Lecture 9 Thermal Notes, 3.054

Thermal Properties of Foams

- Closed cell foams widely used for thermal insulation
- Only materials with lower conductivity are aerogels (tend to be brittle and weak) and vacuum insulation panels
- Low thermal conductivity of foam arises from:
  - low volume fraction of solid
  - high volume fraction of gas with low λ
  - small cell size suppresses convection and radiation (through repeated absorption and reflection)
- Applications: buildings, refrigerated vehicles, LNE tankers
- Foams also have good thermal shock resistance since coefficient of thermal expansion of foam equals to that of the solid; plus the modulus is much lower ($\epsilon = \alpha \Delta T \qquad \sigma = E\alpha\Delta T = \sigma_f$)
  $\Rightarrow$ used as heat shields
- Ceramic foams used as firebrick — ceramic has high T
  — foam - low λ - low heat loss
  — low heat capacity - lowers energy to heat furnace to temperature
  — good thermal shock resistance

Thermal conductivity, λ

- Steady state conduction (T constant with time)

$$\text{Fourier Law:} \quad q = -\lambda \nabla T$$

$$\text{1D} \quad q = -\lambda \frac{dT}{dx}$$

$$\begin{aligned} q &= \text{hect flux [J/(m}^2\text{/s)]} \\ \lambda &= \text{thermal conductivity [W/mK]} \\ \nabla T &= \text{temperature gradient} \\ &= i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z} \end{aligned}$$

- Non-steady heat conduction (T varies with time t)

$$\frac{\partial T}{\partial \tau} = a\,\frac{\partial^2 T}{\partial x^2}$$

$$a = \text{thermal diffusivity} \quad = \frac{\lambda}{\rho\, C_p}$$

$$[\text{m}^2/\text{s}]$$

$$\begin{aligned} \rho &= \text{density} \\ C_p &= \text{specific heat - heat required to} \\ &\quad\ \text{raise the temperature of unit mass by } 1^\circ\text{K} \\ \rho\, C_p &= \text{volumetric heat capacity [J/m}^3\text{K]} \end{aligned}$$

- Values for λ, a Table 7.1

Table 7.1 Thermal conductivities and diffusivities

Material	Thermal conductivity λ(W/m K)	Thermal diffusivity a (m^2/s)
Copper (solid)	384[a]	8.8×10^{-5} [a]
Aluminium (solid)	230[a]	8.9×10^{-5} [a]
Alumina (solid)	25.6[a]	8.2×10^{-6} [a]
Glass (solid)	1.1[a]	4.5×10^{-7} [a]
Polyethylene (solid)	0.35[a]	1.7×10^{-7} [a]
Polyurethane (solid)	0.25[c]	
Polystyrene (solid)	0.15[a]	1.0×10^{-7} [a]
Air	0.025[a]	–
Carbon dioxide	0.016[a]	–
Trichlorofluoromethane (CCl_3F)	0.008[a]	–
Oak ($\rho^*/\rho_s = 0.40$)	0.150[a]	–
White pine ($\rho^*/\rho_s = 0.34$)	0.112[a]	–
Balsa ($\rho^*/\rho_s = 0.09$)	0.055[a]	–
Cork ($\rho^*/\rho_s = 0.14$)	0.045[a]	–
Polystyrene foam ($\rho^*/\rho_s = 0.025$)	0.040[b]	1.1×10^{-6} [b]
Polyurethane foam ($\rho^*/\rho_s = 0.02$)	0.025[b]	9.0×10^{-7} [b]
Polystyrene foam ($\rho^*/\rho_s = 0.029$–0.057)	0.029–0.035[d]	
Polyisocyanurate foam, (CFC-11) ($\rho^* = 32\ kg/m^3$)	0.020[d]	
Phenolic foam, (CFC-11, CFC-113) ($\rho^* = 48\ kg/m^3$)	0.017[d]	
Glass foam ($\rho^*/\rho_s = 0.05$)	0.050[d]	
Glass wool ($\rho^*/\rho_s = 0.01$)	0.042[d]	
Mineral fibre ($\rho^*/\rho_s = 4.8$–$32\ kg/m^3$)	0.046[d]	

All values for room temperature.

References

[a]Handbook of Chemistry and Physics, 66th edn (1985–6) Chemical Rubber Co. ed. R. C. Weast.

