Chapter 5

Coupled Fluids with Heat and Mass Transfer

5.1 November 26, 2003: Coupled Fluids, Heat and Mass Transfer!

Mechanics:

- Congrats to Jenny and David for winning the contest, prize: $5 Tosci's.
- PS8 on Stellar, due Fri 12/5.
- Evaluations next Wednesday 12/3.

Muddy from last time:

- Time smoothing: what are u_x, $\bar{u}_x$, u_x'? They are: the real velocity, the time-smoothed component of velocity, and the fluctuating component of velocity.
- What timescale can you find from the lengthscale of the smallest eddies? How would one go about this? Two timescales are relevant here: one is diffusion timescale ℓ^2/D, which gives mixing time, and the timescale of formation and elimination of these little eddies ℓ^2/ν. When attempting direct numerical simulation of turbulence, this tells how small a timestep one will need (actually, a fraction of this for accuracy); this also describes how long the eddies will last after the mixing power is turned off.

Thermal and solutal boundary layers Types: forced, natural convection; forced today, natural later.

Recall first BL thought experiment on thick polymer sheet extrusion, hot polymer sheet T_∞ and cold water T_s. Now it's happening in a liquid, competing thermal and fluid boundary layers with thicknesses δ_u and δ_T.

Fluid:

$$\delta_u = 5.0\sqrt{\frac{\nu x}{U_\infty}}$$

Thermal if flow uniform, same criterion:

$$\delta_T = 3.6\sqrt{\frac{\alpha x}{U_\infty}}$$

Dimensionless:

$$\frac{\delta_T}{x} = \frac{3.6}{\sqrt{\frac{U_\infty x}{\alpha}}} = \frac{3.6}{\sqrt{\mathrm{Re}_x \mathrm{Pr}}}$$

When is flow uniform? In a solid, or for much larger thermal boundary layer than fluid, so $\alpha >> \nu$, Pr$<< 1$.

Another way to look at it:

$$\frac{\delta_T}{\delta_u} = 0.72\text{Pr}^{-1/2}$$

Large Prandtl number (>.5) means

$$\frac{\delta_T}{\delta_u} = 0.975\text{Pr}^{-1/3}$$

Liquid metals (and about nothing else) have small Pr; mass transfer Pr is almost always large. *E.g.* water $\nu = 10^{-6}\frac{\text{m}^2}{\text{s}} = 10^{-2}\frac{\text{cm}^2}{\text{s}}$, but D is typically around $10^{-5}\frac{\text{cm}^2}{\text{s}}$.

Note: blood platelets diffuse at around $D = 10^{-9}$, but tumbling blood cells not only stir and increase diffusivity, but somehow platelets end up on the sides of blood vessels, where they're needed. I don't fully understand...

Heat and Mass Transfer Coefficients What about h? Start with h_x, then h_L, as before with f_x and f_L. Let $\beta_T = y\sqrt{U_\infty/\alpha x}$, $\theta = T - T_s/T_\infty - T_s$, graph θ vs. β_T gives $\theta = \text{erf}(\beta_T/2)$.

Heat conduction into the liquid:

$$q_y = -k\frac{\partial T}{\partial y} = -k\frac{dT}{d\theta}\frac{d\theta}{d\beta_T}\frac{\partial \beta_T}{\partial y}$$

$$q_y = k(T_s - T_\infty)\frac{1}{\sqrt{\pi}}\sqrt{\frac{U_\infty}{\alpha x}} = h_x(T_s - T_\infty)$$

$$h_x = \frac{k}{\sqrt{\pi}}\sqrt{\frac{U_\infty}{\alpha x}} = \sqrt{\frac{k\rho c_p U_\infty}{\pi x}}$$

Likewise for mass transfer, ρc_p is effectively one, so:

$$h_{Dx} = \sqrt{\frac{DU_\infty}{\pi x}}$$

Next time: average, dimensional analysis, $\delta_T < \delta_u$ case.

5.2 December 1, 2003: Nusselt Number, Heat and Mass Transfer Coefficients

Mechanics:

- Evals Wednesday.

