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3.22 Mechanical Properties of Materials
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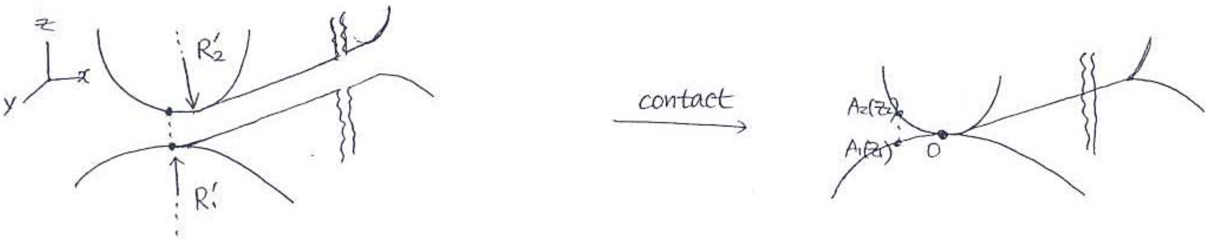
(c) 2-D indentation



In the derivation from PS 2, b in an elliptical contact becomes much larger than a. Thus it becomes a 2-D problem regardless of y.

① • separation distance between A_1 and A_2 is

$$h = z_1 + z_2 = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)x^2 = \frac{1}{2R}x^2 \qquad \text{where } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

• then sum of plastic displacement is

$$\bar{u}_{z1} + \bar{u}_{z2} = \delta - h = \delta - \frac{x^2}{2R} \quad \cdots ⓐ$$

② Local contact stresses

• $$\frac{\partial \bar{u}_{z1}}{\partial x} + \frac{\partial \bar{u}_{z2}}{\partial x} = -\frac{x}{R} = -\frac{2(1-\nu_1^2)}{\pi E_1}\int_{-a}^{a}\frac{p(s)}{x-s}\,ds - \frac{2(1-\nu_2^2)}{\pi E_2}\int_{-a}^{a}\frac{p(s)}{x-s}\,ds \quad \cdots ⓑ$$

where $p(x)$ is a normal pressure acting on the strip $-a \le x \le a$.

• when $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$,

$$-\frac{x}{R} = -\frac{2}{\pi E^*}\int_{-a}^{a}\frac{p(s)}{x-s}\,ds \qquad \therefore \int_{-a}^{a}\frac{p(s)}{x-s}\,ds = \frac{\pi E^*}{2R}x \quad \cdots ⓒ$$

Taken the distribution of pressure for granted,

then
$$p(x) = -\frac{\pi E^*}{2R}\frac{x^2 - a^2/2}{\pi\sqrt{a^2-x^2}} + \frac{P}{\pi\sqrt{a^2-x^2}} \quad \cdots ⓓ$$

To get uniqueness of p, use boundary condition of $p(x=\pm a) = 0$.

$$\therefore p(x=\pm a) = \lim_{x\to\pm a}\frac{1}{\pi\sqrt{a^2-x^2}}\left(-\frac{\pi E^*}{4R}a^2 + P\right) = 0$$

$$\therefore \quad P = \frac{\pi a^2 E^*}{4R} \quad \text{or} \quad a^2 = \frac{4PR}{\pi E^*} \quad \cdots \text{ⓔ}$$

$$p(x) = \frac{2P}{\pi a^2}\sqrt{a^2 - x^2} \quad \cdots \text{ⓕ} \quad (\text{plug ⓔ into ⓓ})$$

- stresses within two solids

$$\begin{cases} \sigma_x = -\dfrac{2z}{\pi}\displaystyle\int_{-a}^{a} \frac{p(s)(x-s)^2}{\{(x-s)^2 + z^2\}^2}\, ds \\ \sigma_z = -\dfrac{2z^3}{\pi}\displaystyle\int_{-a}^{a} \frac{p(s)}{\{(x-s)^2 + z^2\}^2}\, ds \end{cases} \quad \cdots \text{ principal stresses ⓖ}'$$

$\therefore$ in this case

$$\begin{cases} \sigma_x = -\dfrac{p_0}{a}\left\{\dfrac{a^2 + 2z^2}{\sqrt{a^2+z^2}} - 2z\right\} \\ \sigma_z = -\dfrac{p_0}{a}\dfrac{1}{\sqrt{a^2+z^2}} \end{cases} \quad \cdots \text{ principal stresses ⓖ}$$

where p_0 (maximum pressure where $x=0$) $= \dfrac{2P}{\pi a} = \dfrac{4}{\pi}\underbrace{p_m}_{\text{mean pressure}} = \sqrt{\dfrac{PE^*}{\pi R}}$

and principal shear stress

$$\tau_1 = -\frac{2z^2}{\pi}\int_{-a}^{a} \frac{p(s)(x-s)}{\{(x-s)^2 + z^2\}^2}\, ds = -\frac{p_0}{a}\left(z - \frac{z^2}{\sqrt{a^2+z^2}}\right) \quad \cdots \text{ⓗ}$$

- the maximum shear stress

$$\frac{\partial \tau_1}{\partial z} = 0 \longrightarrow \text{then } (\tau_1)_{max} = 0.30\, p_0 \text{ at } \underline{\underline{z = 0.78\, a}}$$

depth for maximum shear stress