

MIT OpenCourseWare
<http://ocw.mit.edu>

3.22 Mechanical Properties of Materials
Spring 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

3.032 Quiz 1
Fall 2007

***DO NOT TURN THIS PAGE OVER AND START THE QUIZ UNTIL YOU
ARE ASKED TO DO SO.***

Guidelines:

1. Show all your work on the sheets included in this stapled document.
 2. Use partial credit to your advantage. If you're running short on time, solve algebraically and then solve numerically (plugging in numbers) later.
 3. If there is not much space given for you to provide an answer, we want you to be brief.
 4. You may not need to use all the information given (e.g., dimensions) to reach your conclusions.
 5. Enjoy your dark chocolate before getting started! It lowers blood pressure (*JAMA*, 2003), contains antioxidants (*Nature*, 2003), and supposedly helps neural synapses responsible for memory to fire faster (*Nature Health*, 2003).
-

NAME (PRINTED):

I agree that this document represents my own independent work on this quiz, using only my own brain, my allowed crib sheet of equations and notes, and my pen / pencil / calculator / protractor / compass / ruler / sliderule.

SIGNATURE:

GOOD LUCK AND ENJOY YOUR COLUMBUS DAY WEEKEND!

A soccer player was slide-tackled by an opponent, which unfortunately caused her tibia (large lower leg bone) to fracture. In this quiz, you will analyze the deformation and repair of this bone as an idealized case of forces applied to a simply-supported solid cylinder made of an isotropic, linearly elastic material in static equilibrium.

Image removed due to copyright restrictions. Please see any any photo of a slide tackle, such as <http://www.geocities.com/sensaydominica/soccer/dombahamas.jpg>

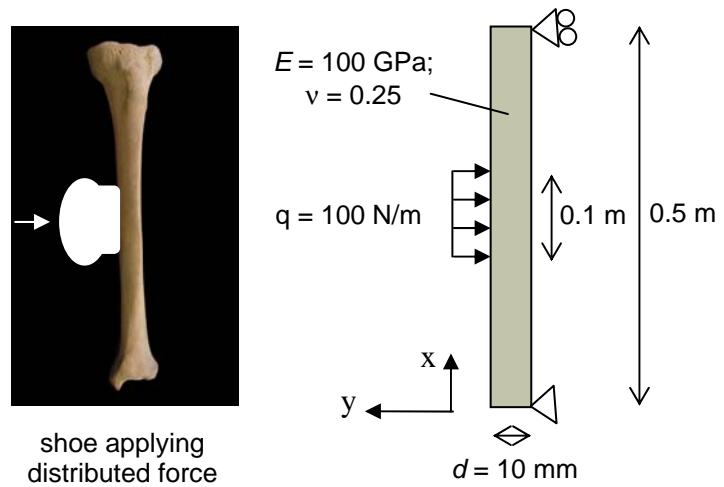


Fig. 1. Idealization of tibia under distributed force.

1. (a) Idealizing the supports as shown in the above figure, **determine the location and magnitude of the maximum bending moment**. Full shear force and bending moment diagrams $V(x)$ and $M(x)$ are not required, but are likely helpful.

(b) **Determine the magnitude of maximum tensile stress** on the bone at this location of maximum bending moment, which is her tibia's flexural strength.

2. To repair the bone fracture, her leg will be placed under compressive force in the simple pin-jointed traction device shown below, by applying a vertical force F_B at pin B.

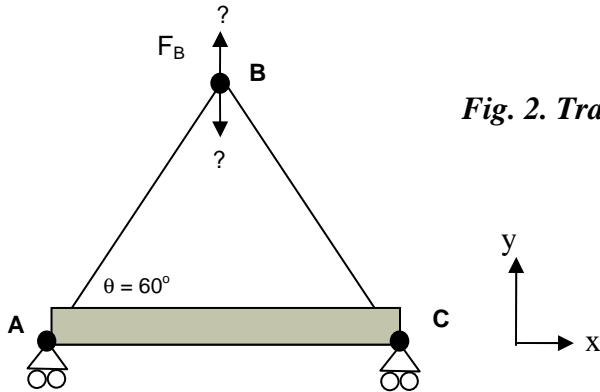
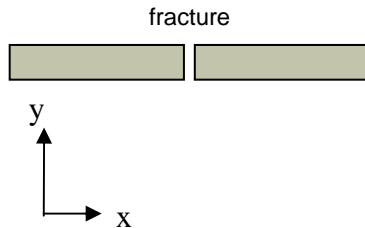


Fig. 2. Traction device to maintain bone in compression.