[b]Patten, G. A. and Skochdopole, R. E. (1962) *Mod. Plast.*, **39**, 149.

[c]Schuetz, M. A. and Glicksman, L. R. (1983) *Proc. SPI 6th International Technical/Marketing Conference*, pp. 332–40.

[d]Glicksman, L. R. (1994) Heat transfer in foams, in *Low Density Cellular Plastics* ed. Hilyard, N. C. and Cunningham, A. Chapman and Hall.

Data for thermal conductivity and thermal diffusivity

Thermal diffusivity, a

- Materials with a high value of a rapidly adjust their temperature to that of surroundings, because they conduct hear rapidly in comparison to their volumetric heat capacity; do not require much energy to reach thermal equilibrium
  e.g. Cu $a = 112 \times 10^{-6}$ m^2/s
  nylon $a = 0.09 \times 10^{-6}$ m^2/s
  wood $a = 0.082 \times 10^{-6}$ m^2/s

Thermal conductivity of a foam, λ^*.

λ^* — contributions from — conduction through solid, λ_s^*
— conduction through gas, λ_g^*
— convection within cells, λ_c^*
— radiation through cell walls and across voids, λ_r^*

$\lambda^* = \lambda_s^* + \lambda_g^* + \lambda_c^* + \lambda_r^*$

- Conduction through solid: $\lambda_s^* = \eta \lambda_s (\rho^*/\rho_s)$ $\quad \eta =$ efficiency factor $\sim 2/3$
- Conduction through gas: $\lambda_g^* = \lambda_g (1 - \rho^*/\rho_s)$

For example, 2.5% dense closed-cell polystyrene foam:

$$\lambda^* = 0.040 \text{ W/mK}; \ \lambda_s^* = 0.15 \text{ W/mK}; \ \lambda_g^* = 0.025 \text{ W/mK (air)}$$

$$\begin{aligned}\lambda_s^* + \lambda_g^* &= 2/3\,(0.15)(0.025) + (0.025)(0.975)\\ &= 0.003 + 0.024\\ &= 0.027\, W/mK\end{aligned}$$

- Most of conductivity comes from conduction through gas
- Foams for isolation blown with low λ_g gases
- Problem with aging — low λ_g gases diffuse out of foam over time, air diffuses in; $\lambda_g^* \uparrow$

Convection within the cell

- Gas rises and falls due to density changes with temperature
- Density changes — buoyancy forces
- Also have viscous forces from drag of gas as it moves past cell wall

Convection is important when Rayleigh number > 1000

$$R_a = \frac{\rho g \beta \, \Delta T_c \, l^3}{\mu a}$$

ρ = density of gas
g = grav. acceleration
β = volume expansion for a gas $= 1/T$ (isobaric)
ΔT_c = temp. diff. across the cell
l = cell size
μ = dynamic viscosity of gas
a = thermal diffusion

Convection

For $R_a = 1000$ **air** $\quad p = p_{\text{atm}} \qquad T = \text{room temp} \qquad \beta = 1/T = 1/300\ (^\circ K^{-1})$.

$$\Delta T_c = 1^\circ K \qquad \mu_{\text{air}} = 2 \times 10^{-5}\ \text{Pa·s} \qquad \rho_{\text{air}} = 1.2\ \text{kg/m}^3$$

$$a_{\text{air}} = 2.0 \times 10^{-5} m^2/s$$

$$\Rightarrow \quad l = 20\ \text{mm}$$

- Convection important if cell size > 20 mm
- Most foams: cell size < 1 mm $\Rightarrow$ convection negligible

Radiation

- Hect flux passing by radiation, q_r^0, from surface at temperature T_1, to one at a lower temperature T_0, with a vacuum between them, is:

  $q_r^0 = \beta_1\, \sigma (T_1^4 - T_0^4) \qquad$ Stefan's law

  $\sigma =$ Stefan's constant $= 5.67 \times 10^{-8}\ \text{W/m}^2\ \text{K}^4$

  $\beta_1 =$ constant (< 1) describing emissivity of the surfaces
  (emitted radiant flux per unit area of sample relative to black body radiator at same temperature and conditions; black body absorbs all energy; black body emissivity =1)