Muddy from last time:

- In the thermal boundary layer with constant velocity, why is $\partial^2 T/\partial x^2 << \partial^2 T/\partial y^2$? That's because $\delta \ll x$, so graph T vs. x and vs. y, show y-deriv is larger.
- Is there a physical meaning behind $\delta_T/\delta_u \propto \mathrm{Pr}^{-1/2}$ and $\delta_C/\delta_u \propto \mathrm{Pr}^{-1/3}$? Yes, see below.

Heat and mass transfer coefficients Recap last time:

- Flow and heat/mass transfer: weakly coupled. So far, all laminar.
- Case 1: much larger thermal(/concentration) boundary layer (Pr<0.1): consider T/C BL to have uniform velocity, use same BL formulation as moving solid motivating example: erf solution, $\delta_T = 3.6\sqrt{\alpha x/U_\infty}$. Here:
$$\delta_C/\delta_u \text{ or } \delta_T/\delta_u = 0.72\mathrm{Pr}^{-1/2}.$$
Physical meaning: grows as sqrt of diffusivity, so ratio is ratio of square roots of diffusivity, which is inverse sqrt(Pr).
- Case 2: smaller thermal(/concentration) boundary layer (Pr>5 or so): consider T/C BL to have linear velocity, smaller velocity means thicker T/C BL. Here:
$$\delta_C/\delta_u \text{ or } \delta_T/\delta_u = 0.975\mathrm{Pr}^{-1/3}.$$
- Moving on, back to case 1, calculated $q|_{y=0}$ from erf solution:
$$q_y = k(T_s - T_\infty)\frac{1}{\sqrt{\pi}}\sqrt{\frac{U_\infty}{\alpha x}} = h_x(T_s - T_\infty) \Rightarrow h_x = \sqrt{\frac{k\rho c_p U_\infty}{\pi x}}.$$
Likewise for mass transfer:
$$h_{Dx} = \sqrt{\frac{DU_\infty}{\pi x}}.$$

Since that's the local, let's integrate for average, neglecting edge effects:

$$q_{av} = \frac{1}{WL}\int_{x=0}^{L} h_x(T_s - T_\infty)W\,dx = h_L(T_s - T_\infty)$$
$$\frac{2(T_s - T_\infty)}{L}\left[\sqrt{k\rho c_p U_\infty x/\pi}\right]_{x=0}^{L} = h_L(T_s - T_\infty)$$
$$h_L = 2\sqrt{\frac{k\rho c_p U_\infty}{\pi L}} = 2h_x|_{x=L}$$

Now for case 2 (high-Prandtl), need different formulation. Dimensional analysis of mass transfer:

$$h_D = f(D_{fl}, U, x, \nu)$$

Five parameters, two base units (cm, s), so three dimensionless. Eliminate x and D. Then one dimensionless is Reynolds (π_U), one is Prandtl (π_ν), what's the third?

$$\pi_{h_D} = \frac{h_D x}{D_{fl}}$$

Looks like the Biot number, right? But it's not, it's actually quite different.

$$\mathrm{Bi} = \frac{h_D L}{D_{solid}} = \frac{L/D_{solid}}{1/h} = \frac{\text{Resistance to conduction in solid}}{\text{Resistance due to BL in liquid}}$$

Uses L=solid thickness, D_{solid}. Heat transfer note: you get one extra dimensionless number, due to heating by viscous friction.

Here, Nusselt #, L=length of plate (in flow direction), the conduction and BL are in the same medium, use D_{liquid}.

$$\mathrm{Nu} = \frac{h_D L}{D_{liquid}} = \frac{L}{D_{liquid}/h_D} \simeq \frac{L}{\delta_C} \text{ or } \frac{L}{\delta_T}.$$

Low-Prandtl fit:

$$\frac{h_L L}{k} = 2\sqrt{\frac{U_\infty L}{\pi\alpha}} = \frac{2}{\sqrt{\pi}}\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/2}$$

Actually, for small to "medium" Pr, slight correction:

$$\mathrm{Nu}_x = \frac{0.564\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/2}}{1+0.90\sqrt{\mathrm{Pr}}}$$

$$\mathrm{Nu}_L = \frac{1.128\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/2}}{1+0.90\sqrt{\mathrm{Pr}}}$$