(a) Determine force in the member F_{AB} in terms of any required reaction forces and moments at A and C and the angle θ , and indicate whether the applied force at B, F_B , should be in the +y or -y direction, as indicated in the x-y axes of the figure, in order to maintain this bone in compression within this device.

(b) The normal stress required to heal bone fracture in static compression is 15 MPa. Determine the magnitude of the axial force in the bone, member AC, that must be applied to achieve this stress if the fracture is oriented as shown:



(c) The critical buckling load of the members AB and BC is 10 kN. Does this limit the amount of compressive force that this device can apply to the bone, which is member AC? Discuss fully but concisely.

3. To reduce swelling of the leg while in the traction device, a hydraulic pressure sleeve (like a blood pressure armband) is applied. This exerts compressive pressure of 10 MPa normal to the bone cylindrical surface:

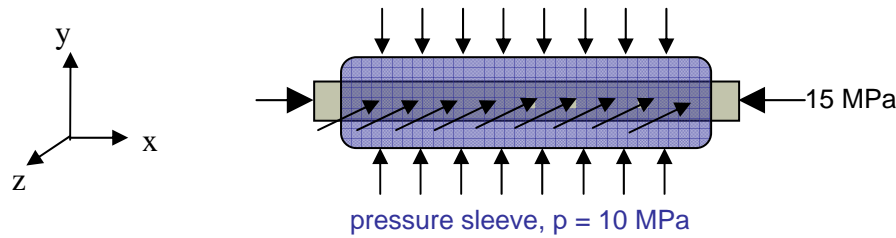
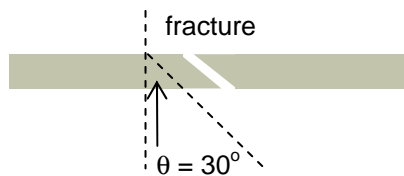


Fig. 3. Superposition of pressure against cylindrical surface of compressed bone.

(a) For the above tibia, **express the full tensor representation of the stress state σ_{ij} of a representative volume element (tiny cube) of the bone**, according to the axes orientation indicated in the figure.

(b) **Determine the magnitude of the normal strain component ϵ_{yy}** under this multi-axial stress state for this isotropic linear elastic continuum.

(c) X-rays show that the fracture line in the bone is inclined 30° counterclockwise from the bone cross-section over which the force F_{AC} acts, as shown. Consider just the x-y plane, and **determine the normal stress on this crack face resulting from the above σ_{ij}** .



4. The actual strength of bone under bending is much higher than the stress you determined in 1(b). **Using concepts covered in 3.032, list at least three (3) key features of the tibia structure and material** that we neglected when we chose to model this problem as a simply-supported solid cylinder made of an isotropic, linearly elastic material.

3.032 Quiz 2
Fall 2007

***DO NOT TURN THIS PAGE OVER AND START THE QUIZ UNTIL YOU
ARE ASKED TO DO SO.***

Guidelines:

1. Show all your work on the sheets included in this stapled document.
 2. Use partial credit to your advantage. If you're running short on time, solve algebraically and then solve numerically (plugging in numbers) later.
 3. If there is not much space given for you to provide an answer, we want you to be brief.
 4. You may not need to use all the information given (e.g., dimensions) to reach your conclusions.
 5. Enjoy your dark chocolate before getting started! It lowers blood pressure (*JAMA*, 2003), contains antioxidants (*Nature*, 2003), and supposedly helps neural synapses responsible for memory to fire faster (*Nature Health*, 2003).
-

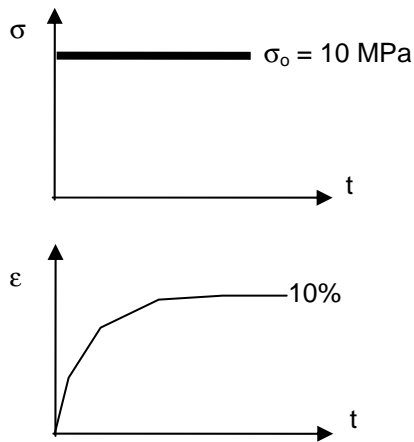
NAME (PRINTED):

I agree that this document represents my own independent work on this quiz, using only my own brain, my allowed crib sheet of equations and notes, and my pen / pencil / calculator / protractor / compass / ruler / sliderule.

