3.35 – Fracture and Fatigue
Problem Set 4 – Solutions
November 4, 2003

Problem 1

a) $1 \text{ psi} = 6.895 \times 10^3 \text{ Pa}$ (stress scale) $(1 \text{ ksi} = 10^3 \text{ psi})$

$T_R = \frac{9}{5} T_c + 491.7$
$T = 273 + T_c$
(Temperature scales)

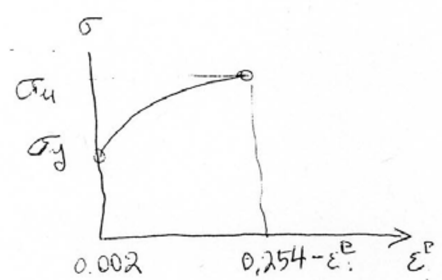
$T_c = -30°C \implies T_R = -24 °R$

Fig. 1 $\Rightarrow$

$$\sigma_y(T_R, \dot{\varepsilon}) = 4.55 \times 10^3 \left(T_R \ln(10^8/\dot{\varepsilon})\right)^{-0.431} \times 10^3 \text{ psi}$$

$$\left.\begin{array}{l}\dot{\varepsilon} = 10^{-2} s^{-1} \\ T_R = -24 °R\end{array}\right\} \Rightarrow \sigma_y^* = 1830 \text{ MPa}$$

$$\left.\begin{array}{l}\dot{\varepsilon} = 5 s^{-1} \\ T_R = -24 °R\end{array}\right\} \Rightarrow \sigma_y^* = 2000 \text{ MPa}$$



↑ engineering definition of yield strain

$\sigma_y = 481$ MPa
$\sigma_u = 642$ MPa
$(\varepsilon_y^p = 0.002$ definition$)$
$(\varepsilon_e$ = elastic strain $= \sigma/E)$

$\left(\varepsilon = \varepsilon^p + \frac{\sigma}{E} = \varepsilon^e + \varepsilon^p \quad \text{total strain}\right)$

Try a power law fit:

$$\frac{\varepsilon}{\varepsilon_y^e} = \left(\frac{\sigma}{\sigma_y}\right)^n \alpha$$

with $0.254 = \varepsilon$ and $\sigma = 642$ MPa
and $E = 200$ GPa

elastic strains:

$\sigma = 642\,\text{MPa} \Rightarrow \varepsilon^e_{ul} = \dfrac{642}{200\times10^3} = 3.21\times10^{-3} = 0.0032$

$\sigma = 481\,\text{MPa} \Rightarrow \varepsilon^e_{y} = \dfrac{481}{200\times10^3} = 0.0024$

plastic strains

$\sigma_y = 481 \Rightarrow \varepsilon^p_y = 0.002$

$\sigma_u = 642 \Rightarrow \varepsilon^p_u = -0.0032 + 0.254 = 0.251$

$\Rightarrow \begin{cases} \alpha \approx 1.0 \\ \text{and} \quad n \approx 16 \qquad > 10 \end{cases}$

Use $n = 10$ from Fig. 3

From Ritchie:

$I_n \approx 4.5$

$$g(n) = \tilde{\sigma}_{\theta\theta}(n)\Big|_{\theta=0}\left[\frac{1-\nu^2}{I_n}\right]^{\frac{1}{n+1}} = \tilde{\sigma}_{yy}(n)\Big|_{\theta=0}\left[\frac{1-\nu^2}{I_n}\right]^{\frac{1}{n+1}}$$

