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3.22 Mechanical Properties of Materials
Spring 2008

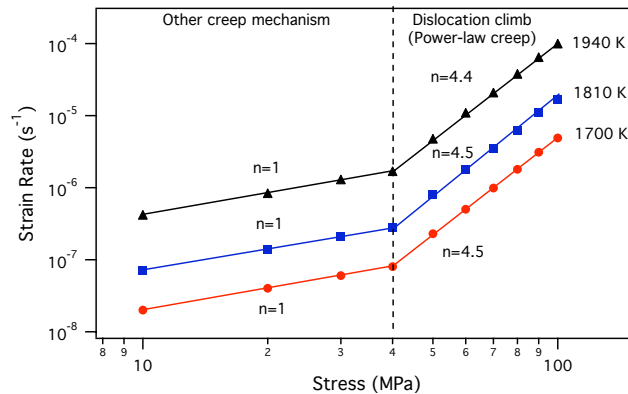
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Problem Set #5

Due: Friday, May 2 by 5:00 PM

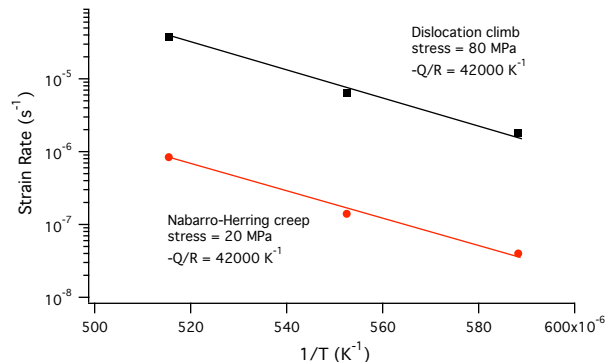
- The data in the supplementary spreadsheet 5-1.xls is obtained from creep experiments on a polycrystalline oxide with a grain size of $10\ \mu\text{m}$.
 - Make a plot of $\log(\text{strain rate})$ versus $\log(\text{stress})$ for the three temperatures. Identify on your plot the regions that correspond to diffusional creep and those that correspond to power-law creep. At what stress and temperature does the transition from dislocation climb to a mechanism that does not require dislocations occur?

Solution: The plot of $\log(\text{strain rate})$ versus $\log(\text{stress})$ for the three temperatures is shown below. We see that there are two regimes that are each linear but that have different slopes. By plotting the data this way, the slope is equal to the exponential dependency n of the strain rate on stress. In the lower stress regime, $n = 1$ corresponding to either Nabarro-Herring or Coble creep. In the higher stress regime, $n = 4.3$ which suggests power law creep (or dislocation climb). At all temperatures, the transition between the two regimes occurs at a stress of $\sim 40\ \text{MPa}$.



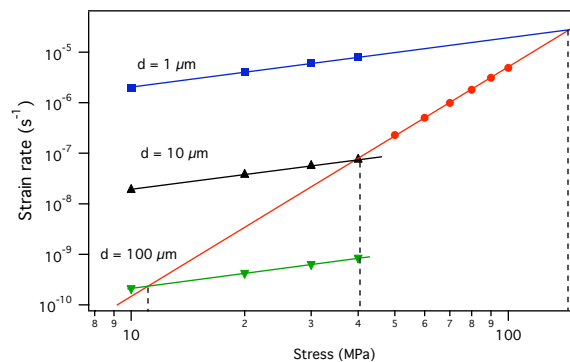
- Does the mechanism that does not require dislocations correspond to Nabarro-Herring creep or to Coble creep?

Solution: The steady-state creep rate of these two mechanisms differs in activation energy and dependency on grain size. Since we are not given any information about grain size, we should consider the activation energy, which is found from the slope of a plot of $\log(\text{strain rate}) T^{-1}$ (shown below). However this only gives us a value of the activation energy for this regime. In order to identify it as Nabarro-Herring or Coble creep, we must compare it against a value of a mechanism we know, such as the dislocation climb region in the plot of part (a). The activation energy for dislocation climb is the activation energy necessary to diffuse atoms through the lattice. This is the same activation energy for Nabarro-Herring creep. As shown in the plot below, we find the slopes of these two regions are the same, so we can say that this regime is Nabarro-Herring creep.



