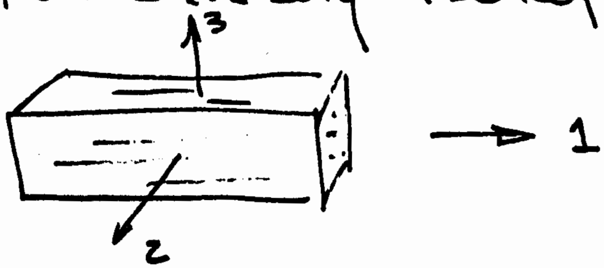
Lamina Constitutive Relations

- Isotropic

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

- Transversely isotropic

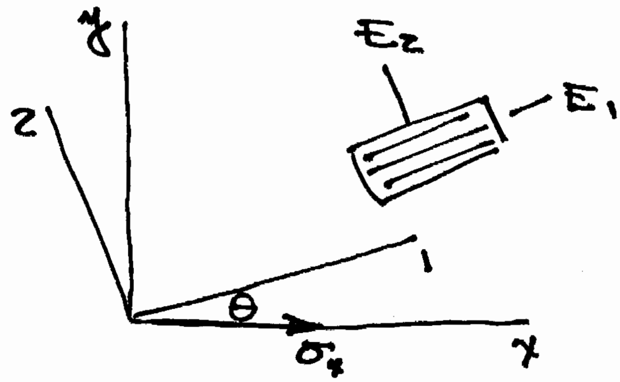


$E_2 = E_3 \neq E_1$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1} \longrightarrow 4 \text{ constants}$$

Transformation of Axes



$G_{12}, \nu_{21}, \nu_{12}$

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}$$

$$\sigma_1 = \sigma_x \cos^2\theta, \quad \sigma_2 = \sigma_x \sin^2\theta, \quad \tau_{12} = \sigma_x \sin\theta\cos\theta$$

$$\epsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\frac{\sigma_2}{E_2} = \frac{\sigma_1}{E_1} - \nu_{12}\frac{\sigma_2}{E_1}$$

$$= \sigma_x \left[\frac{\cos^2\theta}{E_1} - \nu_{12}\frac{\sin^2\theta}{E_1} \right]$$

$$\epsilon_2 = -\nu_{12}\frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2} = \sigma_x \left[-\nu_{12}\frac{\cos^2\theta}{E_1} + \frac{\sin^2\theta}{E_2} \right]$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}} = \frac{\sigma_x}{G_{12}} \sin\theta\cos\theta$$

$$\epsilon_x = \epsilon_1 \cos^2\theta + \epsilon_2 \sin^2\theta + \gamma_{12}\sin\theta\cos\theta$$

$$= \sigma_1 \left[\frac{\cos^4\theta}{E_1} - \nu_{12}\frac{\cos^2\theta\sin^2\theta}{E_1} - \frac{\nu_{12}}{E_1}\cos^2\theta\sin^2\theta + \frac{1}{E_2}\sin^4\theta + \frac{1}{G_{12}}\sin^2\theta\cos^2\theta \right]$$

$$\frac{1}{E_x} = \frac{\epsilon_x}{\sigma_x} = \frac{\cos^4\theta}{E_1} + \frac{\sin^4\theta}{E_2} + \sin^2\theta\cos^2\theta \cdot \left[\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right]$$