SIGNATURE:

GOOD LUCK AND ENJOY YOUR VETERANS' DAY WEEKEND!

1. You have measured the creep response of polystyrene (PS, $T_g = 100\text{C}$) at room temperature, under a constant applied stress $\sigma_o = 10\text{ MPa}$, as shown in Fig. 1a.



(a) Which of the simple LVE models, Maxwell or Kelvin-Voigt, best represents this response. Explain your choice with reference to the governing constitutive equations.

Fig. 1a. Creep response of PS, 25 C.

(b) You have been asked to determine the characteristic time τ of this polystyrene. Explain how you would do this, with clear reference to the governing constitutive equations. You do not need to determine the exact magnitude of τ .

(c) Determine the strain you would expect to induce in this polystyrene under a *constant tensile loading rate* to this polystyrene up to $\sigma = 1\text{ MPa}$ over a loading time much less than this characteristic time τ .

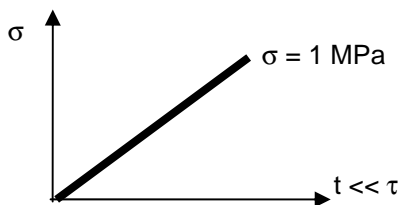


Fig. 1a. Constant loading and stress rate on PS in uniaxial tension.

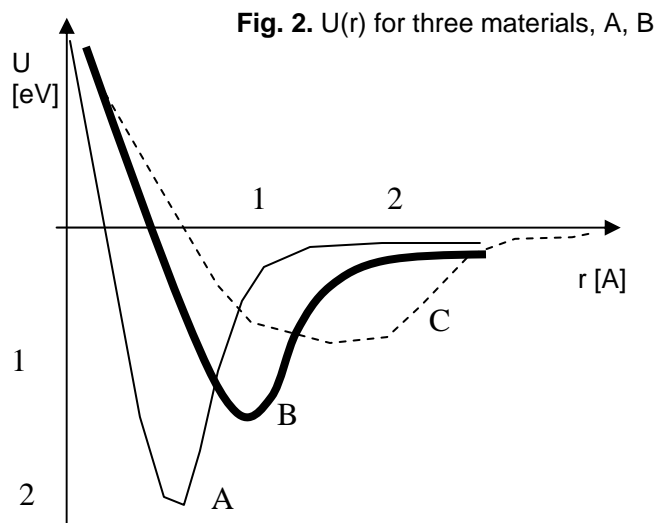
(d) Can you significantly increase the stiffness or effective E of this polymer at room temperature by increasing its molecular weight, which will increase the contour length and persistence length of the polystyrene chains? Explain with reference to mechanisms governing elastic deformation in polymers.

(e) If the strain determined in (c) were too great for the intended engineering application, describe another microstructural modification to increase the material stiffness at room temperature.

2. Compare/contrast materials that exhibit super/pseudoelasticity with those that exhibit rubber/hyper-elasticity:

	Super/Pseudoelastic material	Rubber/Hyperelastic material
Material composition		
Material structure		
Stress-strain response under uniaxial tension (graph)		
Atomistic mechanisms for this σ - ϵ shape		
Deformation governed by total energy/enthalpy or entropy		
Possible existence of dislocations (yes/no)		
Mechanical energy dissipation upon unloading (yes/no)		
Plausible engineering applications		

3.



(a) Given three interatomic potentials U_A , U_B and U_C in Fig. 2, which one is best representative of a metal, a ceramic, and an amorphous polymer deformed at temperatures $< T_g$. Assume there is only one of each material class, and explain your reasoning.

(b) Construct (draw) the yield surface you feel most accurately captures the conditions required to *ensure yielding of material B*. Explain your answer and choice completely but concisely.

(c) The yield strength of material B is $\sigma_y = 100$ MPa, and is required to reversibly deform under a state of pure shear $\tau_{12} = \tau_{21} = 60$ MPa (and all $\sigma_{ii} = 0$). Determine whether this is possible, noting that the principal stress state may be useful.

(d) Explain one physical, structural, *and* mechanical property would you require to identify a new crystalline material that would have a higher yield strength than material B, and why this would this property would correlate with increased yield strength. Be brief!

