Honeycomb-like materials in nature: wood

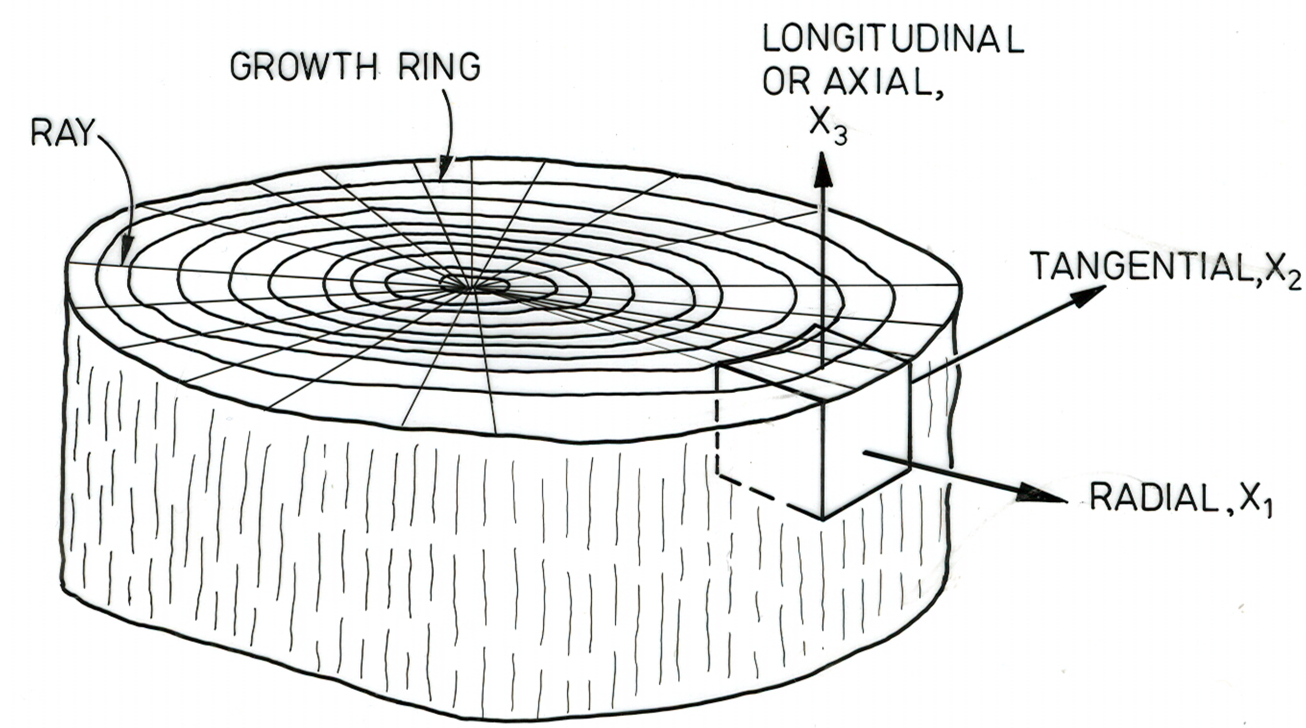
- "materials" derives from Latin "materies, materia" means wood or trunk of a tree
- old Irish – names of first letters of the alphabet refer to woods

A	alem	=	elm
B	beith	=	birch
C	coll	=	hazel
D	dair	=	oak

Wood - structure

- orthotropic (if neglect curvature of growth rings)
- ρ^*/ρ_s ranges from 0.05 (balsa) to 0.80 (lignum vitae)
- trees have cambial layer, beneath bark
- cell division @ cambial layer
  - new cells on outer part of cambial layer → bark
  - " " " inner " " " " → wood

Wood structure



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

- living plant cells - plasma membrane + protoplast
- living cells secrete plant cell wall - analogous to extracellular matrix in animal tissues
- in trees, cells lay down cell wall over a few weeks, then die
- always retain a cambial layer of cells

Cellular structure: softwoods

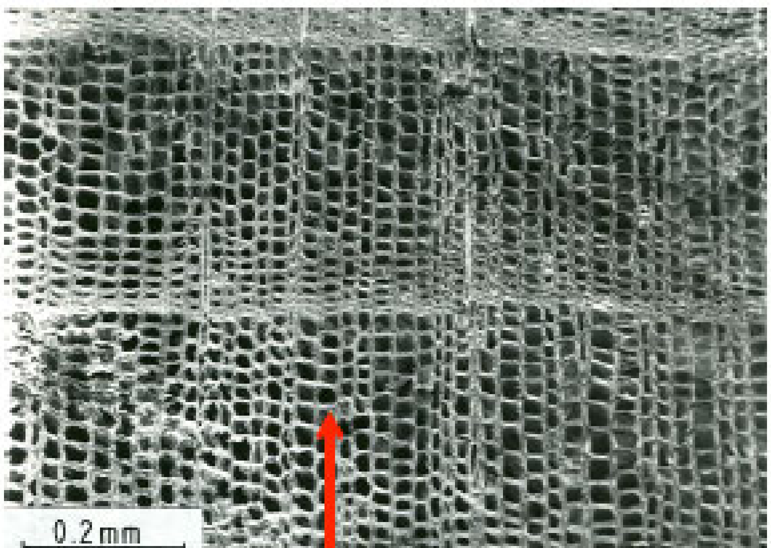
- tracheids - bulk of cells (90%), provide structural support
  - have holes in cell wall for fluid transport (pits)
  - ~ 2.5 - 7.0 mm long; 20 - 80 μm across; t = 2-7 μm