High: (>0.6): nice derivation in W^{3}R chapter 19:

$$\mathrm{Nu}_x = 0.332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{0.343}$$

$$\mathrm{Nu}_L = 0.664\mathrm{Re}_L^{1/2}\mathrm{Pr}^{0.343}$$

Just as there are more correlations for f (friction factor), lots more correlations for various geometries etc. in handout by 2001 TA Adam Nolte. Summarize: flow gives Re, props give Pr, gives Nu, gives h (maybe Bi).

5.3 December 3: Natural Convection

Mechanics:

- Course evals today!

Muddy from last time:

- What's the relationship between h_x or h_L and the friction factor? Hmm... Meaning: heat transfer coefficient, kinetic energy transfer coefficient. Types: local, global/average. Laminar flow variation: both$\sim 1/\sqrt{x}$, integral$\sim \sqrt{x}$, average$\sim 1/\sqrt{x}$. Laminar $f_L = 2f_x|_{x=L}$, $h_L = 2h_x|_{x=L}$. Dimensionless: $f = f(\text{Re})$, Nu=f(Re,Pr). Different correlations for different geometries.

- Other Nusselt numbers from sheet by Adam Nolte. (Note for Re=0 with a sphere...)

Natural convection Hot stuff rises, cold stuff sinks. Obvious examples: radiators, etc. Strongly-coupled equations:

$$\frac{D\rho}{Dt} + \nabla \cdot \vec{u} = 0$$

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \eta \nabla^2 \vec{u} + \rho \vec{g}$$

$$\frac{DT}{Dt} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho c_p}$$

Full coupling comes in the ρ in the fluid flow equations.

Volumetric thermal expansion coefficient:

$$\beta = -\frac{1}{\rho}\frac{d\rho}{dT}, \ \rho - \rho_0 = \beta(T - T_0)$$

Note relation to 3.11 thermal expansion coeff:

$$\alpha = \frac{1}{L}\frac{dL}{dT}$$

$$\beta = -\frac{1}{\rho}\frac{d\rho}{dT} = -\frac{V}{M}\frac{d(M/V)}{dT}$$

$$d(1/V) = -dV/V^2, \ dV = d(L^3) = 3L^2 dL$$

$$\beta = V\frac{dV}{V^2 dT} = \frac{3L^2 dL}{V dT} = \frac{3}{L}\frac{dL}{dT} = 3\alpha.$$

Word explanation: heat a solid cube, length increases 1% in each direction, volume increases 3%. Both have units $1/K$. Ideal gases:

$$\rho = \frac{P}{RT}, \ \beta = -\frac{1}{\rho}\frac{d\rho}{dT} = -\frac{RT}{P}\left(-\frac{P}{RT^2}\right) = 1/T.$$

Also $\beta_C = -\frac{1}{\rho}\frac{d\rho}{dC}$.

Simplest case: vertical wall, T_s at wall, T_∞ with density ρ_∞ away from it, x vertical and y horizontal for consistency with forced convection BL. Assume:

1. Uniform kinematic viscosity $\nu = \nu_\infty$.

2. Small density differences: ρ only matters in ρg term, otherwise ρ_∞ for convective terms.

3. Steady-state.

4. Boussinesq approx: $p \simeq -\rho_\infty g x + const$, obvious away from BL, no pressure difference across BL to drive flow.

5. Also with small density diff: $\Delta\rho/\rho = \beta\Delta T$ (ρ is roughly linear with T).
6. No edge effects (z-direction).

With assumptions 1 and 2, get momentum equation:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = \nu_\infty \nabla^2 \vec{u} + \frac{1}{\rho_\infty} \left(\rho \vec{g} - \nabla p\right).$$

Now for x-momentum, steady-state (assumption 3), assumption 4 gives:

$$\vec{u} \cdot \nabla u_x = \nu_\infty \nabla^2 u_x + \frac{-\rho g + \rho_\infty g}{\rho_\infty}$$

Now assumptions 5 and 6, x-momentum becomes:

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \nu_\infty \nabla^2 u_x + g\beta(T - T_\infty)$$

With $T_s > T_\infty$ and $g_x = -g$, this gives driving force in the positive-x direction, which is up, like it's supposed to. Okay, that's all for today, more next time.