Poisson ratio: $\nu = 0.33$

$\Rightarrow g(n) = 2.1 \qquad (n=10)$

This can be found from Fig. 3 by normalizing

$$\frac{\sigma_{yy}}{\sigma_y} = \left[\frac{r}{(K/\sigma_y)^2}\right]^{-\frac{1}{1+n}} g(n) \quad ; \quad n = 10$$

$$K_{IC} = \sqrt{l_0^*}\ \frac{(\sigma_y^*)^{\frac{n+1}{2}}}{\sigma_y^{\frac{n-1}{2}}}\ g(n)^{-\frac{n+1}{2}}$$

s 1 $\Rightarrow$

$\sigma_y(T, \dot{\varepsilon})$ = as in previous equations

This includes a critical stress criterion at a microstructural length l_0^* ($\sim 2 d_g$, twice the grain size) and a small-scale HRR stress field (lower shelf K_{IC})

$$l_0^* \approx 2 \times 25\,\mu m = 50\,\mu m$$

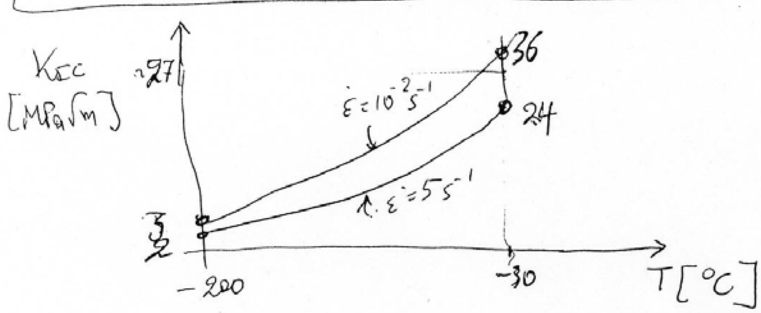
$$\boxed{K_{IC} = 1.2 \times 10^{-4}\ \frac{\sigma_y^{*\,5.5}}{\sigma_y^{4.5}}\ MPa\sqrt{m}}$$

For $\dot{\varepsilon} = 10^{-2}\,s^{-1}$

$\Rightarrow$ $K_{IC} = 6.4 \times 10^{-4}\,(273 + T_C)^{1.94}\ MPa\sqrt{m}$

For $\dot{\varepsilon} = 5\,s^{-1}$

$\Rightarrow$ $K_{IC} = 4.2 \times 10^{-4}\,(273 + T_C)^{1.94}\ MPa\sqrt{m}$



(b) Upper shelf region : $0 < T_c < 200°C$

Failure micromechanism is void growth and coalescence.

$$K_{Ic} = A\sqrt{l_0^* \varepsilon_f^* \sigma_y E(1-\nu^2)}$$

The equivalent plastic strain, ε_f^*, is related to the crack-tip opening displacement, δ. Using the results of Ritchie (Fig. 3) (or Fig. 4)

$$\varepsilon_f^* = 0.35\frac{\delta_c}{x} \approx 0.35\frac{\delta_c}{l_0^*}$$

($l_0^* = 50\ \mu m$) ↳ spacing between voids

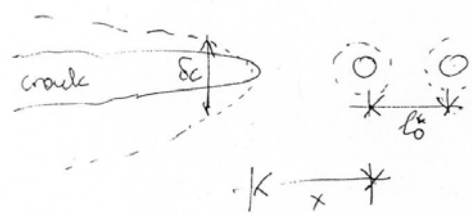
Elastic strain limit : $\frac{\sigma_y}{E} \approx 0.0025$

$$\delta_c = 0.6\frac{K_{Ic}^2}{E\sigma_y}$$

($E \approx 200 GPa$)

$K_{Ic} = 2.2\sqrt{l_0^* \varepsilon_f^* E\sigma_y}$

$$\Rightarrow K_{Ic} = 43.6\sqrt{\sigma_y^2 \varepsilon_f^* l_0^*} \quad [MPa\sqrt{m}]$$