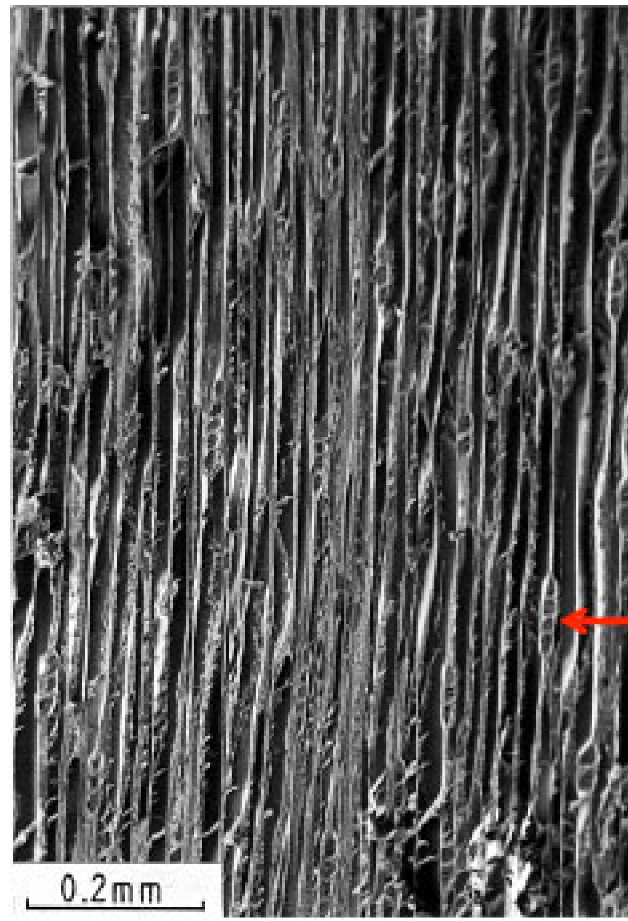
- rays - radial arrays of smaller parenchyma cells that store sugars

Cellular structure: hardwoods

- fibers provide structural support; 35-70 % of cells
- vessels - sap channels - conduction of fluids 6-55% of cells
- rays - store sugars; 10-30% of cells

Softwood: Cedar





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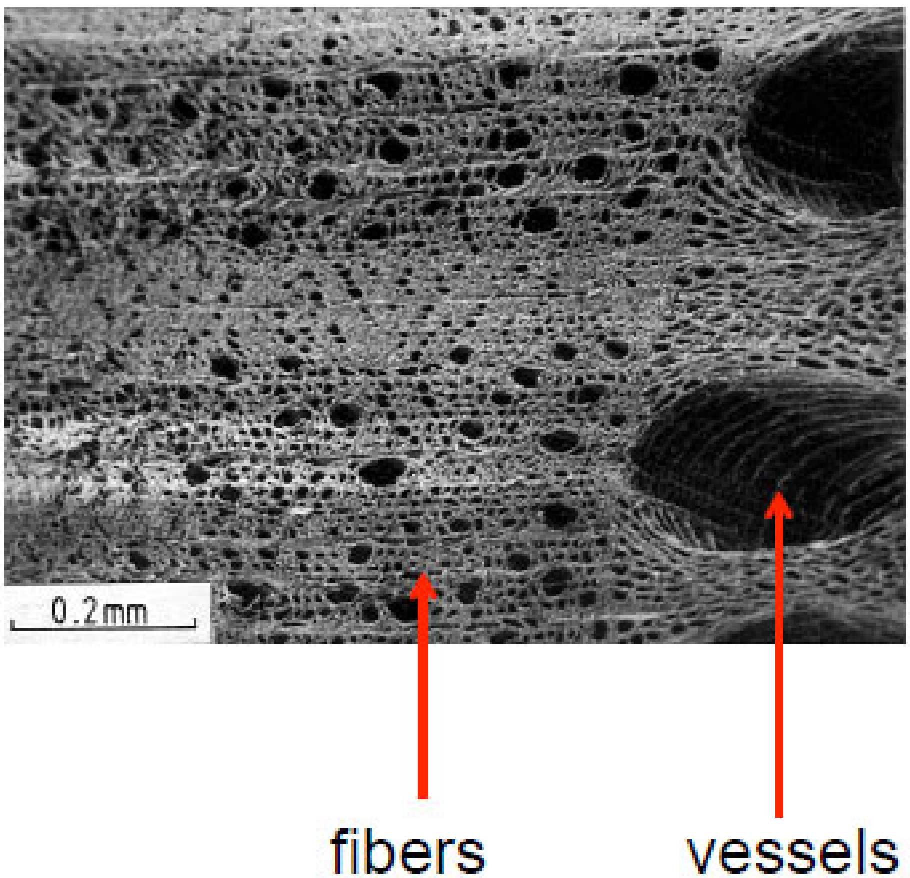
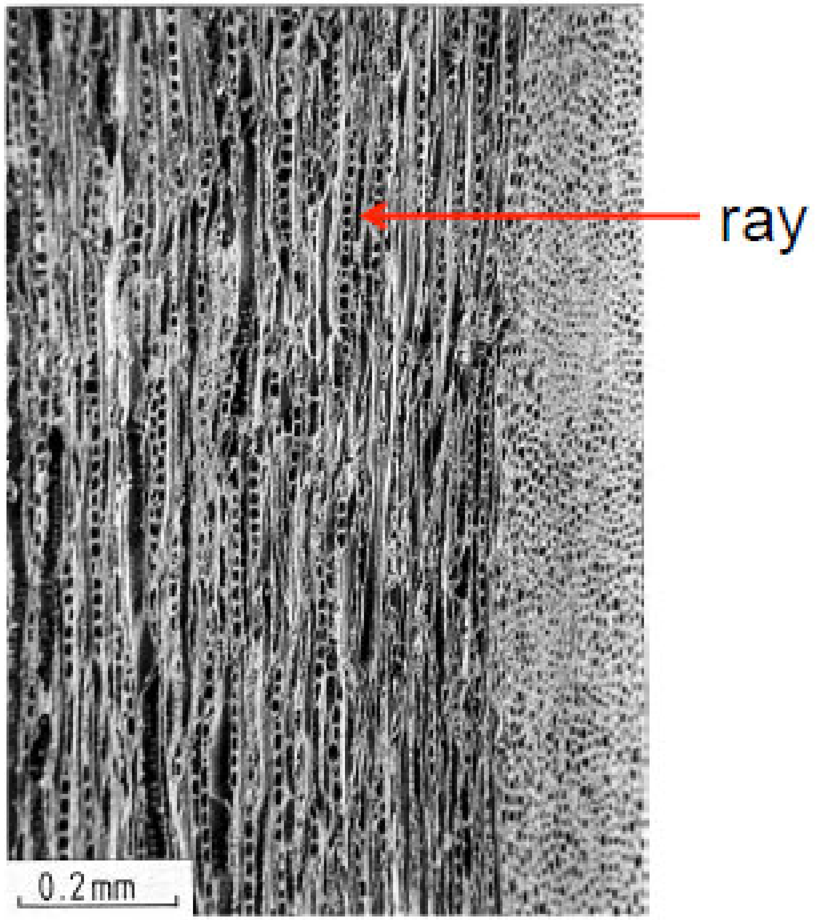
Hardwood: Oak