- (c) Plot the strain rate versus stress at 1700 K for this material at the grain size of 1 μm , 10 μm and 100 μm . How does the change in grain size affect the transition point between dislocation climb and the mechanism you named in part (b)?

Solution: See the graph below. We assume that by changing the grain size we remain within the Nabarro-Herring creep regime. Because the strain rate of Nabarro-Herring goes as d^{-2} (d = grain size), the Nabarro-Herring portion of the curve will increase or decrease 100 fold for an order of magnitude decrease or increase, respectively, in grain size. The dislocation climb region however will remain unchanged since it has no dependency on grain size. Therefore the stress at which the dominant mechanism transitions from Nabarro-Herring creep to dislocation climb increases with decreasing grain size.



- (d) Using the constitutive equations for Nabarro-Herring and Coble creep, show how the barrier in a deformation mechanism map between the two would change with grain size.

Solution: This question is really asking how the temperature and stress where the two mechanisms are the same varies with grain size. We can find this by equating the strain rates of Nabarro-Herring and Coble creep (e.g. $\dot{\epsilon}_{\text{NH}} = \dot{\epsilon}_{\text{C}}$). Doing this, we find

$$k_{\text{NH}} \frac{\sigma}{d^2} \left[D_{\text{o,L}} \exp \left(\frac{-Q_{\text{L}}}{RT} \right) \right] = k_{\text{C}} \frac{\sigma}{d^3} \left[D_{\text{o,GB}} \exp \left(\frac{-Q_{\text{GB}}}{RT} \right) \right]$$

where k is an empirical constant, d is the grain size, σ is the applied stress, D_o is a diffusional constant, Q is the activation energy, R is the gas constant, and T is the temperature. If we solve this for T , then we have the temperature at which the strain rates of Nabarro-Herring and Coble creep are the same. Doing this gives

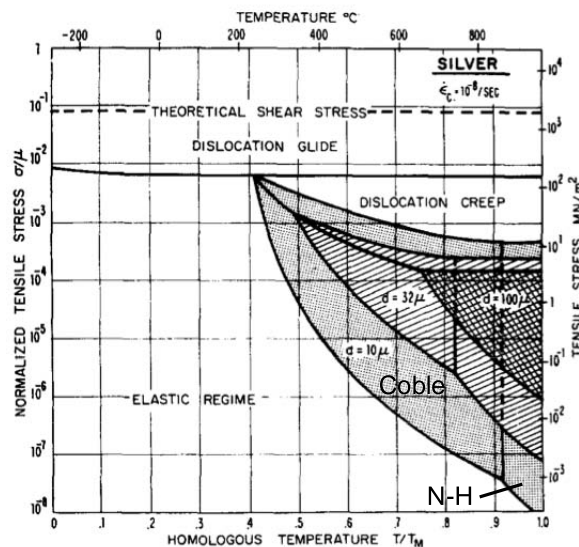
$$T = \frac{Q_{GB} - Q_L}{R \ln \left(\frac{D_{o,GB} k_C}{D_{o,L} k_{NH} d} \right)}$$

Therefore, if we increase the grain size, the boundary between these two regimes will move to lower temperatures.

We also see from this analysis that the stress terms cancel, which means the boundary is a vertical line and therefore the grain size has no effect on the position of the barrier along the stress axis.

- (e) Schematically draw a deformation mechanism map (σ/G vs. T/T_m) for this material with a grain size of $10 \mu\text{m}$. Clearly label the mechanism that dominates in the different regimes. Using the information you learned in parts (c) and (d), show on this deformation mechanism map how the boundaries between these regimes would change with a change in grain size to $1 \mu\text{m}$ and $100 \mu\text{m}$.

