Session #24: Homework Solutions

Problem #1

To increase its corrosion resistance, chromium (Cr) is diffused into steel at 980°C. If during diffusion the surface concentration of chromium remains constant at 100%, how long will it take (in days) to achieve a Cr concentration of 1.8% at a depth of 0.002 cm below the steel surface? (D_o = 0.54 cm^2/s; E_A = 286 kJ/mol)

Solution

A solution to Fick's second law for the given boundary conditions is:

$\frac{c}{c_s} = 1 - \text{erf}\frac{x}{2\sqrt{Dt}}$, from which we get $\text{erf}\frac{x}{2\sqrt{Dt}} = 1 - 0.018 = 0.982$

From the error function tables, 0.982 is the erf of 1.67. This means that

$$\frac{0.002}{2\sqrt{Dt}} = \frac{0.001}{\sqrt{Dt}} = 1.67$$

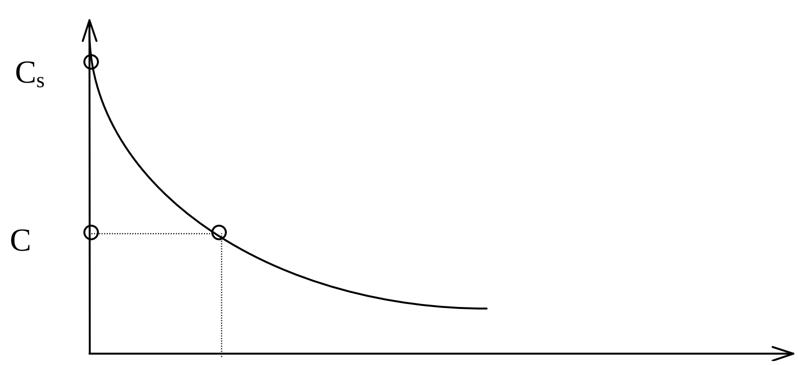
$$D = D_o e^{\left(\frac{-286\times10^5}{8.314\times1253}\right)} = 6.45\times10^{-13}\ cm^2/s$$

$$\therefore\ t = \frac{0.001^2}{1.67^2\times6.45\times10^{-13}} = 5.56\times10^5\ \text{sec} = 6.4\ \text{days}$$

Problem #2

By planar diffusion of antimony (Sb) into p-type germanium (Ge), a p-n junction is obtained at a depth of 3×10^{-3} cm below the surface. What is the donor concentration in the bulk germanium if diffusion is carried out for three hours at 790°C? The surface concentration of antimony is held constant at a value of 8×10^{18} cm^{-3}; $D_{790°C} = 4.8 \times 10^{-11}$ cm^2/s.

Solution



$$\frac{c}{c_s} = \text{erfc}\,\frac{x}{2\sqrt{Dt}} \;=\; \text{erfc}\,\frac{3\times 10^{-3}}{2\sqrt{Dt}} = \text{erfc}(2.083)$$

$$\frac{c}{c_s} = 1 - \text{erf}(2.083), \quad \therefore \; 1 - \frac{c}{c_s} = 0.9964$$

$$\frac{c}{c_s} = 3.6\times 10^{-3}, \quad \therefore \; c = 2.88\times 10^{16}\ \text{cm}^{-3}$$

The donor concentration in germanium is $2.88 \times 10^{16}/\text{cm}^3$.

Problem #3

You wish to dope a single crystal of silicon (Si) with boron (B). The specification reads 5×10^{16} boron atoms/cm^3 at a depth of 25 μm from the surface of the silicon. What must be the effective concentration of boron in units of atoms/cm^3 if you are to meet this specification within a time of 90 minutes? Assume that initially the concentration of boron in the silicon crystal is zero. The diffusion coefficient of boron in silicon has a value of 7.23×10^{-9} cm^2/s at the processing temperature.

Solution

$$c(x,\,t) = A + B\ \text{erf}\,\frac{x}{2\sqrt{Dt}}\,;\; c(0,\ t) = c_s = A\,;\; c(x,\ 0) = c_i = 0$$

$$c(\infty,\,t) = c_i = 0 = A + B \;\rightarrow\; A = -B$$

$$\therefore\; c(x,\,t) = c_s - c_s\text{erf}\,\frac{x}{2\sqrt{Dt}} = c_s\text{erfc}\,\frac{x}{2\sqrt{Dt}} \;\rightarrow\; 5\times 10^{16} = c_s\text{erfc}\,\frac{25\times 10^{-4}}{2\sqrt{7.23\times 10^{-9}\times 90\times 60}}$$

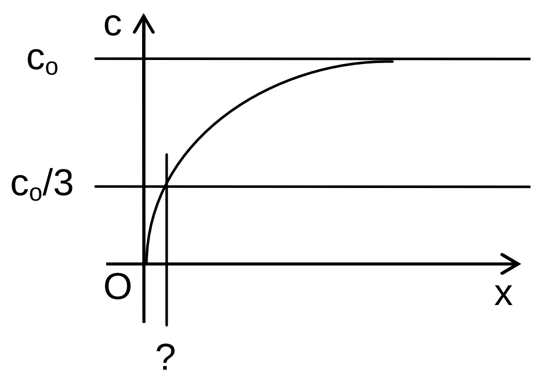
$$\therefore\; c_s = \frac{5\times 10^{16}}{\text{erfc}\,\dfrac{25\times 10^{-4}}{2\sqrt{7.23\times 10^{-9}\times 5400}}} = 6.43\times 10^{16}\ \text{cm}^{-3}$$

$$\text{erfc}(0.20) = 1 - \text{erf}(0.20) = 1 - 0.2227 = 0.7773$$

Problem #4

A slab of plate glass containing dissolved helium (He) is placed in a vacuum furnace at a temperature of 400°C to remove the helium from the glass. Before vacuum treatment, the concentration of helium is constant throughout the glass. After 10 minutes in vacuum at 400°C, at what depth from the surface of the glass has the concentration of helium decreased to 1/3 of its initial value? The diffusion coefficient of helium in the plate glass at the processing temperature has a value of 3.091×10^{-6} cm^2/s.

Solution



$$c = A + B\ \text{erf}\,\frac{x}{2\sqrt{Dt}}\ ;\ c(0,\ t) = 0 = A\ ;\ c(\infty,\ t) = c_o = B$$

$$\therefore\ c(x,\ t) = c_o \text{erf}\,\frac{x}{2\sqrt{Dt}}$$

What is x when $c = c_o/3$?

$$\frac{c_o}{3} = c_o \text{erf}\,\frac{x}{2\sqrt{Dt}} \ \rightarrow\ 0.33 = \text{erf}\,\frac{x}{2\sqrt{Dt}}\ ;\ \text{erf}(0.30) = 0.3286 \approx 0.33$$

$$\therefore \frac{x}{2\sqrt{Dt}} = 0.30 \ \rightarrow\ x = 2 \times 0.30 \times \sqrt{3.091 \times 10^{-6} \times 10 \times 60} = 2.58 \times 10^{-2}\ \text{cm} = 258\mu\text{m}$$

3.091SC Introduction to Solid State Chemistry
Fall 2009