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Structure: cell wall

- fiber reinforced composite
- cellulose fibers in matrix of lignin / hemicellulose
- 4 layers, each with fibers at different orientation
- Between 2 cells: middle lamella

Cell wall properties

- similar in different species of wood

$\rho_s = 1500\ kg/m^3$

$E_{sA} = 35\ GPa$

$E_{sT} = 10\ GPa$

$\sigma_{ysA} = 350\ MPa$

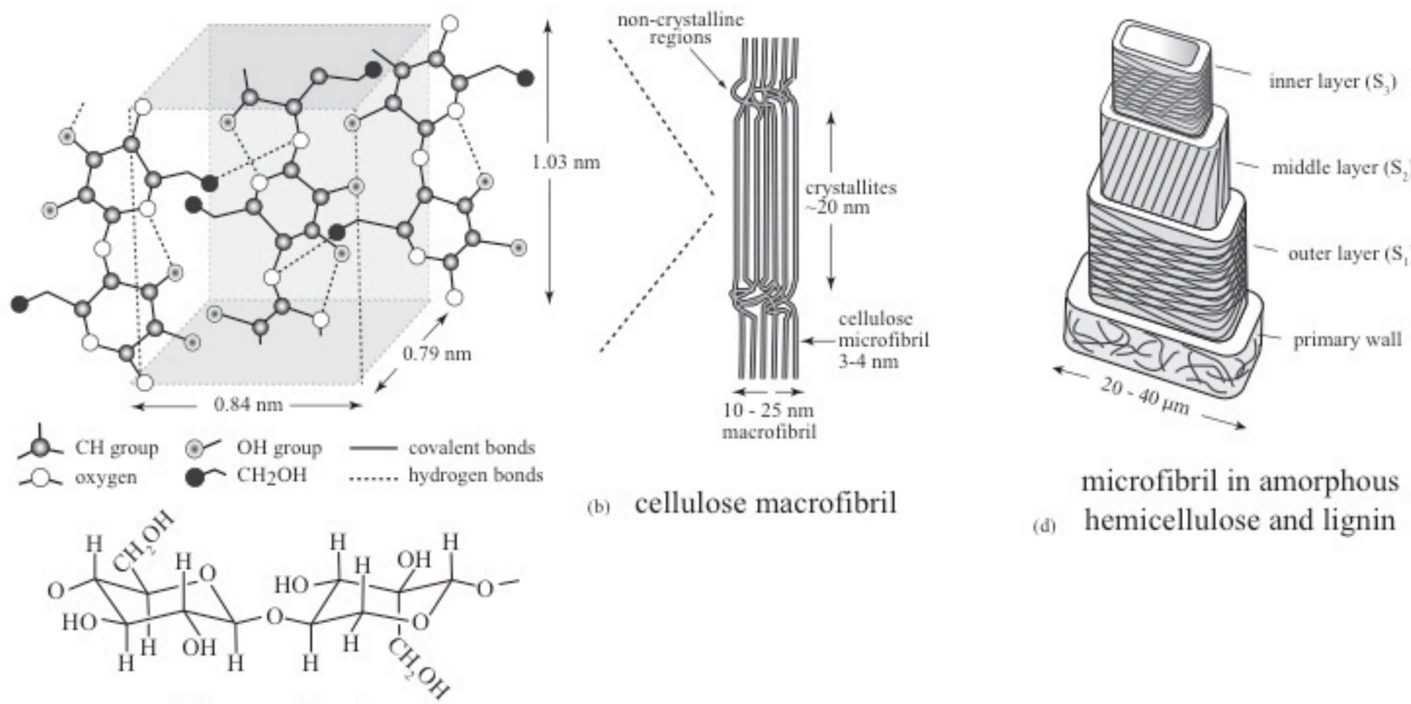
$\sigma_{ysT} = 135\ MPa$

(Note cellulose: $E \sim 140\ GPa$
$\sigma_y \sim 750\ MPa$)

A = axial direction

T = transverse direction

Wood Structure



(a) cellulose molecule

(b) cellulose macrofibril

(d) microfibril in amorphous hemicellulose and lignin

Stress-strain curves

- $\sigma - \epsilon$ curves resemble those for honeycombs
- mechanisms of deformation most easily identified on low density balsa
- curves + images for balsa
- tangential loading: formation of plastic hinges in bent cell walls
- radial loading: rays act as reinforcing; plastic yielding in cell walls
  - starts at platens + moves inward
- axial loading: axial def$^{\underline{n}}$ of cell walls; then break end caps
  serrations correspond to each layer of end caps breaking.

