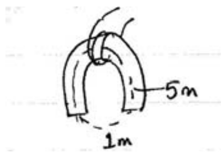
3.15 - Problem Set 7 Solutions

Problem 1

a.



Ampere's Law says:

$$\int H dl = ni$$
$$5H_{core} + H_{gap} = 10^4$$

Force we need to exert is $2 \cdot 10^4$ N.
Max. magnetization of car $= B_s = 1$T. $B = \mu_0(H + M)$.
So:

$$M_s = \frac{1}{\mu_0} B_s = \frac{1}{4\pi 10^{-7}} \text{A/m} = 0.08 \cdot 10^7 \text{A/m or} 800 \text{kA/m}$$

Force on car is: $\mu_0 H_{gap} M_s V = 2 \cdot 10^4 N$ so we need $H_{gap} = \frac{2 \cdot 10^4}{\mu_0 M_s V}$.
Volume of steel is: $\frac{500\text{kg}}{2.5\text{g/cm}^3} = \frac{500}{25000} m^3 = 0.2 m^3$.
$H_{gap} = \frac{2 \cdot 10^4}{0.2} = 10^5$ A/m or 100 kA/m.

This is quite a small field so should be possible to achieve.
From Ampere: if $H_{gap} = 10^5$ A/m, $10^4 = 5H_{core} + 10^5$.
Max value of $H_{core} = \frac{1}{\mu_0} B_{core}$, choose *soft, high* B_s core like Fe-Si.

$$B_{core} = 2.1\text{T}$$
$$H_{core} = \frac{2.1}{4\pi 10^{-7}} \text{A/m} = \frac{1}{6} 10^7 \text{A/m} = 1.67 \cdot 10^6 \text{A/m}$$
$$10^4 = 10^5 + 5 \times \frac{2.1}{4\pi 10^{-7}} = 10^5 + \frac{10}{12} \cdot 5 \times 10^7 \approx 8.5 \cdot 10^6 \text{A}$$

So need current of 850A.

A lot of current needed, because the magnet is large and $\oint H dl$ is big $\Rightarrow$ get hot.

Problem 2

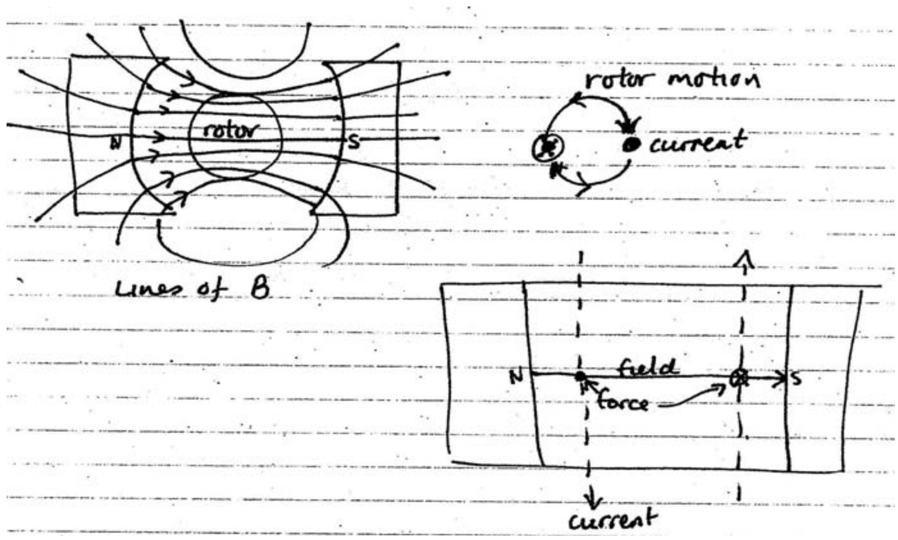
a.

Current flows into page; magnetic field is horizontal, so a vertical force rotates the rotor.

$$F = B_i l$$

Sliding contact needed because after a 180° rotation, you would have a force in the other direction, so it would just rock back and forth.

b.



c.

Max torque when it is placed as shown in the original diagram. When it has gone through 90° the torque is less because the field is weaker. Also the field is more tangential so the force has a (useless) radial component change thus by redesigning magnet to give more uniform field, or add more wires to rotor.

d.

Rotot is magnetic to concentrate the lines of B: High μ_r. You need it to be extremely soft so the field lines always go left $\rightarrow$ right. Need to avoid hysteresis loss. High resistivity ($\Rightarrow$ laminations) for eddy current less.

e.

As the car brakes, the wheell is connected to a rotor. Movement of the rotor generates a voltage across the coil - same as a generator or an inductive read head.

$$\text{Voltage} = -nd\phi/dt, \phi = B_i A$$

When you draw a current out of the coil, this produces a force opposite to the motion, i.e. it leads to braking.

Problem 3

a.

There are several factors that can influence coercivity. Magnetocrystalline anisotropy will affect the magnetic direction, and the coercivity because H depends on how easily walls can form (and how easily they move.) Domain wall energy $\propto \sqrt{K}$.

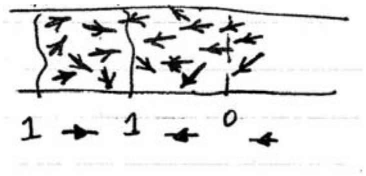
However, for these particles, they are very small so they are probably below the single-domain size. So the material cannot sustain domain walls. This means the magnetism needs to reverse by *rotation* (all the magnetic moments reverse at once). This depends on K because you have to pull the moments away from the easy axis.

If the particles are too small, thermal energy kT can cause reversal of magnetism spontaneously. This occurs if $kV \approx 60kT$ (undesirable). [These particles are actually too big for this to be important.]

(Extra note - we didn't talk about this, but the elongated shape of the particles contributes to anisotropy because magnetism lies along long axis of particle.)

b.

Bit would look like this with multiple particles per bit field.



So bit size depends on *partilce size.* These are $\approx 250\mu$m long, so bit length will be at least 1 μm.

In contrast hard disk has $\approx$ 5 nm grains and bit length $\approx$ 50cn, 20 times more. Track width also narrower.

c.

Ferrimagnet. Fe^{3+} is ... $3d^5$, Fe^{2+} is $3d^6$.
Fe_3O_4 is $5 - (5+4) = 4\mu_B$ per formula unit.
If 8 formula units occupy $(0.8\text{nm})^3 = 0.51\text{nm}^3 = 5.1\times10^{-28}\text{m}^3$ then net moment $= \frac{8\times4\times(9.27\cdot10^{-24}\text{Am}^2)}{5.1\cdot10^{-28}\text{m}^3} = 580\text{kA/m} = M_{sat}$.