Radiation

- If put foam between two surfaces, heat flux is reduced, since radiation is absorbed by the solid and reflected by cell walls

- Attenuation $q_r = q_r^0 \exp(-K^* t^*)$ Beer's law
  K^* = extinction coefficient for foam
  t^* = thickness of foam

- For optically thin walls and struts ($t < 10\mu m$) (transparent to radiation)

$$K^* = (\rho^*/\rho_s)\, K_s$$

- Heat flux by radiation then:

$$q_r = \lambda_r^* \frac{dT}{dx}$$

$$q_r = \beta_1\, \sigma(T_1^4 - T_0^4)\, \exp\left[-(\rho^*/\rho_s)K_s\, t^*\right] = \lambda_r^* \frac{dT}{dx}$$

- Obtain λ_r using some approximations

Approximations:

$$\frac{dT}{dx} \approx \frac{T_1 - T_0}{t^*} = \frac{\Delta T}{t^*}$$

$$T_1^4 - T_0^4 \approx 4\,\Delta T\,\bar{T}^3 \qquad \bar{T} = \left(\frac{T_1 - T_0}{2}\right)$$

$$q_r = \beta_1 \sigma 4\,\Delta T\,\bar{T}^3 \exp\left[-(\rho^*/\rho_s) K_s\, t^*\right] = \lambda_r^* \,\frac{\Delta T}{dx}$$

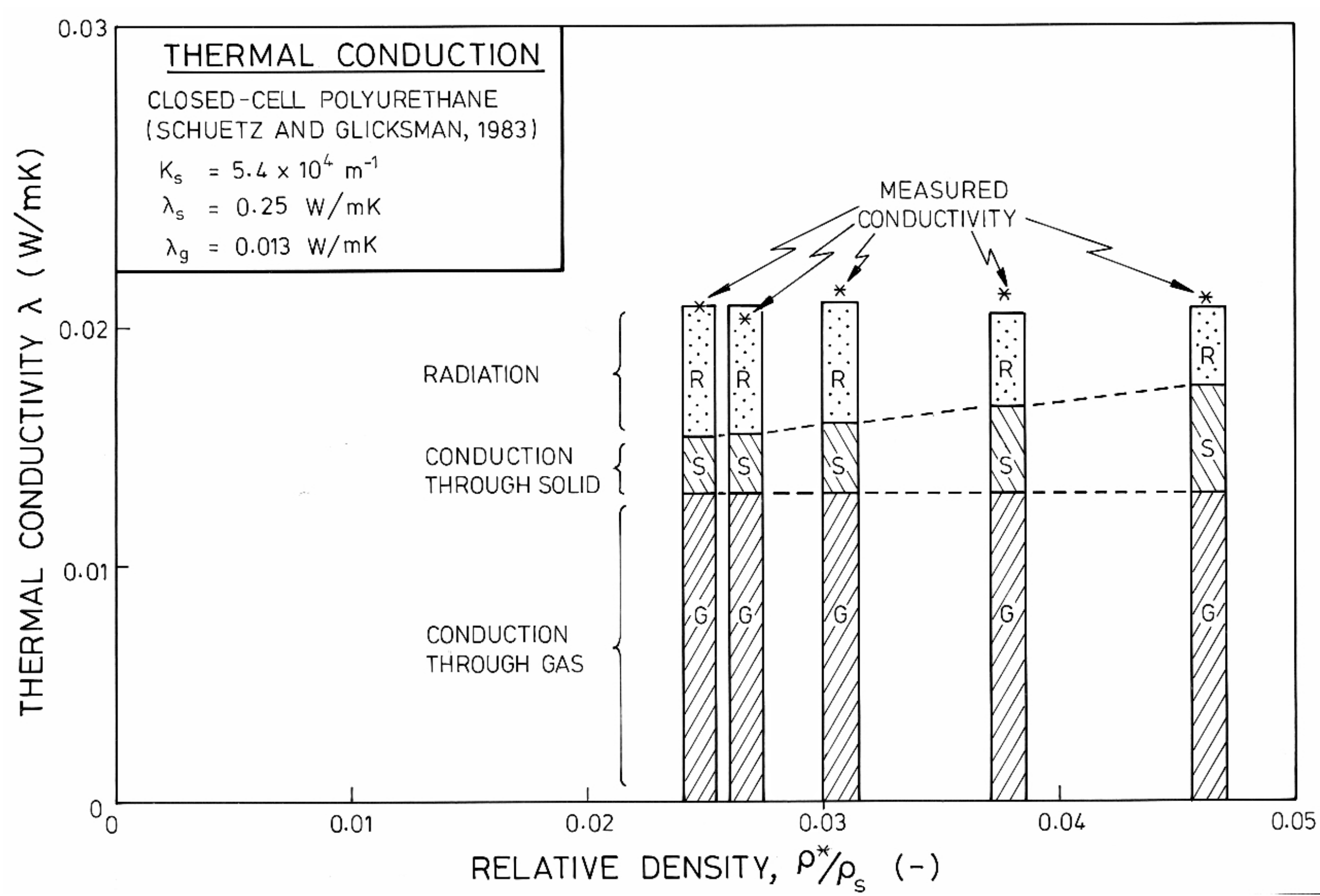
$$\lambda_r^* = 4\beta_1 \sigma \bar{T}^3\, t^* \exp\left[-(\rho^*/\rho_s) K_s\, t^*\right]$$

