

3.185 Problem Set 5

Advanced Heat Transfer

Due Monday October 27, 2003

1. Electrical resistivities of titanium-aluminum alloys at 800K are given below. Estimate the thermal conductivity for each alloy at that temperature. (10)

At. % Al	Resistivity, $\mu\Omega\text{cm}$
0	112
3	140
6	165
11	190
33	210

2. Spreadsheet finite difference model of 1-D unsteady conduction (30)

The design for some optical device calls for a poly(methyl methacrylate) (PMMA, a.k.a. plexiglass) plate 5 mm thick at 30°C to be immersed in a hot liquid at 80°C. Because the index of refraction will change with temperature, the designer wants to know how the temperature varies across the plate as a function of time.

Let x represent the distance into the PMMA plate from one side.

Data:

- PMMA thermal conductivity: $0.21 \frac{\text{W}}{\text{m}\cdot\text{K}}$
- PMMA density: $1190 \frac{\text{kg}}{\text{m}^3}$
- PMMA specific heat: $1470 \frac{\text{J}}{\text{kg}\cdot\text{K}}$
- Fluid heat transfer coefficient: $9000 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$

Source: http://www.asahi-kasei.co.jp/plastic/e/technical/pmma/bussei_doukou.htm

- (a) Calculate the Biot number for this situation. What kind of boundary condition approximation can you make at the two sides of the plate? (4)
- (b) For the 1-D explicit form of the finite difference method, give the temperature of an interior node in terms of the mesh Fourier number and the temperatures of the adjacent nodes at the previous timestep. (5)
- (c) For a mesh with six evenly-spaced nodes in the x -direction (so $n = 5$ intervals), what is the maximum allowed time step size Δt in the explicit scheme? How does this change for eleven nodes ($n = 10$)? (5)
- (d) Use the spreadsheet provided on the website to calculate the temperature profiles at times from zero to the steady-state timescale using the finite difference method with $n = 5$. (8)
- (e) Repeat the finite difference calculation in part 2d for $n = 10$ (on the second sheet). (8)

3. Electron beam centrifugal atomization of metal (35)

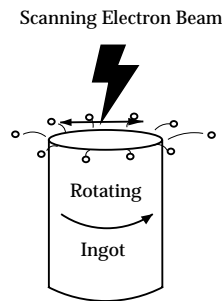
Introduction In class, we discussed ultrasonic gas atomization, which is one method for producing a spray from a liquid. The resulting size distribution is quite broad, that is, there is a wide range of spray droplet sizes that result from that process, and when they solidify, the resulting powder is a mixture of spheres of various sizes.

For some materials applications, it's much better to have a powder with a narrow distribution, that is, with most of the droplets having the same size. One way to achieve this is by centrifugal atomization, in which we rotate a solid cylinder and melt it at a controlled rate so droplets break off at a size determined by the balance between centrifugal and surface tension forces:

$$R \sim \sqrt{\frac{\gamma}{\rho\omega^2 r}}$$

where R is the droplet size, γ is the surface tension, ρ the liquid density, ω the rotation rate and r the distance from the rotation axis where the liquid droplet breaks free.

An arrangement which produces this result is pictured below. The cylinder, called the ingot, is held vertically and rotated quickly while melting slowly from the top such that a thin film of liquid is accelerated out to the edges, where the liquid breaks into droplets.



The problem For this vertical atomization arrangement, we would like to calculate two things:

- The amount of heat needed to melt and atomize at a certain rate.
- The temperature distribution in the rotating ingot at steady-state.

We will use titanium as the atomized material here, which has the following properties:

- Melting point: $1667^\circ\text{C} = 1940\text{ K}$
- Radiative emissivity: 0.55
- Thermal conductivity: $20\frac{\text{W}}{\text{m}\cdot\text{K}}$
- Density: $4700\frac{\text{kg}}{\text{m}^3}$
- Molar mass: $0.0479\frac{\text{kg}}{\text{mol}}$
- Heat capacity: $700\frac{\text{J}}{\text{kg}\cdot\text{K}}$
- Heat of fusion: $300\frac{\text{kJ}}{\text{kg}}$
- Vapor pressure constants: $A = 23200\text{K}$, $B = 11.74$, $C = -0.66$, $D = 0$

$$\log_{10} p_v(\text{torr}) = -\frac{A}{T} + B + C \log_{10} T(+DT)$$

- Heat of vaporization: $\Delta H_e = 9.2\frac{\text{MJ}}{\text{kg}}$

Source: E. Brandes, ed., *Smithells Metals Handbook* (6th edition), Boston: Butterworth & Co., 1983.

- (a) Assuming the chamber is cold and black, and that the liquid film is all at the melting point, estimate the radiative heat loss from the top surface of the ingot. (4)
- (b) Assuming ideal Langmuir evaporation into a vacuum, calculate the evaporation rate and heat loss due to evaporation. (5)
- (c) If we would like to melt and atomize at a rate of 1 cm of ingot per minute, what is the required power density of the heat source? You may neglect losses from the sides of the ingot, but include energy required to heat the titanium from 300 K to its melting point, and to melt it, and the losses in parts 3a and 3b. (6)
- (d) Is the process more energy-efficient if it goes faster or slower? (2)
- (e) The ingot bottom temperature and initial temperature are both 300 K. If the ingot is 1 m long, can it be considered semi-infinite? When the process reaches steady-state, what is the relationship between temperature and distance from the top of the ingot? (7)
- (f) Use your answer from part 3e to calculate the heat flux into the top of the solid ingot. Which of the energy components from part 3c does this relate to? (Radiative/evaporative losses, heat of fusion, heat of raising the titanium to its melting point) (7)
- (g) Suppose the ingot were turned on its side, and hit on the top by a fixed (not scanning) electron beam while spun like a rolling pin. Give at least one advantage or disadvantage this different form of the process would have vs. that pictured above. (4)

4. Radiative Cooling of an Aluminum Cube (25 points)

An anodized aluminum object at 1000 K, which we'll model as a cube with size 0.1 m, is placed in a cold (much lower temperature), black enclosure. Its bottom is resting on an insulated surface, so it cools by radiation from its other five sides.

- (a) Express the net radiative heat flux from the cube surface as a function of cube surface temperature. You may assume the surroundings are at a low enough temperature that their emission back to the cube is negligible. (5)
- (b) Write an expression for the radiative “heat transfer coefficient”, which is the heat flux divided by the surface temperature, using the grey body approximation, and an “environment” temperature of zero. (5)
- (c) Calculate the maximum Biot number for radiative cooling of this cube at between 1000 K and 400 K. What approximation may we use for the temperature profile across the cube? (5)
- (d) Calculate the time required for this object to cool from 1000 K to 400K by radiation alone. (10)

Aluminum data:

- Thermal conductivity: $k = 238 \frac{\text{W}}{\text{m}\cdot\text{K}}$
- Density: $\rho = 2700 \frac{\text{kg}}{\text{m}^3}$
- Heat capacity: $c_p = 917 \frac{\text{J}}{\text{kg}\cdot\text{K}}$
- Anodized aluminum emissivity: $\epsilon = 0.85$