5.4 December 5: Wrapup Natural Convection

Mechanics:

- Test 2: before max=90, mean 75.38, std. dev 12.23; after max=100, mean 95.76, std. dev 6.37.

Muddy from last time:

- D'oh! Left too early...

Last time: assumptions led to equation:

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \nu_\infty \nabla^2 u_x + g\beta(T - T_\infty)$$

One more assumption, $\delta_u \ll x$, gives:

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \nu_\infty \frac{\partial^2 u_x}{\partial y^2} + g\beta(T - T_\infty)$$

New dimensional analysis:

$$h = f(x, \nu, k, \rho c_p, g\beta, T_s - T_\infty)$$

Seven params 4 base units (kg, m, s, K); 3 dimless params. Again Pr (dim'less ρc_p), Nu (dim'less h), this time Grashof number (dim'less β).

$$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

Forced convection: $\mathrm{Nu} = f(\mathrm{Re}, \mathrm{Pr})$.
Natural convection: $\mathrm{Nu} = f(\mathrm{Gr}, \mathrm{Pr})$.
Detour: recall falling film

$$u_x = \frac{g \sin\theta (2Lz - z^2)}{2\nu}$$

$$u_a v = \frac{g \sin\theta L^2}{3\nu}$$

$$\mathrm{Re} = \frac{u_a v \delta}{\nu} = \frac{g \cos\beta \delta^3}{3\nu^2}$$

So Gr is a natural convection Reynolds number, determines the rate of growth of the BL.

Graphs of dimensionless $T = (T - T_\infty)/(T_s - T_\infty)$, dimensionless $u_x = \mathrm{Re}_x / 2\sqrt{\mathrm{Gr}_x}$ vs. $y/\sqrt[4]{Gr_x}$ on P&G p. 232 corresponding to dimensional graphs in W^{3}R p. 313. Explain velocity BL is always at least as thick as thermal BL, but thermal can be thinner for large Pr.

Forced convection: $\delta \propto \sqrt{x}$
Natural convection: $\delta \propto \sqrt[4]{x}$
Note: in P&G p. 232 plots, Pr=0.72 corresponds to air.
Another Gr interpretation: dimensionless temperature gradient; for $\theta = \frac{T - T_\infty}{T_s - T_\infty}$:

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \theta} \frac{\partial \theta}{\partial \frac{y}{x} \sqrt[4]{\frac{Gr_x}{4}}} \frac{1}{x} \sqrt[4]{\frac{Gr_x}{4}} = (T_s - T_\infty) f(\mathrm{Pr}) \frac{1}{x} \sqrt[4]{\frac{Gr_x}{4}}$$

Note velocity squared proportional to driving force in pipe flow, kinda same here; heat trans proportional to square root of velocity. Hence $\mathrm{Re}_x \propto \sqrt{\mathrm{Gr}_x}$ for velocity, $\mathrm{Nu_x} \propto \sqrt{\mathrm{Re_x}} \propto \sqrt[4]{\mathrm{Gr}_x}$.

Transition to turbulence determined by Ra=GrPr, boundary at 10^9. Laminar, Ra between 10^4 and 10^9:

$$\frac{\mathrm{Nu}_L}{\sqrt[4]{\mathrm{Gr}_L/4}} = \frac{0.902 \mathrm{Pr}^{1/2}}{(0.861 + \mathrm{Pr})^{1/4}}$$

Special for $0.6 < \mathrm{Pr} < 10$, laminar:

$$\mathrm{Nu}_L = 0.56(\mathrm{Gr}_L \mathrm{Pr})^{1/4}$$

Turbulence, Ra between 10^9 and 10^{12} (p. 259):

$$\mathrm{Nu}_L = \frac{0.0246\mathrm{Gr}_L^{2/5} Pr^{7/15}}{(1 + 0.494\mathrm{Pr}^{2/3})^{2/5}}$$

Again, velocity$^{0.8}$ in a way, sorta like turbulent forced convection boundary layers.