Solution: In part (c), we showed that the boundary between Nabarro-Herring creep and dislocation climb shifts to higher stresses with decreasing grain size (and vice versa). (This is also true for Coble creep). In part (d), we showed that the boundary between Nabarro-Herring and Coble creep shifts to higher temperature with decreasing grain size (and vice versa). Therefore the deformation mechanism map for $1 \mu\text{m}$, $10 \mu\text{m}$, and $100 \mu\text{m}$ schematically looks similar to the one show below (taken from M.F. Ashby, *Acta Metall.* **20**, 887 (1972))



Courtesy of Elsevier, Inc., <http://www.sciencedirect.com>. Used with permission.

- (f) How would the creep activation barrier change (higher or lower?) and the deformation mechanism map change in this material if it was an amorphous oxide? Explain. How would the activation barrier (higher or lower?) be different between an amorphous oxide and amorphous metal? Explain.

Solution: The activation energy is related to the difficulty associated with “rearranging” atoms locally in the structure. It is easier to rearrange atoms in amorphous materials because of the higher free volume. Therefore the activation energy in an amorphous oxide would be lower than that in a crystalline oxide with the same chemical composition. Similarly, in an oxide the bonding is covalent, which is stronger than the metallic bonds found in a metal. Therefore the activation energy in the amorphous metal would be lower than that in an amorphous oxide.

With regards to the deformation mechanism map, we would no longer see regions of power-law, Nabarro-Herring, or Coble creep; regimes associated with atomic movements in a polycrystalline atomic structure. Instead, other mechanisms will be activated that contribute to the viscous flow of the amorphous glass.

2. Ice is a hydrogen-bonded crystalline material that creeps appreciably, especially under the high stresses and potentially high temperatures characteristic of polar ice caps and glaciers.

- (a) Consider a portion of a polar ice cap that is under an in-plane equibiaxial compressive stress of 10 MPa at -50°C . What is the maximum shear stress imposed on the ice by this stress state?

Solution: Using Mohr’s circle, the maximum shear stress is the average of 10 and 0 MPa, or 5 MPa.

- (b) Referring to the deformation mechanism map below, what is the strain rate $d\gamma/dt$ we would expect this polar ice cap region to exhibit under the conditions defined in (a)? Clearly state any assumptions of ice cap geometry and relevant physical/mechanical properties. (A cool, relatively recent paper on ice mechanics is Elvin et al., *J. Eng. Mech.* Jan (1998); check out the author affiliations.)

Solution: From the given paper and other sources, G of ice is approximately 3 GPa. Interestingly, this is similar to the stiffness of amorphous, glassy polymers. The normalized shear stress is then $5\text{ MPa}/3\text{ GPa} = 10^{-3}$. At the given temperature, the ice is in a regime of power-law creep dominated by dislocation climb.

- (c) Assuming the applied stress on the ice cap remains constant, and the polar ice cap ambient temperature increases to the highest magnitude currently predicted by climatologists, what is the % difference in creep rate with respect to a current temperature of -50°C ?

Solution: The creep rate at the given temp is about $10^{-8}/\text{s}$. If the polar regions of the Earth warmed by 10°C , a severe prediction by climatologists, the creep rate would increase about an order of magnitude and could shift into the diffusional flow regime.

- (d) Assuming the ice cap is already in stage II creep in 2008 and this worst-case temperature is achieved this year, how long will it take for a polar ice cap of this grain size and thickness of 50 cm to shrink in in-plane dimensions by 50%?