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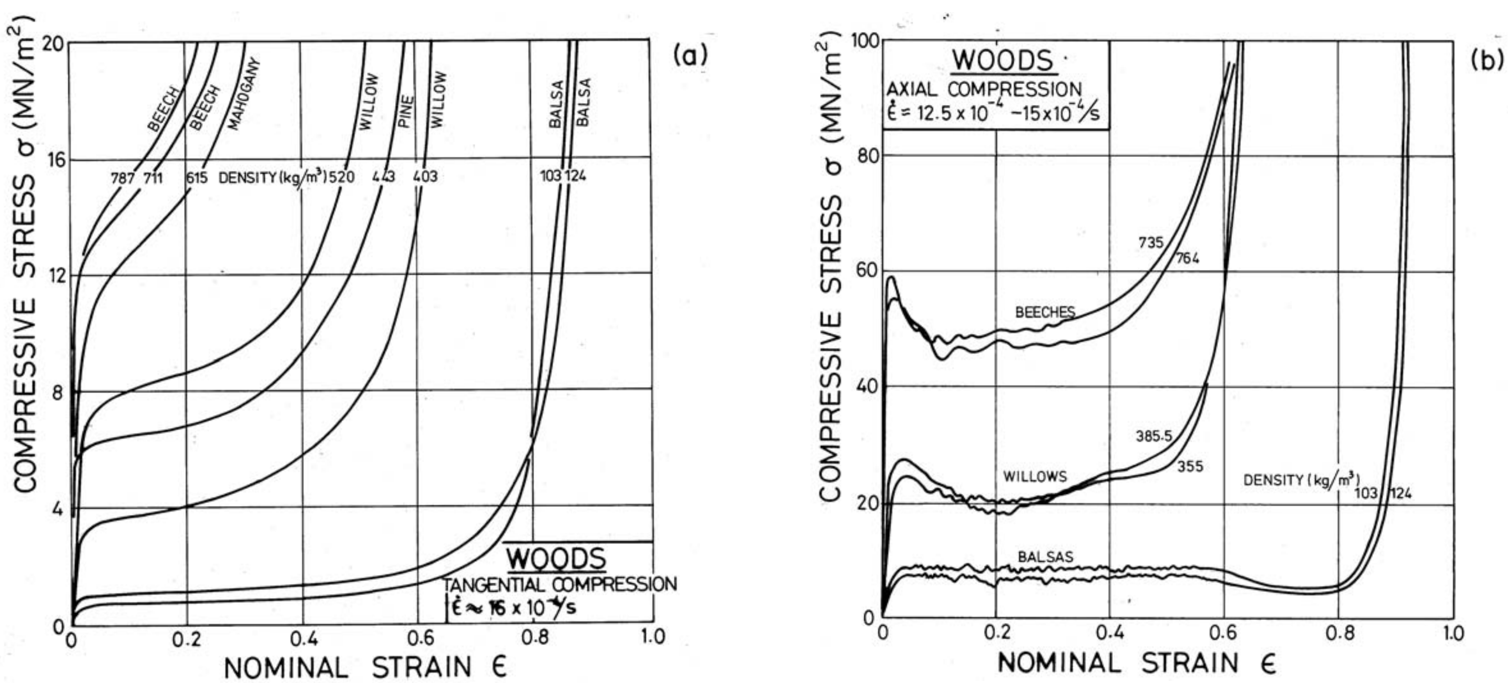
failure by plastic buckling + formation of kink bands also observed

- denser species

  Douglas fir - tangential, radial compression

  Norway spruce - axial compression

Stress strain curves



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Balsa

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Balsa: Tangential

Figure removed due to copyright restrictions. See Figure 4: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

Figure removed due to copyright restrictions. See Figure 7: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

Balsa: Radial

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Balsa: Axial

Figure removed due to copyright restrictions. See Figure 6: Easterling, K. E., R. Harrysson, et al. "On the Mechanics of Balsa and Other Woods." *Proceedings The Royal of Society. A* 383, no. 1784 (1982): 31-41.

Douglas Fir: Tangential Comp

Douglas fir: Radial comp.

Norway spruce: Axial comp

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Data for wood

$$E^*/E_s \propto \rho^*/\rho_s \quad \text{(axial)}$$

$$E^*/E_s \propto (\rho^*/\rho_s)^3 \quad \text{(tangential; radial somewhat stiffer)}$$

$$\sigma^*/\sigma_{ys} \propto (\rho^*/\rho_s) \quad \text{(axial)}$$

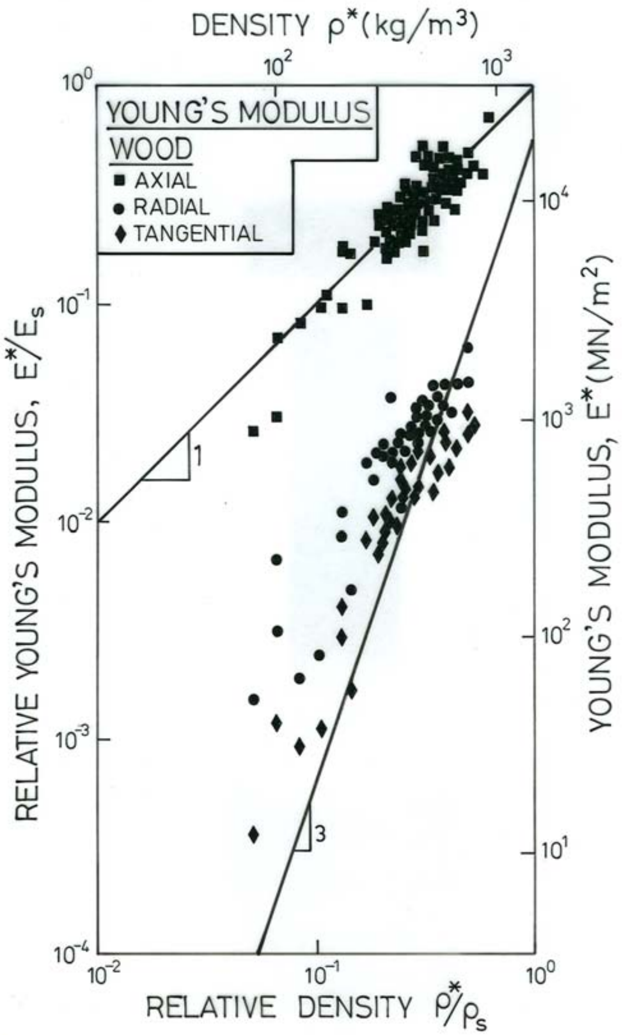
$$\sigma^*/\sigma_{ys} \propto (\rho^*/\rho_s)^2 \quad \text{(tangential/radial)}$$