- Crystal structure property:
- Microstructural property:
- Mechanical property:

(e) The application in (c) is for a solar cell, for which Si is being considered. Explain which of the above strengthening mechanisms in (d) that you'd rule out to maximize efficiency (%light converted to electricity) of Si that is used in solar cells.

3.032 Quiz 3
Fall 2007

***DO NOT TURN THIS PAGE OVER AND START THE QUIZ UNTIL YOU
ARE ASKED TO DO SO.***

Guidelines:

1. Show all your work on the sheets included in this stapled document.
 2. Use partial credit to your advantage. If you're running short on time, solve algebraically and then solve numerically (plugging in numbers) later.
 3. If there is not much space given for you to provide an answer, we want you to be brief.
 4. You may not need to use all the information given (e.g., dimensions) to reach your conclusions.
 5. Enjoy your dark chocolate before getting started! It lowers blood pressure (*JAMA*, 2003), contains antioxidants (*Nature*, 2003), and supposedly helps neural synapses responsible for memory to fire faster (*Nature Health*, 2003).
-

NAME (PRINTED):

I agree that this document represents my own independent work on this quiz, using only my own brain, my allowed crib sheet of equations and notes, and my pen / pencil / calculator / protractor / compass / ruler / sliderule.

SIGNATURE:

GOOD LUCK AND ENJOY YOUR VETERANS' DAY WEEKEND!

1. Wiki modifications

(a) Correct this entry:

“Precipitation Hardening

Main article: [Precipitation strengthening](#)

In most binary systems, alloying above a concentration given by the phase diagram will cause the formation of a second phase. A second phase can also be created by mechanical or thermal treatments. The particles that compose the second phase precipitates act as pinning points in a similar manner to solutes, though the particles are not necessarily single atoms.”

(b) This entry is true, but explain the atomistic origin of this increased strength in amorphous polymers:

“Mixing polymers is another method of increasing strength, particularly with materials that show crazing preceding brittle fracture such as atactic [polystyrene](#) (APS). For example, by forming a 50/50 mixture of APS with polyphenylene oxide (PPO), this embrittling tendency can be almost completely suppressed, substantially increasing the fracture strength ^[5].”

(c) How can the following statement be true, when we know that a perfect single crystal has theoretical strength on the order of GPa?

“As shown in Figure 1 and the equation above, work hardening has a half root dependency on the number of dislocations. The material exhibits high strength if there are either high levels of dislocations (greater than 10^{14} dislocations per m^2) or no dislocations. A moderate number of dislocations (between 10^{-7} and 10^{-9} dislocations per m^2) typically results in low strength.”

2. Samuel Evans' family attempted to cook a turkey in their oven on Thanksgiving Day 2007, but the heating element suspended at the bottom of the oven catastrophically failed. It shattered into several pieces when the element dropped to the oven floor, making it difficult to determine the cause of material failure. Here you will determine the most likely cause and mechanism of fracture in the heating element. Relevant properties of the materials are listed on the board.

(a) Idealizing the heating element as a simply supported beam (Fig. 1a) that is loaded by the force due to gravity (i.e., distributed load due to its mass), determine the static shear force and bending moment at the cantilever support.

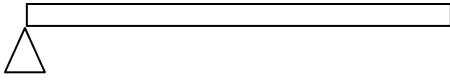


Fig. 1a. Heating element of length $L = 20$ cm and mass = 200 g/cm.

(b) The heating element is comprised of three materials: a copper wire core, an oxide-based ceramic jacket, and a conductive steel jacket (Fig. 1b). Determine the magnitude and orientation of the maximum normal stress and the maximum shear stress due to thermal heating to a typical turkey-cooking temperature of 350°F (177°C or 450K) in the Cu core, idealizing Cu as a linear elastic, isotropic continuum. Assume the heating element is at room temperature when not in use.

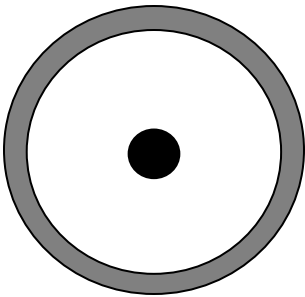


Fig. 1b. Heating element cross-section of diameter $D = 5$ mm. Center = Cu core; white jacket = oxide; outer rim = steel jacket.