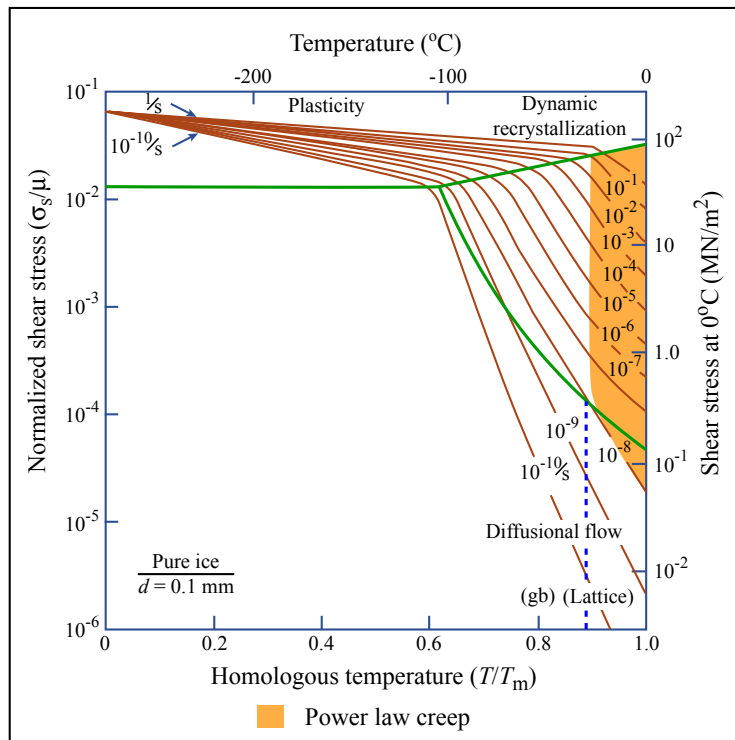


Figure by MIT OpenCourseWare.

Figure 1: Deformation mechanism map of pure ice.

Solution: For a total strain of 0.5 and a creep rate of $10^{-9}/s$, it would take about 109 s, or about 64 years.

3. What are the units for energy release rate? Explain thoroughly, yet concisely, what is meant by “rate” in this context? Explain how this relates to the material property of toughness. In general, rank the classes of materials (metallic, ceramic, and polymeric) from the highest to lowest energy release rate by using specific, cited examples from literature.

Solution: The units of energy release rate are MPa m (or more generally, energy per unit distance). The energy release rate is not a change of energy with respect to time, but is instead the rate of change in potential energy with crack area. It is therefore a value that represents the energy required to extend the crack. If this energy is small, the potential energy to increase the crack length is small, and thus the toughness of the material is low. Metals will have the highest energy release rate followed by polymers and then ceramics.

4. Calculation of the stress as a function from distance from an elliptical through-thickness crack tip of width $2a$ and tip radius R was given in class but not derived, as $\sigma = \sigma_o(1 + 2\sqrt{\frac{a}{R}})$. The derivation includes solution of a biharmonic equation of elasticity, the wise selection of an appropriate Airy stress function χ that maintains continuity and strain compatibility across the crack face, and complex variables; this is the same approach used to solve the self-stress around an edge dislocation. You could do it, but it’s typically treated in an advanced mechanical behavior class. To put things in perspective, this was solved in 1913 by Inglis, when analytical elastic mechanics was the only game in town.

- (a) The solution is a twist on the so-called Eshelby inclusion problem, or the effects of

stress due to a spherical hole (or particle) of radius $R = a$ inside a material deformed under uniaxial stress σ_o . It is derived in polar coordinates r (distance from hole center) and θ (in-plane angle around hole center), and has the nice result of being bounded at the maximum to some finite stress. Let's work on part of this to understand why the stress concentration is bounded to be lower than infinity for a circular flaw: The stress distribution σ_{ij} given by this internal defect is:

$$\sigma_{rr} = \frac{1}{2}\sigma_o\left(1 - \frac{R^2}{r^2}\right)\left\{1 + \left(1 - \frac{3R^2}{r^2}\right)\cos 2\phi\right\}$$

$$\sigma_{\theta\theta} = \frac{1}{2}\sigma_o\left(1 + \frac{R^2}{r^2}\right)\left\{1 + \left(1 + \frac{3R^4}{r^4}\right)\cos 2\phi\right\}$$

$$\sigma_{r\theta} = \frac{-1}{2}\sigma_o\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\phi$$

Given that you expect the stress to be maximized along the equator of the hole (i.e., orthogonal to the loading axis), use the above expressions to find the magnitude of the maximum stress component in terms of only the applied stress σ . In other words, what is the numerical, maximum stress concentration factor due to this circular flaw? Graphing σ_{ij} may help illustrate this.