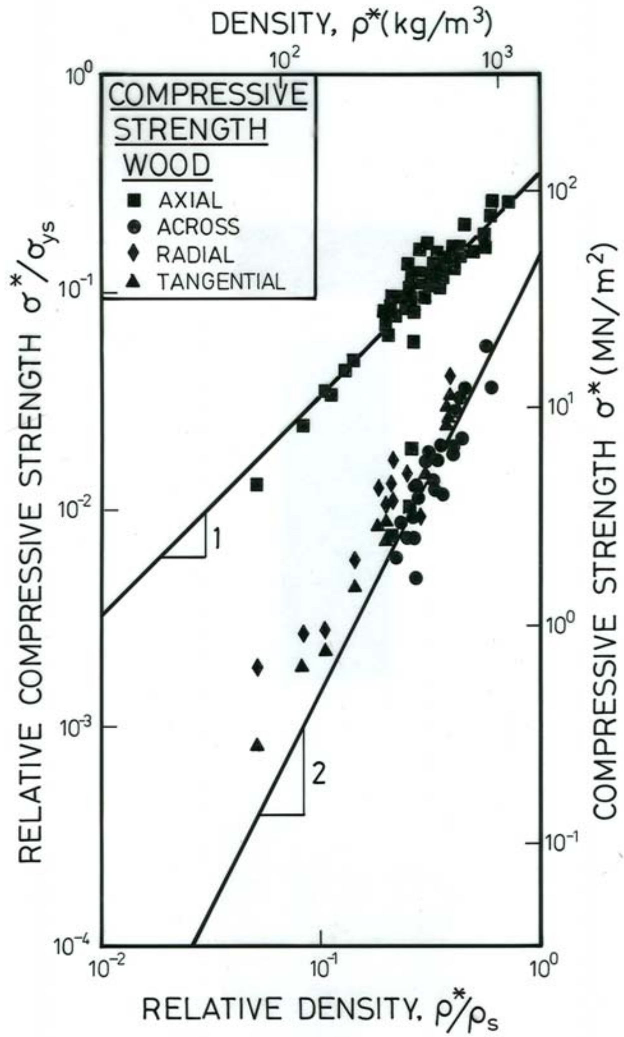
$\nu^*_{RT} \sim 0.5 - 0.8$ $\nu^*_{RA} \sim 0.02 - 0.07$ $\nu^*_{AR} \sim 0.25 - 0.5$

$\nu^*_{TR} \sim 0.2 - 0.6$ $\nu^*_{TA} \sim 0.01 - 0.04$ $\nu^*_{AT} \sim 0.35 - 0.5$

Modelling wood properties

- very simplified model - first order
- does not attempt to capture finer details (eg. softwoods vs. hardwoods)
- cell wall has been modelled as fiber composite; it is itself anisotropic
- we normalize all properties with respect to E_s, σ_{ys} axial
- constant of proportionality also reflects cell wall anisotropy





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Model for wood microstructure

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Linear elastic moduli

- tangential loading – model as honeycomb – cell wall bending

  $E^*_T / E_s \propto (\rho^*/\rho_s)^3$

  - rays, end caps act to stiffen wood – data lie slightly above $(\rho/\rho_s)^3$

- radial loading – rays act as reinforcing plates + are higher density than fibers

  V_R = volume fraction of rays

  $R = (\rho^*/\rho_s)_{rays} / (\rho^*/\rho_s)_{fibers} \approx 1.1$ to 2

  $E^*_R = V_R R^3 E^*_T + (1-V_R) E^*_T$

  $\approx 1.5 E^*_T$

  - E^*_R slightly larger than E^*_T; $\propto (\rho/\rho_s)^3$

- axial loading
  - axial deformation in cell wall

  $E^*_A / E_s \propto (\rho^*/\rho_s)$

- explains, to first order,
  - density dependence
  - anisotropy

Modelling - Poisson's ratios

		Model	
ν^*_{RT}	$= 0.5 - 0.8$	1	constraining effect of rays + end caps
ν^*_{TR}	$= 0.2 - 0.6$	1	
ν^*_{RA}	$= 0.02 - 0.07$	0	
ν^*_{TA}	$= 0.01 - 0.04$	0	
ν^*_{AR}	$= 0.25 - 0.5$	ν_s	data close to $0.4 \sim \nu_s$
ν^*_{AT}	$= 0.35 - 0.5$	ν_s	

Modelling - compressive strength

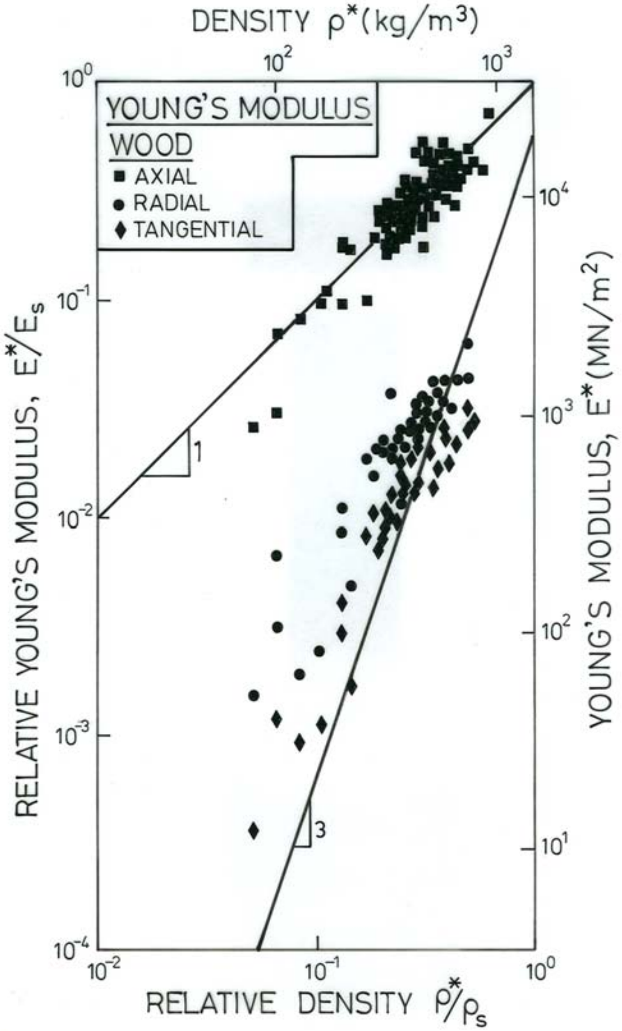
- tangential loading - bending, plastic hinges $\sigma^*_T / \sigma_{ys} \propto (\rho^*/\rho_s)^2$
- radial loading - $\sigma^*_R = V_R R^2 \sigma^*_T + (1 - V_R) \sigma^*_T$