5.5 December 8: Wrapup Natural Convection, Streamfunction and Vorticity

Mechanics:

- Final exam Monday 12/15 in 4-149. Discuss operation, incl. closed/open sections, new diff eq, essay.

Muddy from last time:

- What were we supposed to get out of the last lecture? Pretty much the list given: how natural conv BLs work, calculate $h_{(D)L}$ using Nu_L, $\delta_u \geq \delta_T$ or δ_C, natural BLs grow more slowly, velocity and temperature profiles.
- What direction is velocity? Dominant velocity is in x-direction, which is vertical; upward for hot wall, downward for cold. What's the difference between velocity in the BL, far from it? Far from it, velocity is zero.
- Why $\delta_u \geq \delta_T$? Hot region lifts (or cold region sinks) fluid, so all of the hot/cold region (thermal BL) will be moving (in the velocity BL). For large Pr, $\nu > \alpha$, so the momentum diffusion happens faster, thin thermal and thick velocity.
- Dimensionless curves: crazy non-intuitive axis value $\frac{y}{x}\sqrt[4]{\mathrm{Gr}_x/4}$! Well, not much worse than Blassius: u_x/U_∞ vs. $\beta = y\sqrt{U_\infty/\nu x} = \frac{y}{x}\sqrt{\mathrm{Re}_x}$. But I'll give you that the dimensionless velocity is a bit odd.
- Where do these things come from? Okay. Concretize:

$$\frac{\frac{u_x x}{\nu}}{2\sqrt{\mathrm{Gr}_x}} = \frac{1}{2}\frac{u_x x}{\nu}\sqrt{\frac{\nu^2}{g\beta\Delta T x^3}} = \frac{u_x}{\sqrt{g\beta\Delta T x}} \Rightarrow u_{x,max} = f(\mathrm{Pr})\sqrt{g\beta\Delta T x}.$$

$$\frac{\delta_u}{x}\sqrt[4]{\frac{\mathrm{Gr}_x}{4}} = f(\mathrm{Pr}) \Rightarrow \delta_u = f(\mathrm{Pr})\frac{x}{\sqrt[4]{\mathrm{Gr}_x/4}} = \sqrt{2}f(\mathrm{Pr})\sqrt[4]{\frac{x^4\nu^2}{g\beta\Delta T x^3}} = \sqrt{2}f(\mathrm{Pr})\sqrt[4]{\frac{x\nu^2}{g\beta\Delta T}}.$$

These two results are consistent with: $u_{x,max} \propto \text{thickness}^2$, forced convection $\Delta u_x/\Delta y$ goes as $1/\sqrt{\mathrm{Re}_x}$.

Other geometries: Raylegh-Bernard cells in inversion for GrPr greater than 1000. Solutal buoyancy too, dissolving salt cube.

$$\beta_C = -\frac{1}{\rho}\frac{d\rho}{dC}.$$

Special: nucleate boiling, film boiling, h vs. T with liquid coolant.

If time: BL on rotating disk: $u \propto r$, so uniform BL. Pretty cool.

Now can calculate (estimate) heat/mass transfer coefficients for forced and natural convection, laminar or turbulent. (D'oh! Forgot this closing part after the muddy stuff.)

Stream Function and Vorticity Vorticity introduced in turbulence video, measure of local rotation, definition:

$$\omega = \nabla \times \vec{u}$$

2-D scalar, 3-D vector. Some formulations give 2-D NS in terms of u_x, u_y, ω. Also, vorticity particle methods: bundles of vorticity moving, combining, annihilating.

Other application: crystal rotation in semisolid rheology.

Stream function, for incompressible flow where $\nabla \cdot \vec{u} = 0$:

$$u_x = \frac{\partial \Psi}{\partial y},\ u_y = -\frac{\partial \Psi}{\partial x}$$

Collapses velocity components into one parameter. Look at $\Psi = Ax$, $\Psi = By$, $\Psi = Ax + By$, $\Psi^2 = x^2 + y^2$. Cool.