Solution: There are several approximations of this type, based on the assumed displacement field $\mathbf{u}(r)$ from which the strain tensor ϵ_{ij} and then the stress tensor σ_{ij} are computed. The stress intensity factor that magnifies the applied stress σ_o can be evaluated at $R = r = a$ (i.e., at the perimeter of the hole) and $\theta = \phi = 0$ (for assumed coordinate axes centered at the hole center), such that $\sigma_{\theta\theta} = 2\sigma_o$ and $\sigma_{rr} = 0$, for the case of biaxial tension applied to a thick material containing a through-thickness hole.

For the uniaxial applied stress case above, $\sigma_{\theta\theta} = 5\sigma_o$ for $R = r = a$ and $\theta = 0$:

$$\sigma_{\theta\theta} = \frac{1}{2}\sigma_o(1 + 1)\left\{1 + (1 + 3) * 1\right\}$$

and $\sigma_{rr} = \sigma_{r\theta} = 0$. Other estimates of the maximum, taken as the derivative with respect to r or θ give $\sigma_{\theta\theta} = 3\sigma_o$. Both approaches for uniaxial and biaxial tension show that the stress at a blunted crack or a hole is not infinite, but only a couple or a few times the applied stress. This is why sharp cracks are much more deleterious than blunted cracks, in terms of applied stress required to fracture a defected material.

- (b) Now let us assume the crack is a through-thickness, sharp crack as Griffith considered, and the material is amorphous silica for which others have reported γ_{surf} and E . What is the fracture stress σ_f if this silica initially contained defects of length $2a = 1$ mm?

Solution: The Griffith solution simply balances the energetic penalty of new surface area to the energetic benefit of relieved strain energy. The values I found for γ_{surf} and E of silica are 3.5 to 5.3 J/m² [S.M. Wiederhorn. J. Am. Ceram. Soc. 52: 99 (1969)] and 68 GPa (my own measurements, but also handbook values), respectively. I assumed

a value of 4.25 J/m^2 for surface tension, as this is the average of reported values. With these values and initial internal crack of half-length 0.5 mm , the fracture stress σ_f is computed as:

$$\sigma_f = \sqrt{\frac{2E\gamma_s}{\pi a}} = 19 \text{ MPa}.$$

Note that surface tension is actually often computed by applying Griffith's criterion to fracture experiments, rather than measuring γ_s directly!

- (c) If your application required a fracture strength of 1 GPa and you could chemically modify any internal flaw surfaces in the above silica via chemical vapor deposition, what would the target surface tension be? Is this reasonably achievable?

Solution: To attain a 100-fold increase in fracture stress, we can compute the target γ_s from the same equation:

$$\gamma_s = \frac{\pi a \sigma_f^2}{2E} = 11,550 \text{ J/m}^2 = 12 \text{ kJ/m}^2 = 12 \text{ kN/m}.$$

This is far greater than the surface tensions typically measured for synthetic materials, and suggests that the only way to achieve such high fracture strengths in silica is to make the material flaw-free or change the environment from ambient air in a way that increases the effective surface tension with the surrounding ambient fluid. Both approaches are attempted in practice.

5. Please see comments and address new questions related to special topics wiki, which will be posted by Friday 4/25 night (after DMSE social). Also note that this will be the final wiki pset assignment. Next week you will be preparing a short presentation for the class, to be delivered on May 8 (R) or May 13 (T).