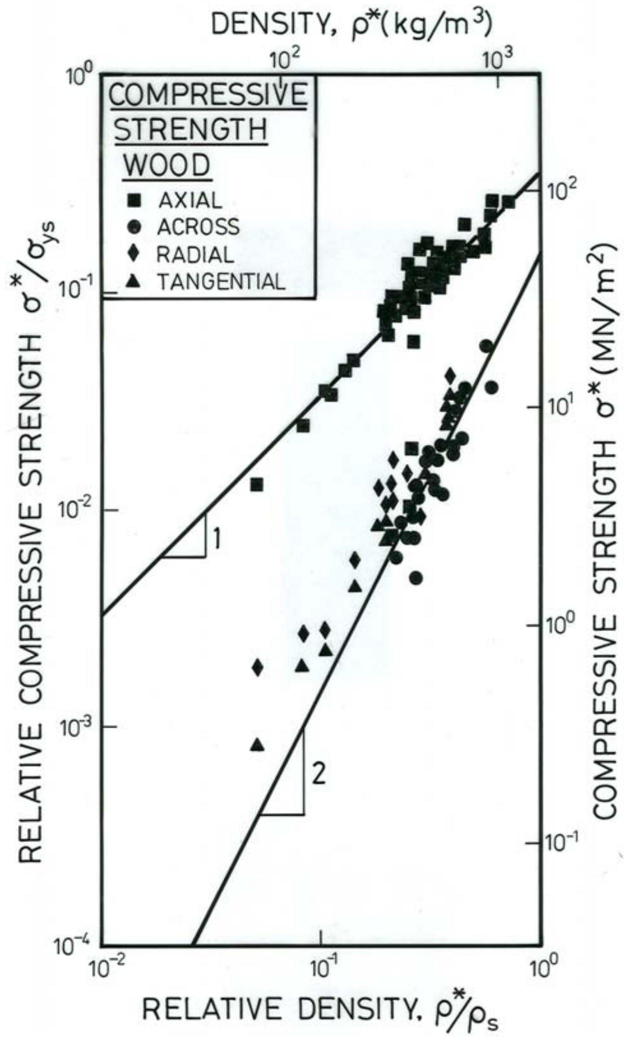
  balsa : $V_R \sim 0.14$ $R \sim 2$ $\sigma^*_R = 1.4 \sigma^*_T$

  higher density woods - R smaller

  σ^*_R slightly larger than σ^*_T; both $\propto (\rho^*/\rho_s)^2$
- axial loading - initial failure by axial yield (then end cap fracture, or buckling)

  $\sigma^*_A / \sigma_{ys} \propto \rho^*/\rho_s$





Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

Modelling: cell wall + cellular structure

- cell wall can be modelled as a fiber composite

  cellulose E ~ 140 GPa   lignin/hemicellulose E ~ 2 GPa

  composite upper + lower bounds give envelope at right of figure

  measured values for $E_{s\,Axial}$ = 35 GPa $E_{s\,transverse}$ = 10 GPa

- can also show cellular solids model on same plot

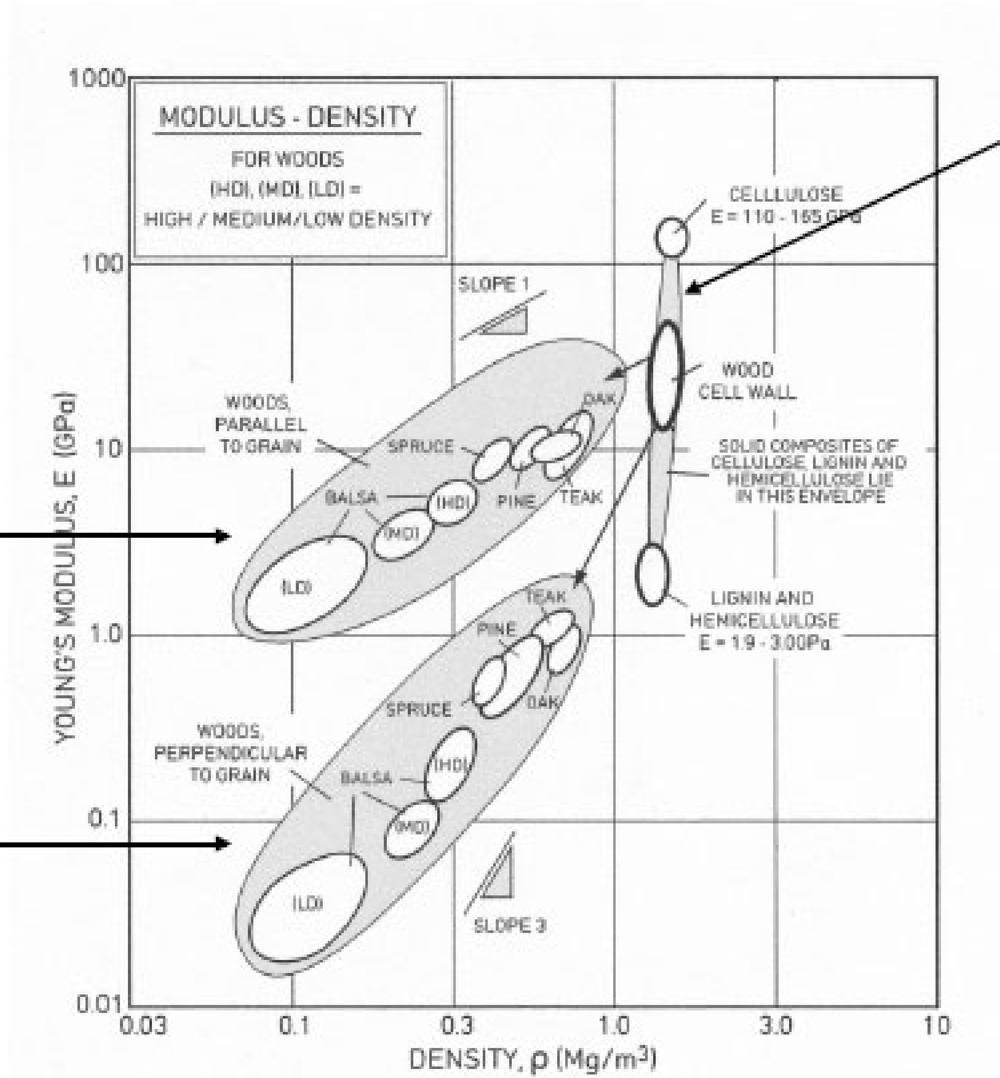
- overall, plot shows how wood hierarchical structure, density variation give wood moduli that vary by a factor of 1000