Gradient is normal to flow direction. Streamlines: curves of constant Ψ, parallel to flow direction. If spaced apart same difference in Ψ, then

$$|\vec{u}| \propto \text{distance between streamlines}$$

Aero-astros look out at wing and see streamline, Mech Es see structure, Mat Scis see a giant fatigue specimen...

Visualizing 2-D flows, giving approximate regions of large and small velocity. DON'T CROSS THE STREAMS!

Concept: flow separation, difference between jet and inlet. Breathing through nose. (D'oh! Forgot to mention breathing through the nose.)

Decisions... Finish the term with the Bernoulli equation, or continuous flow reactors? Bernoulli wins the vote.

5.6 December 10, 2003: Bernoulli, Semester Wrapup

TODO: get rooms for review sessions!

Mechanics:

- Review sessions: me Friday 2 PM, Albert Sunday evening.

No muddy cards from last time.

Bernoulli Equation W^3R chap 6: control volume integral derivation based on first law of thermodynamics. Interesting, I do somewhat different, based on Navier-Stokes; I like to think mine is more straightforward, but you can read W^3R if needed.

Also called "inviscid flow". Motivation: tub with hole, pretty close to zero friction factor, velocity is infinity? No. Something other than viscosity limits it.

Navier-Stokes, throw out viscous terms:

$$\rho\frac{D\vec{u}}{Dt} = -\nabla p + \rho\vec{g}$$

Change coordinates to local streamline frame: $\hat{s}$ in direction of flow, $\hat{n}$ in direction of curvature (perpendicular in 2-D, complicated in 3-D).

Flow only in s-direction, s-momentum equation for $\vec{g} = -g\hat{z}$:

$$\rho\left(\frac{\partial u_s}{\partial t} + u_s\frac{\partial u_s}{\partial s}\right) = -\frac{\partial p}{\partial s} + \rho g_z\frac{\partial z}{\partial s}$$

Steady-state, constant ρ:

$$\frac{\partial\left(\frac{1}{2}\rho u_s^2\right)}{\partial s} + \frac{\partial p}{\partial s} - \rho g_z\frac{dz}{ds} = 0$$

Integrate along a streamline:

$$\frac{1}{2}\rho V^2 + p + \rho g z = constant$$

In other words:

$$KE + P + PE = constant$$

This is the Bernoulli equation.

Example 1: draining tub with a hole in the bottom. Set $z = 0$ at the bottom: PE=ρgh at top, P at bottom corner is that plus atmospheric pressure, $\frac{1}{2}\rho V^2$ beyond outlet (further accelerating). Potential energy becomes pressure $\Delta P = \rho gh$, then becomes kinetic $V = \sqrt{2gh}$.

Illustrate how changes with long tube h_2 down from bottom: ρgh at top, P_0 at base in corner, $\frac{1}{2}\rho V^2 + P_1$ at base over spout, $\frac{1}{2}\rho V^2 - \rho gh_2$ at tube end. Three equations in three unknowns. Solves to $P_1 = \rho gh$, $V^2 = 2g(h + h_2)$, $P_1 = -\rho gh_2$. Can also fill in the table...

Conditions:

- No shear or other losses (not nearly fully-developed)
- No interaction with internal solids, etc.
- No heat in or out, mechanical work on fluid (pumps, etc.)
- No sudden expansion (jet$\longrightarrow$turbulent dissipation, separation complicates stuff)
- No turbulence
- No combustion (mixing$\longrightarrow$effective viscosity)
- *Yes* sudden contraction.

Note time-to-drain problem on final of three years ago (that was the "derive and solve a new equation" problem of 2000), tendency for diff eqs and thought problems...

Semester summary You've come a very long way! Mentioned linear to multiple nonlinear PDEs, understanding of solution. More generally, learned to start with a simple conservation relation: accum = in - out + gen, turn into really powerful results, on macro or micro scale, for diffusion, thermal energy, mass, momentum, even kinetic energy.

Covered all topics in fluid dynamics and heat and mass transfer, in MechE, ChemE, aero-astro. If want to go on, take graduate advanced fluid dynamics or heat/mass transfer, will be bored in undergrad class.