$$\text{as } \rho^*/\rho_s \downarrow \quad \lambda_r^* \uparrow$$

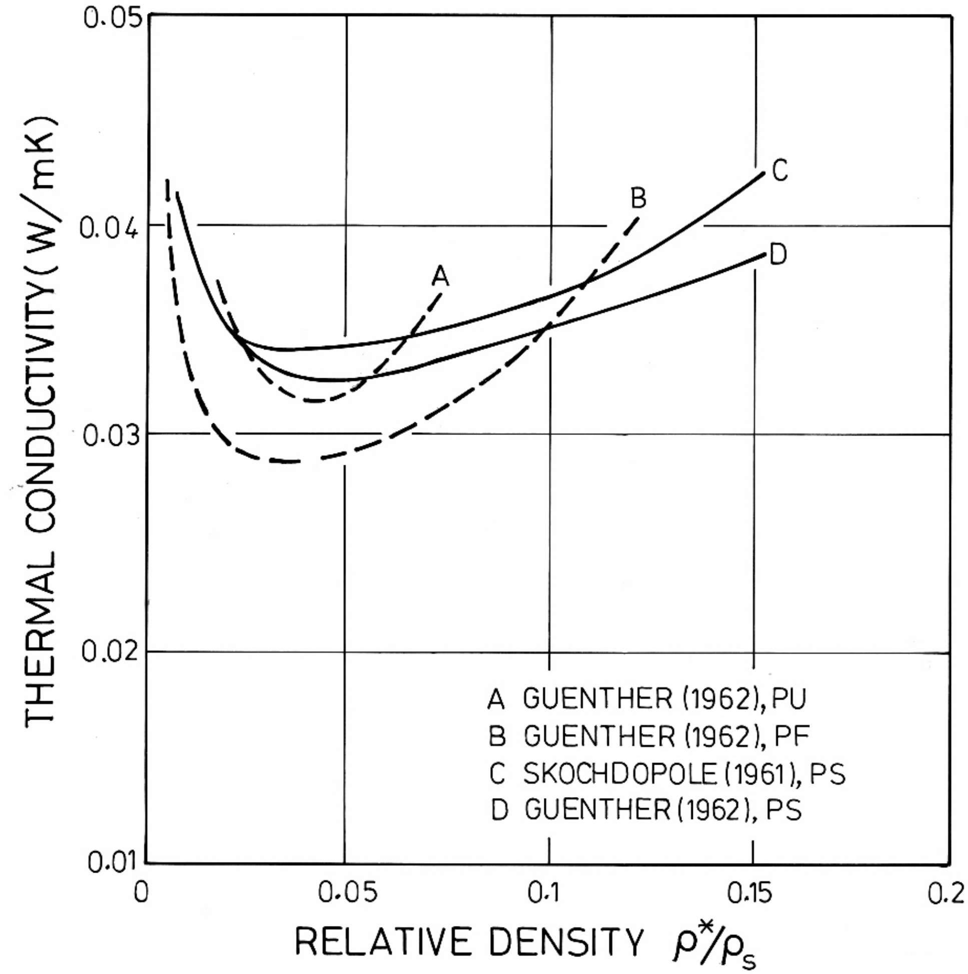
Thermal conductivity

- Relative contributions of λ_s^*, λ_g^*, λ_r^* shown in Fig. 7.1
  - largest contribution λ_g^*
- λ^* plotted against relative density Fig. 7.2
  - minimum between ρ^*/ρ_s of 0.03 and 0.07
  - at which point λ^* only slightly larger than λ_s^*
  - at low ρ^*/ρ_s, λ^* increases - increasing transparency to radiation (also, walls may rupture)
  - tradeoff: as ρ^*/ρ_s goes down, λ_s^* goes down, but λ_r^* goes up

