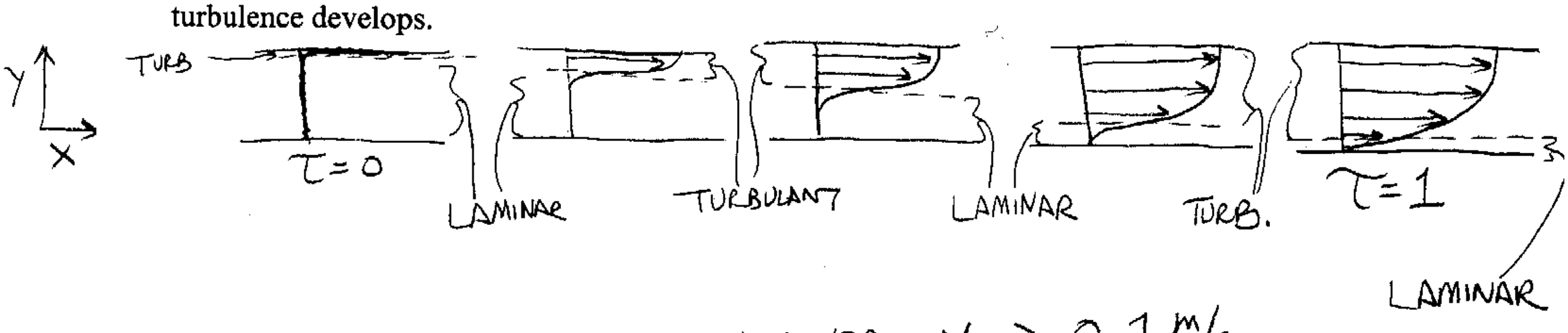
$V_{CRIT} = 0.1$ m/s. GIVEN THAT $V_o = 1$ m/s, THERE MUST BE A TURBULENT LAYER.

2 **Part B:** In light of your answer to part A, draw a schematic of what you think the average flow profile might look like in the steady state. Explain your drawing in one sentence or less



TURBULENT FLOWS ARE MORE EFFICIENT AT TRANSPORTING MOMENTUM BECAUSE THE FLOW GOES UP & DOWN, AS WELL AS THE FLOW IN THE X DIRECTION. ALSO, TURBULANT FLOWS CAN SUSTAIN STEEP SHEAR GRADIENTS, FOR THE SAME REASON.

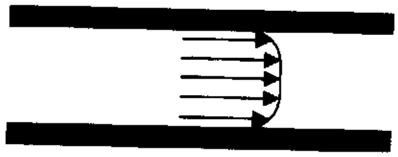
2 **Part C:** Now let's talk about the transient problem. Draw a series of flow profiles for this system as a function of time. Please draw at least a few pictures at various dimensionless times ranging from $\tau = 0$ to 1. Include in your pictures something that indicates where and when turbulence develops.



- TURBULANCE OCCURS WHEREVER $V_x > 0.1$ m/s
- TO SATISFY BOUNDARY CONDITIONS $V_x = V_o$ AT $y = L$ AND $V_x = 0$ AT $y = 0$ FOR ALL $\tau \geq 0$.
- AS τ INCREASES, MOMENTUM GRADUALLY DIFFUSES FROM THE TOP PLATE TO THE BOTTOM PLATE.

Problem #6: Snubbed

Here is a picture of a flow profile in a pressurized tube:



The profile is NOT parabolic; it is "snubbier" than would normally be expected from our classical laminar flow analysis.

Question: In the absence of any other information, can you explain why the flow profile is not parabolic?

Explain several possible answers to this question, in each case explaining how the stated answer would lead to a "snubbier" flow profile

1 **Answer A: Turbulence.**

TURBULANT FLOWS CAN SUSTAIN STEEPER SHEAR GRADIENTS, SO THE EDGES ARE STEEP, AND THE RANDOMNESS OF THE FLOW AVERAGES OUT IN THE MIDDLE TO CAUSE SHALLOWER GRADIENTS. THE WALLS OF THE TUBE LIMIT FLOW IN THE Y DIRECTION, SO NEAR THE WALLS, THE AVERAGE IS BIASED, LEADING TO A SHARP VELOCITY DROP.

2 **Answer B: Temperature effects.**

IF THE WALLS ARE HOT & THE FLUID IS COLD, THEN THE VISCOSITY AT THE WALLS WILL BE LOWER AND CAN SUPPORT STEEP GRADIENTS IN VELOCITY. THE FLUID IN THE MIDDLE IS STILL COLD, SO IT HAS A HIGHER VISCOSITY & SHALLOWER GRADIENTS.

2 **Answer C: The fluid is not Newtonian.**

FOR NON-NEWTONIAN FLOWS, THE MOMENTUM FLUX IS $\tau_{yx} = -\mu \left(\frac{\partial v_x}{\partial y}\right)^m$. $m<1$ FOR MOST MATERIALS.

$\frac{\partial v_x}{\partial y}$ IS SMALL IN THE MIDDLE OF THE FLOW (BY SYMMETRY), AND RAISING IT TO A SMALL POWER MAKES IT EVEN SMALLER, FLATTENING THE PROFILE. ALTERNATIVELY, $m<1$ IMPLIES "SHEAR THINNING", & AREAS OF HIGH SHEAR (I.E., NEAR THE WALLS) WILL SEEM TO HAVE A LOWER VISCOSITY THAN AREAS OF LOW SHEAR (I.E., IN THE MIDDLE).