(c) Rank these three materials in terms of the maximum stress generated in each, due to heating the element, justifying your answer with calculations as needed.

(d) Which of these materials is the stiffest, and why? Which of these materials is the strongest, and why? Which of these materials is the toughest, and why? Here, “why” means “what is the atomistic origin of this mechanical property?”.

(e) If Fig. 1d represents the deformation mechanism map of the copper wire core, is creep a likely source of failure? Here, the stress on the Cu is due to the total weight of the element and the thermal stress due to heating.

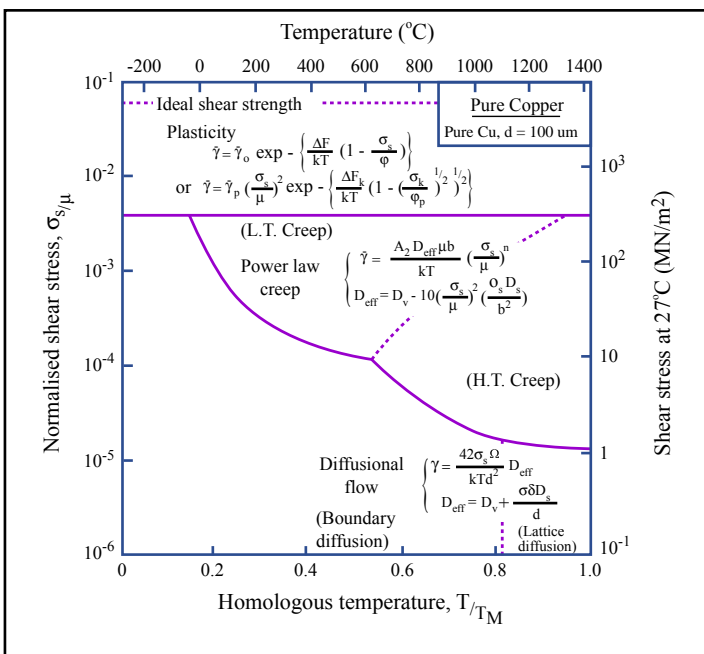


Fig. 1d. Deformation mechanism map for Cu of similar microstructure and processing to Cu core of heating element.

Figure by MIT OpenCourseWare.

(f) Based on your identification of the dominant deformation mechanism under static stress, and how would you propose to modify the Cu microstructure to decrease the rate of creep deformation in the core?

(g) Identify the material best modeled by Griffith’s fracture law, and determine whether this combined stress due to mass and temperature was sufficient to fracture this material.

(h) In actuality, the heating element undergoes cyclic heating that of high amplitude and low frequency. Let's assume the oven is used once a week to cook a large piece of meat or casserole. Graph the cyclic stress $\sigma(t)$ for each of the three materials on the same graph. Clearly identify values of σ_a , $\Delta\sigma$, and ν .

(i) What is the stress at the outer surface of the steel jacket, and why is it that value?

(j) Idealize the cyclic heating as sinusoidal with the stress amplitude identified in (h). Let us assume that the Cu core was visually inspected for surface cracks before the heating element was constructed, and that these thermal stresses are sufficient to propagate a pre-existing crack. Determine the expected lifetime of the Cu wire core due to this thermal fatigue, in units of weeks.

(k) Sketch a representative fracture surface that you would expect to see if the Cu core of the heating element failed via fatigue.

(l) Explain why the Cu might fail via thermal fatigue even if the applied stresses due to weight and temperature were below the fracture stress of Cu measured under uniaxial tension.

(m) Briefly describe one of the two concepts of designing materials and structures to extend fatigue failure lifetimes (one being more conservative than the other). According to this material design principle, describe how you would redesign the heating element materials and/or structure to increase N_f .

(n) The failure surface of the heating element is available for you to view in class. Based on your observations and above calculations, justify your identification of the most likely failure mechanism of this heating element. Note that you may decide creep, fracture, or fatigue failure was not the cause, but must present a well-justified alternative mechanism.

CONGRATULATIONS ON COMPLETING 3.032 AND YOUR FALL 2007 SEMESTER!
HAVE A GREAT BREAK.

USE THIS PAGE FOR EXTRA PROBLEM SOLVING AS REQUIRED.

USE THIS PAGE FOR EXTRA PROBLEM SOLVING AS REQUIRED.