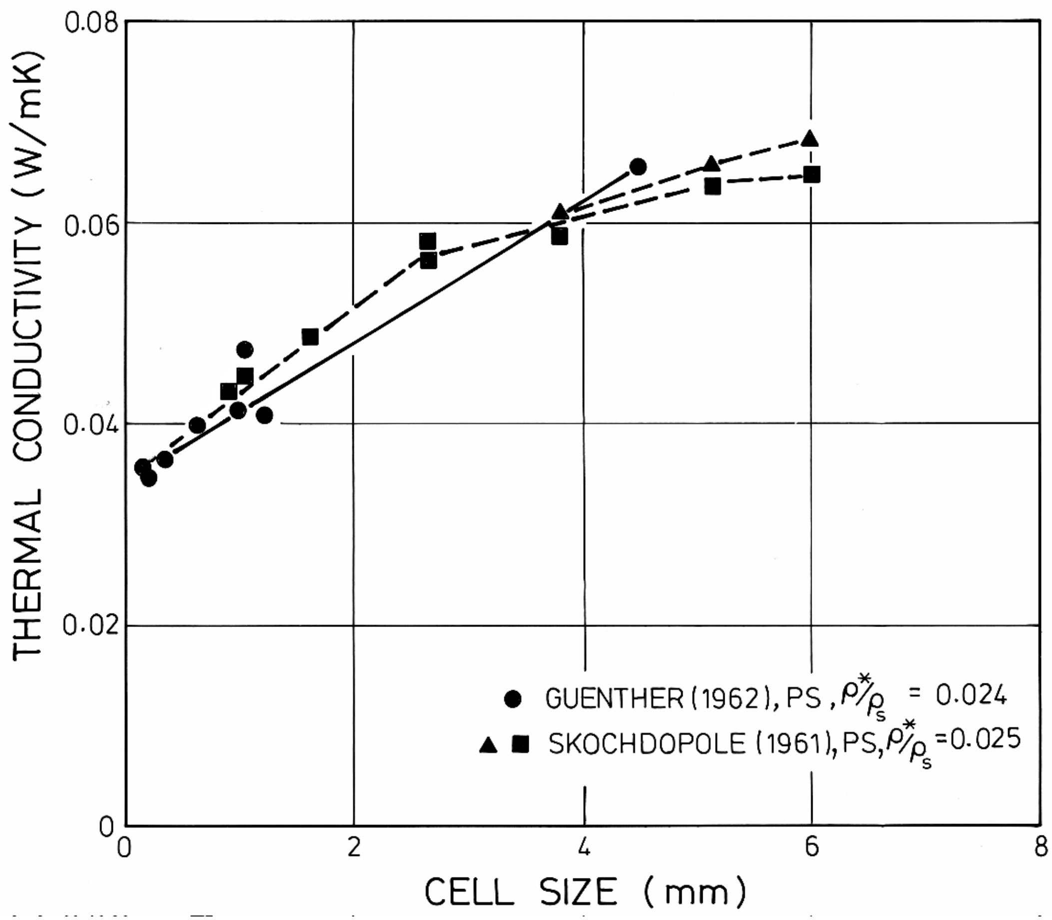
Thermal Conductivity



Cond. Vs. Relative Density



Cond. vs. Cell Size



Gibson, L. J., and M. F. Ashby. *Cellular Solids: Structure and Properties*. 2nd ed. Cambridge University Press, © 1997. Figure courtesy of Lorna Gibson and Cambridge University Press.