- can make similar plot for strength

Wood: Honeycomb Models

$$\frac{E^*}{E_{s\,along}} = \frac{\rho^*}{\rho_s}$$

$$\frac{E^*}{E_{s\,across}} = \left(\frac{\rho^*}{\rho_s}\right)^3$$



Cell wall: Fiber Composite Model

Gibson, L. J., and M. F. Ashby. *Cellular Materials in Nature and Medicine.* 2nd ed. Cambridge University Press, © 2010. Figure courtesy of Lorna Gibson and Cambridge University Press.

Wood: Honeycomb Models

Diagram removed due to copyright restrictions. See Figure 5b: Gibson, L. J. "The Hierarchical Structure and Mechanics of Plant Materials." *Journal of the Royal Society Interface* 9 (2012): 2749-66.

Material selection

- for a beam of a given stiffness, P/δ, length, l, square cross-section with edge length, t, what material minimizes the mass of the beam?

$$m = \rho t^2 l$$

$$\delta = \frac{Pl^3}{CEI} \qquad \frac{P}{\delta} = \frac{CEt^4}{l^3} \qquad t^2 = \left[\left(\frac{P}{\delta}\right)\frac{l^3}{CE}\right]^{1/2}$$

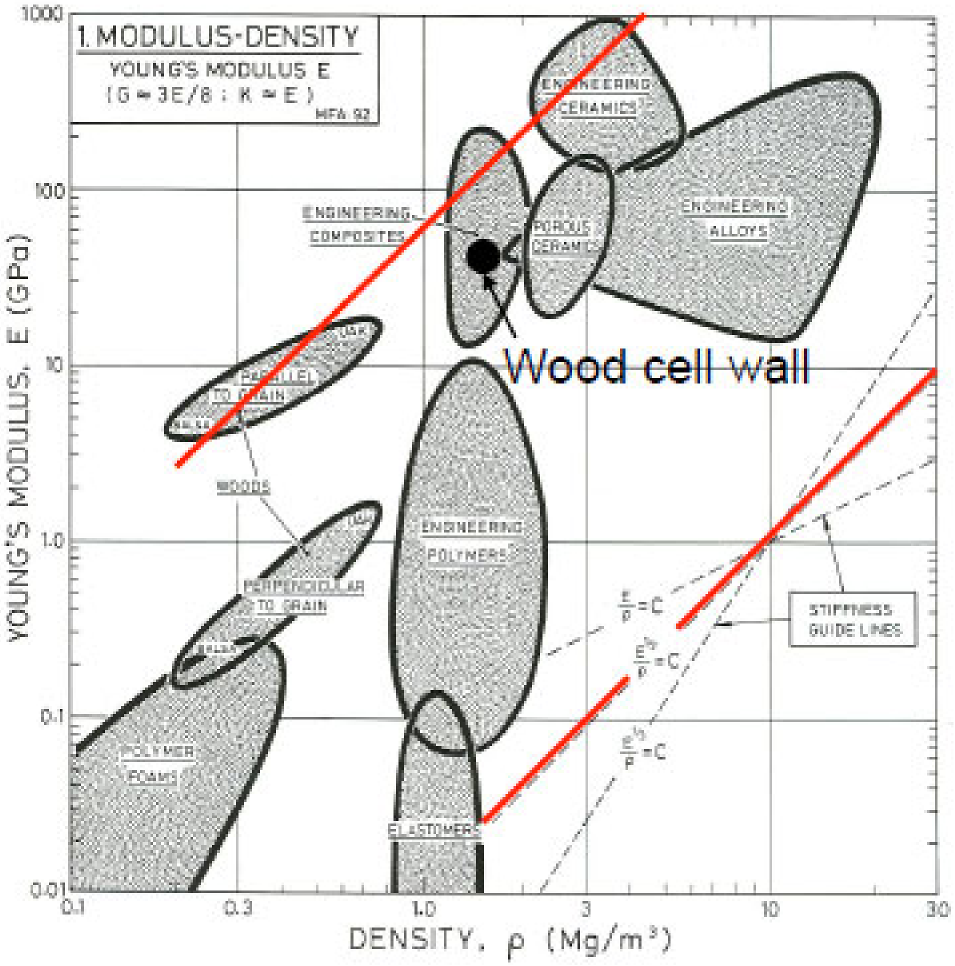
$$m = \rho\left[\left(\frac{P}{\delta}\right)\frac{l^3}{CE}\right]^{1/2} l$$

to minimize mass, choose material with min. $\rho/E^{1/2}$ or maximize $E^{1/2}/\rho$.

