3.044 MATERIALS PROCESSING

LECTURE 14

1-D Fluid Flow

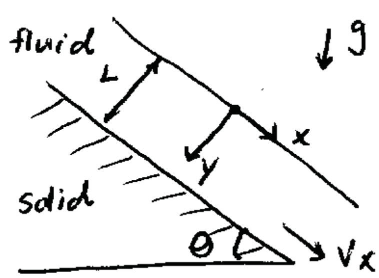
Newton's Law of Viscosity: $\boxed{\tau_{yx} = -\mu \frac{\partial V_x}{\partial y}}$

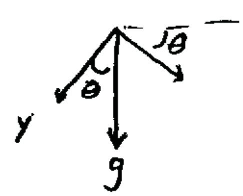
Momentum Balance: $\frac{\partial (\rho V_x)}{\partial t} = \mu \frac{\partial^2 V_x}{\partial y^2} + F_x$

Assume Incompressible: $\boxed{\frac{\partial V_x}{\partial t} = \nu \frac{\partial^2 V_x}{\partial y^2} + \frac{F_x}{\rho}}$

Let's Explore Body Force: e.g. gravity
$\Rightarrow$ no gravity for horizontal flow, more relevant for inclined flow

Falling Film:





Body Force:

$$g_x = |\bar{g}| \ \sin\theta$$
$$g_x = g \ \sin\theta$$
$$F_x = \rho \ g \ \sin\theta$$

Date: April 4th, 2012.

Steady State Flow Eq.

$$0 = \frac{\partial V_x}{\partial t} = \nu \frac{\partial V_x}{\partial y} + \frac{F_x}{\rho}$$
$$0 = \frac{\partial V_x}{\partial t} = \nu \frac{\partial V_x}{\partial y} + g \sin\theta$$

Solve:

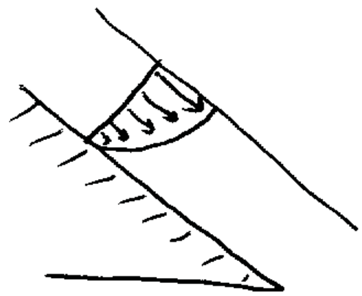
$$\frac{\partial^2 V_x}{\partial y^2} = \frac{-g\sin\theta}{\nu}$$
$$\int \mathrm{d}\left(\frac{\partial V_x}{\partial y}\right) = \int -\frac{g}{\nu}\sin\theta\,\mathrm{d}y$$
$$\int \mathrm{d}V_x = \int \left(-\frac{g}{\nu}\sin\theta\, y + A\right)\mathrm{d}y$$
$$\boxed{V_x = -\frac{g}{2\nu}\sin\theta\, y^2 + Ay + B}$$

Boundary Conditions:

$$@y = 0, \frac{\partial V_x}{\partial y} = 0 \Rightarrow \tau_{yx} = 0$$
$$@y = L, V_x = 0$$

Plug in B.C:

$$\left.\frac{\partial V_x}{\partial y}\right|_{y=0} = \left(-\frac{g}{\nu}\sin\theta\right)0 + A = 0 \Rightarrow \boxed{A = 0}$$
$$V_x|_{y=L} = -\frac{g}{2\nu}\sin\theta\, L^2 + 0(y) + B = 0 \Rightarrow \boxed{B = -\frac{g}{2\nu}\sin\theta\, L^2}$$
$$\boxed{V_x = -\frac{g\sin\theta}{2\nu}\left(L^2 - y^2\right)}$$



Maximum velocity occurs at $y = 0$:

$$V_{max} = \frac{g \sin\theta}{2\nu} L^2$$

Average velocity:

$$V_{avg} = \frac{\int_0^L V_x \, \mathrm{d}y}{\int_0^L \, \mathrm{d}y} = \frac{g \sin\theta}{3\nu} L^2$$

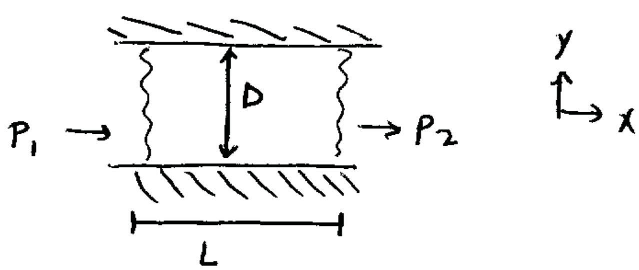
$$\boxed{V_{avg} = \frac{2}{3} V_{max}}$$

Net Flow Rate:

$$Q = V_{avg} \cdot A$$

$\Rightarrow$ where A is the cross-sectional area of flow

Pressure Driven Flow:



Force Balance:

$$F_{net} = (P_1 - P_2)\, D \cdot W$$

Force Per Volume:

$$F_p = \frac{(P_1 - P_2)\, D \cdot W}{(D \cdot W \cdot L)}$$
$$F_p = \frac{\Delta P}{L}$$

Full Equation for Flow:

$$\frac{\partial V_x}{\partial t} = \nu \frac{\partial^2 V_x}{\partial y^2} + \frac{F_x}{\rho} + \frac{\Delta P}{\rho L}$$

1-D Fluid Flow Equation:

$$\boxed{\frac{\partial V_x}{\partial t} = \nu \frac{\partial^2 V_x}{\partial y^2} + \frac{F_x}{\rho} - \frac{1}{\rho}\frac{\partial P}{\partial x}}$$

Assume Body Force is Zero:

$$\frac{\partial V_x}{\partial t} = \nu \frac{\partial^2 V_x}{\partial y^2} + \overbrace{\frac{F_x}{\rho}}^{=0} - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

Steady State:

$$\frac{\partial^2 V_x}{\partial y^2} = \frac{1}{\nu \rho}\frac{\Delta P}{L}$$

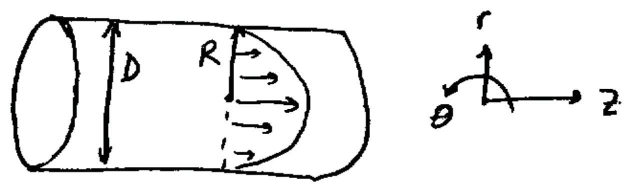
Boundary Conditions:

$$@y = 0, V_x = 0$$
$$@y = D, V_x = 0$$

Pressurized Flow Between Horizontal Plates:

$$\boxed{V_x = \frac{\Delta P}{8\mu L}\left(D^2 - 4y^2\right)}$$

Pressurized Flow In a Tube:



$$\boxed{V_z = \frac{\Delta P}{4\mu L}\left(R^2 - r^2\right)}$$

$$\boxed{V_{avg} = \frac{\int_0^R V_z \, 2\pi r \, \mathrm{d}r}{\int_0^R 2\pi r \, \mathrm{d}r} = \frac{\Delta P R^2}{8\mu L}}$$

Summary and Comparison:

	Chemical Diffusion	**Heat Conduction**	**Fluid Flow**
Conserved Quantity	moles of solute	joules of energy	kg$\frac{\mathrm{m}}{\mathrm{s}}$ of momentum
Local Density of It	c	$(\rho c_p)\, T$	$(\rho)\, \bar{V}$
Flux (1-D)	Fick's 1st: $j = -D\frac{\partial c}{\partial x}\left[\frac{mol}{m^2\, s}\right]$	Fourier: $q = -k\frac{\partial T}{\partial x}\left[\frac{J}{m^2\, s}\right]$	Newton: $\tau_{yx} = -\mu\frac{\partial V_x}{\partial y}\left[\frac{kg}{m\, s^2}\right]$
Conservation Equation (1-D)	$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}(j) + G$	$\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial}{\partial x}(q) + \dot{q}$	$\frac{\partial(\rho V_x)}{\partial t} = -\frac{\Delta P}{L} - \frac{\partial}{\partial x}\tau_{yx} + F_x$
Diffusion Equation (1-D)	$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} + G$	$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{c_p \rho}$	$\frac{\partial V_x}{\partial t} = \nu\frac{\partial^2 V_x}{\partial y^2} + \frac{F_x}{\rho} - \frac{1}{\rho}\frac{\partial P}{\partial x}$
Diffusivity	D $\frac{m^2}{s}$	$\alpha = \frac{k}{c_p \rho}\frac{m^2}{s}$	$\nu = \frac{\mu}{\rho}\frac{m^2}{s}$
Flux (3-D)	$j = -D\nabla C$	$q = -k\nabla T$	$\tau = -\mu\left(\nabla\bar{V} + \left(\nabla\bar{V}\right)^T\right)$
Diffusion Equation (3-D)	$\frac{\partial c}{\partial t} = D\nabla^2 c + G$	$\frac{\partial T}{\partial t} = \alpha\nabla^2 T + \frac{\dot{q}}{\rho c_p}$	$\frac{\partial \bar{V}}{\partial t} = \nu\nabla^2\bar{V} + \frac{\bar{F}}{\rho}\frac{\Delta P}{\rho}$

Plate Glass

· 1500-1600 modified glass blowing

· 1700 cast large blocks + grinding + polishing

· 1930 continuous rolling + grinding + polishing ($\sim$25%)

· 1970 Pilkington Process: pour molten glass onto a **liquid** mold
$\Rightarrow$ need liquid with **special properties**

3.044 Materials Processing
Spring 2013