λ^* plotted against cell size Fig. 7.3

- λ^* increases with cell size
- Radiation reflected less often

Note: aerogels

- Pore size $< 100nm$
- Mean free path of air at ambient pressure = 68 nm
  $\rightarrow$ average distance molecules move before collision with another molecule
- Aerogels — pore size < mean free path of air — reduced conduction through gas

Specific hear C_p

- Specific heat — energy required to raise temperature of unit mass by unit temperature

  $C_p^* = C_{\text{ps}} \qquad [\text{J/kg}\cdot\text{ K}]$

Thermal expansion coefficient

$\alpha^* = \alpha_s \qquad$ (consider foam as framework)

(but if closed-cell foam cooled dramatically — gas can freeze, collapsing the cells; or if heated — gas expands, increasing the internal pressure and strains)

Thermal shock resistance

- If material subjected to sudden change in surface temperature - induces thermal stresses at surface, plus cracking and spalling
- Consider material at T_1 dropped intp water at T_2 ($T_1 > T_2$)
  - Surface temperature drops to T_2, contracting surface layers
  - Thermal strain $\epsilon_T = \alpha\,\Delta T$
- If surface bonded to underlying block of material - constrained to original dimensions

$$\sigma = \frac{E\,\alpha\Delta T}{1-\nabla} \qquad \text{in the surface}$$

- Cracking/spalling when $\sigma = \sigma_f$

$$\Delta T = \sigma_f \frac{1-\nu}{E\alpha} = \text{critical } \Delta T \text{ to just cause cracking}$$

- For foam: (open cells)

$$\Delta T_c^* = \frac{0.2\,\sigma_{\mathrm{fs}}(\rho^*/\rho_s)^{3/2}(1-\nu^*)}{E_s\,(\rho^*/\rho_s)^2\alpha_s} = \frac{0.2}{(\rho^*/\rho_s)^{1/2}}\,\frac{\sigma_{\mathrm{fs}}(1-\nu)}{E_s\alpha_s} = \frac{0.2}{(\rho^*/\rho_s)^{1/2}}\,\Delta T_{cs}$$

- As foam density goes down, ΔT_c^* goes up   firebrick - porous ceramic

Case study: optimization of foam density for thermal insulation

- There is an optimal foam density for a given thermal insulation problem
- Already saw λ^* has a minimum as a $f(\rho^*/\rho_s)$
- Typically, have a constraint on the foam thickness, t^*, t^* =constant

$$\lambda^* = \frac{2}{3}(\rho^*/\rho_s)\lambda_s + (1 - \rho^*/\rho_s)\lambda_g^* + 4\beta_1\sigma\bar{T}^3 t^*\ exp[-K_s(\rho^*/\rho_s)t^*]$$

- What is optimum ρ^*/ρ_s for a given t^*?