- material selection chart: plot $\log E$ vs $\log \rho$
- line of constant $E^{1/2}/\rho$ shown in red on plot
- materials with largest values of $E^{1/2}/\rho$ at upper left of plot
- woods have similar values of $E^{1/2}/\rho$ as engineering composites
- note that tree trunks, branches, loaded primarily in bending.
- also note, from models, $\frac{(E^*)^{1/2}}{\rho^*} = \frac{E_s^{1/2}}{\rho_s} \cdot \left(\frac{\rho_s}{\rho}\right)^{1/2}$

  ⇒ performance index for wood higher than that for the solid cell wall
- similarly for strength in bending

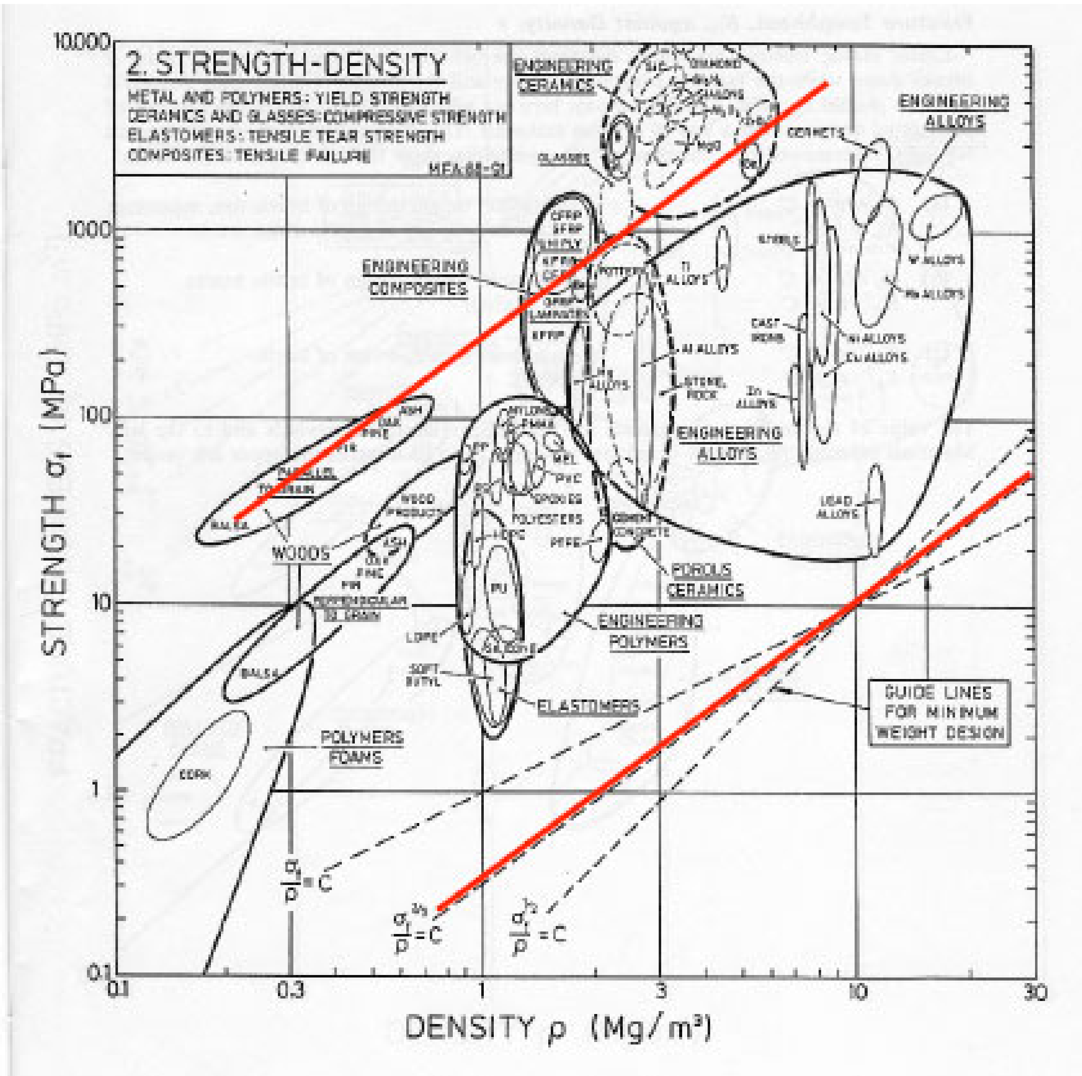
Wood in Bending: $E^{1/2}/\rho$



$$\frac{\left(E^*\right)^{1/2}}{\rho^*} = \frac{\left(E_s\right)^{1/2}}{\rho_s}\left(\frac{\rho_s}{\rho^*}\right)^{1/2}$$

Stiffness performance index for wood in bending is similar to that for best engineering composites

Wood in Bending: $\sigma_f^{2/3}/\rho$



$$\frac{\left(\sigma_f^*\right)^{2/3}}{\rho^*} = \frac{\left(\sigma_{ys}\right)^{2/3}}{\rho_s}\left(\frac{\rho_s}{\rho^*}\right)^{1/3}$$

Strength performance index for wood in bending is similar to that for best engng composites

Wood Use in Design

Historical example: 17th century wooden ships

- colonial times, importance of navies to colonial powers
- used particular species for different parts of ship, based on their properties
- oak - used for much of the hull, ribs, knees, planking ⇒ dense wood; stiff + strong
  - "straight oak" - straight pieces, cut from trunk
  - "compass oak" - curved pieces from trunk + branch, so that grain runs along curved, cut piece ⇒ max E, σ*
    - used for knees, wing transom - curved pieces of ship hull.

- eastern white pine - British Royal Navy used for masts, imported from New England
  - England had run out of tall straight trees for masts
  - strategic resource - ship speed, size depended on size of mast + sail area
  - Eastern white pine known for straight, tall trunks; some over 100' tall
- lignum vitae - densest wood; acts as own lubricant
  - used in block + tackle
  - also used in clock gears ~1760 — H4 1759 lost 5 seconds in 81 days at sea
  - John Harrison's chronometer - story of Longitude, Dava Sobel

Figure removed due to copyright restrictions. See *The international book of wood*. Bramwell, M, ed. Artists House, 1982. pp 186-87.

Modern example: glue laminated timber

- glue long pieces of wood, typically 1-2" thick, together
- select strips to avoid defects (eg. knots)
- glu-lam has better mechanical properties than sawn lumber.
- also, can make curved members by using curved molds & clamps during bonding process

  ⇒ grain runs along the curve - exploits high stiffness + strength of wood along the grain

  ⇒ architecturally attractive

Image of graceful glued-laminated timber arch bridge removed due to copyright restrictions. See Figure 13: *Engineered Wood Products: A Guide for Specifiers, Designers and Users*. Smulski, S., ed. PFS Research Foundation, 1997.

Engineered Wood Products: A Guide for Specifiers, Designers and Users, S. Smulski Ed. PFS Research Foundation, 1997

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