Also done some computation; for more depth with or without programming experience, try 22.00J/3.021J! (Shameless plug...)

Thank Albert for a terrific job as a TA!

Last muddy questions

- What is the relevance of the boundary layer thickness to the Bernoulli equation? The boundary layer is a region where there is quite a bit of shear, and sometimes turbulence. If it is thin relative to the size of the problem (e.g. relative to the diameter of the tube), then most of the fluid will have negligible shear.

- Why such a wierd coordinate system in Bernoulli example 2? Why not just make $z = 0$ at the bottom of the tube? You could do that too, and it would work equally well, it just differs by a constant in the potential energy; the way we did it is just more consistent with the first example:

Point	KE	P	PE
1	~ 0	p_{atm}	$\rho g(h_1 + h_2)$
2	~ 0	$p_{atm} + \rho g h_1$	$\rho g h_2$
3	$\rho g(h_1 + h_2)$	$p_{atm} - \rho g h_2$	$\rho g h_2$
4	$\rho g(h_1 + h_2)$	p_{atm}	0

Batch and Continuous Flow Reactors For those interested.

Basic definitions, motivating examples. Economics: batch better for flexibility, continuous for quality and no setup time (always on).

Two types: volumetric and surface reactors. Volume V, generation due to chemical reaction; we'll discuss first-order $A \longrightarrow B$, so

$$G = -kC_A$$

For a volume batch reactor, start with $C_{A,in}$, dump into reactor, it goes:

$$\text{accum} = \text{generation}$$

$$V\frac{dC_A}{dt} = -VkC_A$$

$$\ln(C_A) = -kt + A$$

$$\frac{C_{A,out}}{C_{A,in}} = \exp(-kt)$$

For mass transfer-limited surface batch reactor, say

$$\text{accum} = \text{out}$$

$$V\frac{dC_{A,out}}{dt} = -Ah_dC_A$$

$$\frac{C_{A,out}}{C_{A,in}} = \exp\left(-\frac{h_D A}{V}t\right)$$

Two extremes in continuous reactor behavior with flow rate Q: plug flow and perfect mixing.

Plug flow is like a mini-batch with $t_R = V/Q$, draw plug in a pipe, derive:

$$\frac{C_{A,out}}{C_{A,in}} = \exp\left(-\frac{kV}{Q}\right)$$

With a surface, the Vs cancel, left with

$$\frac{C_{A,out}}{C_{A,in}} = \exp\left(-\frac{h_D A}{Q}\right)$$

Perfect mixing: in, out, gen, no accum, out at $C_{A,out}$ reactor conc:

$$0 = QC_{A,in} - QC_{A,out} - kVC_{A,out}$$

$$\frac{C_{A,out}}{C_{A,in}} = \frac{Q}{Q+kV} = \frac{1}{1+\frac{kV}{Q}}$$

With area:

$$\frac{C_{A,out}}{C_{A,in}} = \frac{1}{1+\frac{h_D A}{Q}}$$

Say target conversion is 0.01, given volume V, homogeneous with constant k.

- Batch:

  $$t_R = \frac{1}{k}\ln(C_{A,in}/C_{A,out}) = \frac{4.6}{k}$$

  prodection rate is

  $$\frac{V}{\frac{4.6}{k} + t_{change}} = \frac{kV}{4.6 + kt_{change}}$$

- Plug:

  $$Q = \frac{kV}{\ln(C_{A,in}/C_{A,out})} = \frac{kV}{4.6}$$

  Better than batch, likely better quality too, less flexible.

- Perfect mixing:

  $$Q = \frac{kV}{C_{A,in}/C_{A,out} - 1} = \frac{kV}{99}$$

  Much smaller than either of the others!

Dead zones and effective volumes!
How to tell: tracers, Peclet number.
Other examples: catalytic combustion (that dimensional analysis problem in PS3), alveoli/breathing (continuous/batch mixed). Batch: generally better conversion in same volume (see why); continuous: consistent quality, no setup time.
Steelmaking: batch, but folk want to make continuous.