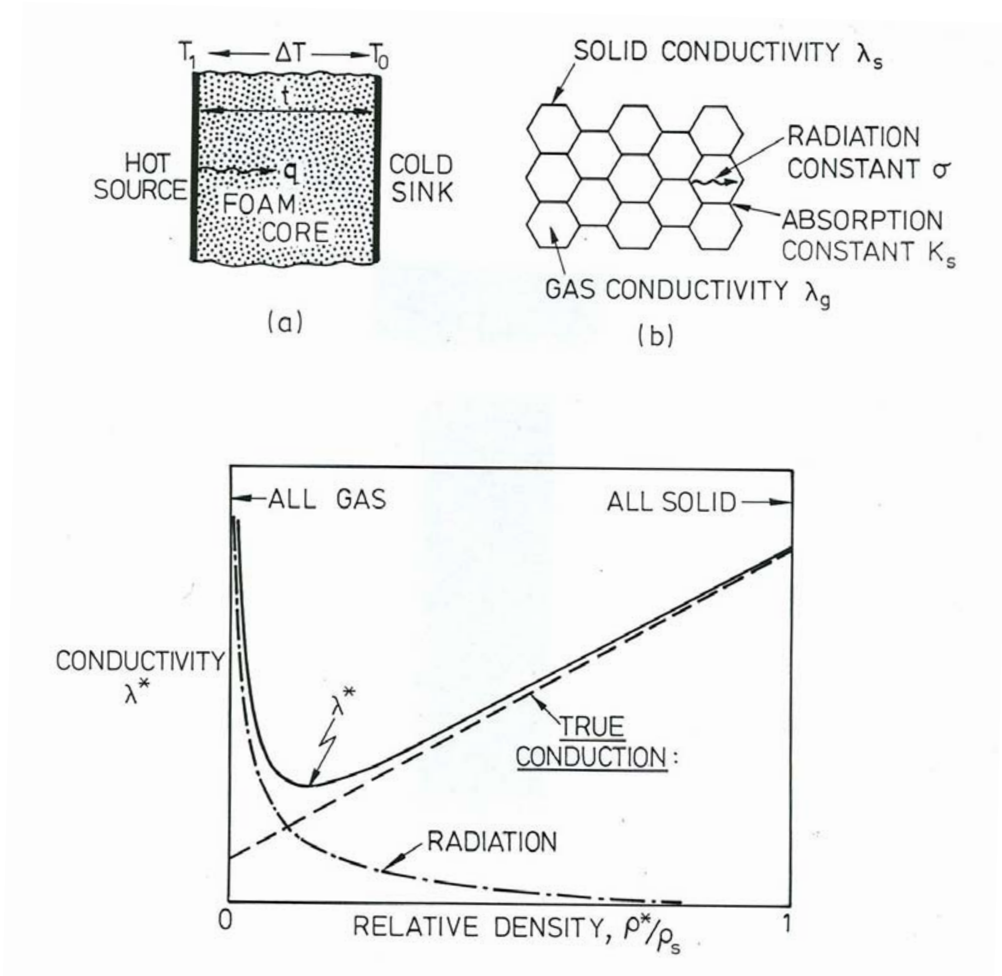
$$\frac{d\lambda^*}{d(\rho^*/\rho_s)} = 0 \Rightarrow (\rho^*/\rho_s)_{\text{opt}} = \frac{1}{K_s t^*}\ \ln\left[\frac{4K_s\beta_1\sigma\bar{T}^3\, t^{*2}}{\frac{2}{3}\lambda_s - \lambda_g}\right]$$

- As given thickness t^* increases, $(\rho^*/\rho_s)_{\text{opt}}$ decreases
- As $\bar{T}$ increases, $(\rho^*/\rho_s)_{\text{opt}}$ increases

e.g.	coffee cup	$t^* = 3\text{mm}$	$(\rho^*/\rho_s)_{\text{opt}} = 0.08$
	refrigerator	$t^* = 50\text{mm}$	$(\rho^*/\rho_s)_{\text{opt}} = 0.02$

(see PP slide Table 7.3 for data used in calculations)

Case Study:
Optimization of Relative Density



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Case Study: Optimum Relative Density

Table 7.3 Data for optimization case study

Extinction coefficient of solid polymer, K_s	$5.67 \times 10^4\ m^{-1}$
Emissivity factor, β_1	0.5
Conductivity of solid polymer, λ_s	0.22 W/m K
Conductivity of gas, λ_g	0.02 W/m K
Mean temperature, $\bar{T}$	300°K
Stefan's constant, σ	$5.67 \times 10^{-8}\ W/m^2K^4$

Case study: insulation for refrigerators

- Insulation reduces energy cost, but has a cost itself
- Total cost is the cost of insulation plus the cost of energy lost by hear transfer through walls
- Objective function: minimize total cost
- given: x=thickness of insulation $\qquad C_M$=cost of insulation/mass
  ΔT=temp. diff. across insulation $\qquad C_E$=cost of energy / joule
  t_l=design life of refrigerator $\qquad C_T$=total cost/area

$$C_T = x\,\rho^* C_M + \lambda\,\frac{\Delta T}{x}\,t_l C_E \qquad (\text{heat flux } q = \lambda\frac{\Delta T}{x}\ \frac{\mathrm{J}}{\mathrm{m^2 s}})$$

Define: $\quad M_1 = \dfrac{1}{\rho^* C_M} \qquad M_2 = \dfrac{1}{\lambda}$

$$\frac{C_T}{x} = \frac{1}{M_1} + \left[\frac{\Delta T}{x^2} t_l C_E\right]\frac{1}{M_2}$$

- The terms are equal when:

$$M_2 = \underbrace{\left[\frac{\Delta T}{x^2} \, t_l \, C_E \right]}_{\text{coupling constant}} M_1$$

- Family of parallel straight lines of constant value $\frac{\Delta T}{x^2} \, t_l \, C_E$

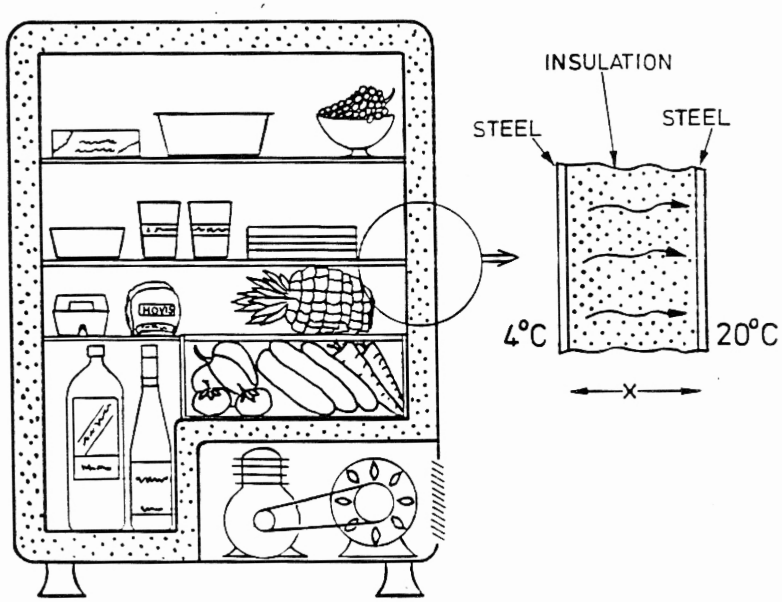
- Fig. 13.11 $\quad \Delta T = 20^\circ \quad x = 10\text{mm} \quad C_E = 0.01/\mu\text{J}$

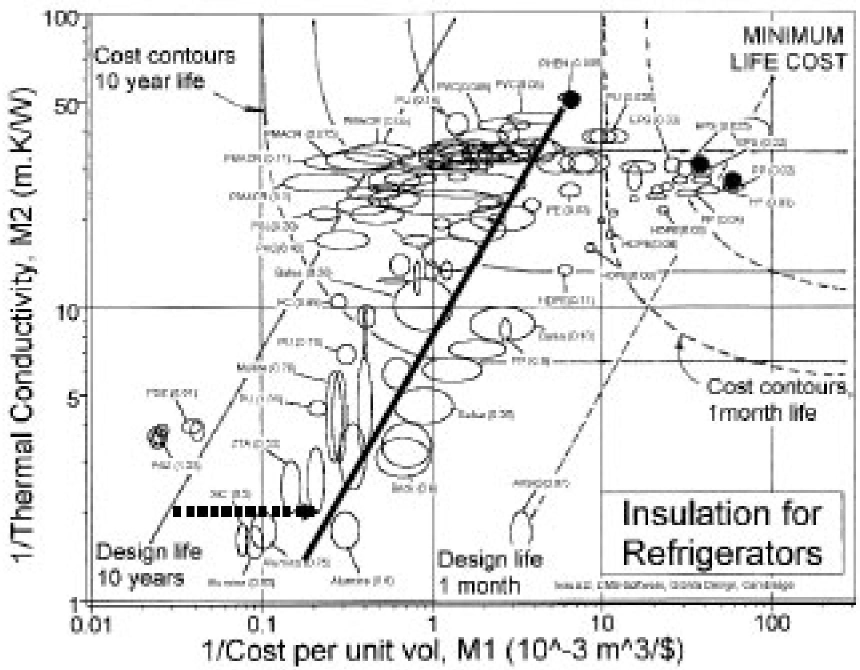
  Two lines for $t_2 = 10$ years and $t_l = 1$ month
  (note error in book $t_l = 10$ years line should be moved over)

- Also plotted a set of curved contours - plots of C_T/x:
  - As move up and to the right of plot, the value of C_T/x decreases

- For $t_l = 10$ years $\Rightarrow$ phenolic foam $\rho^* = 0.035$ Mg/m^3
  For $t_l = 1$ month $\Rightarrow$ EPS $\rho^* = 0.02$ Mg/m^3
  PP $\rho^* = 0.02$ Mg/m^3

Case Study: Insulation for Refrigerators





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3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2015