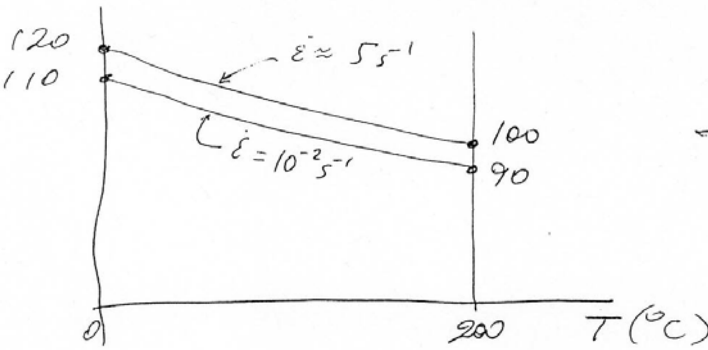
$\varepsilon_f^* \rightarrow \bar{\varepsilon}_f$
Fig. 2 gives the ratio of the hydrostatic stress versus the effective stress: $\sigma_m/\bar{\sigma}$.

$\frac{\delta_c}{l_0^*} \approx 0.09$

From Figure 4 $\Rightarrow \begin{cases} \frac{x}{\delta_t} \rightarrow \frac{x}{\delta_c} : \bar{\varepsilon}_p \sim 0.46\frac{\delta_t}{x} - 0.2 \\ \bar{\varepsilon}_p \rightarrow \varepsilon_p^* \quad \text{(from curve fit)} \end{cases}$

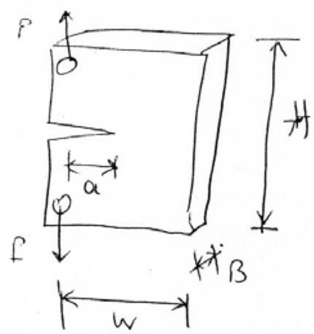
σ_y is as in question (a)

$\begin{cases} \sigma_m/\bar{\sigma} \gtrsim 1.2 \\ \text{Fig. 2} \Rightarrow \bar{\varepsilon}_f \approx 0.24 \end{cases}$



Note the <u>opposite</u> <u>effect</u> from the <u>low-sclt</u> region!

(c) A typical example of a single crack specimen geometry is a CT-specimen



(from ASTM:
$H \sim 1.2\,W$
$\frac{a}{W} > 0.5$)

Valid K_{Ic} test: $a, B, (W-a), H > 2.5 \left(\frac{K_I}{\sigma_y}\right)^2$

Valid J_{Ic} test: $a, B, (W-a), H > 25 \underbrace{\frac{K_{IC}^2 (1-\nu^2)}{\sigma_y E}}_{\frac{J_{IC}}{\sigma_y}}$

For valid K_{Ic} tests for all temperatures and strain rates, we must maximize $\left(\frac{K_{Ic}}{\sigma_y}\right)^2$.

This occurs in the upper-shelf region under static conditions. Using the previous results:

$$\max\left(\frac{K_{Ic}}{\sigma_y}\right)^2 \simeq 0.14 \quad \Rightarrow \quad a, B, (W-a) > 36 \text{ cm}$$

- For valid J_{Ic} tests for all temperatures and strain rates (assuming that $E \sim 200 GPa$ and $\nu \sim 0.33$ independent of temperature), we must maximize $\frac{K_{Ic}^2}{\sigma_y}$. This occurs in the upper-shelf region under static conditions. Using the previous results:

$$\max\left(\frac{K_{Ic}^2}{E\sigma_y}\right) = 0.032$$

$$\Rightarrow \quad a, B, (W-a) > 0.79 \text{ cm}$$

(d) Neutron irradiation :

- increases the yield strength
- reduces the strain hardening exponent
- critical fracture stress unaffected

$$\Rightarrow \begin{cases} \sigma_y^* \text{ constant} \\ n \text{ increasing} \\ \sigma_y \text{ increasing} \end{cases}$$

Leiter-self fracture :

$$K_{IC} = \sqrt{l_0^*} \, \frac{\left(\sigma_y^*\right)^{\frac{n+1}{2}}}{\sigma_y^{\frac{n-1}{2}}} \, g(n)^{-\frac{n+1}{2}} \qquad \text{with } n \geq 1$$

Microstructural size l_0^* constant (assumed)

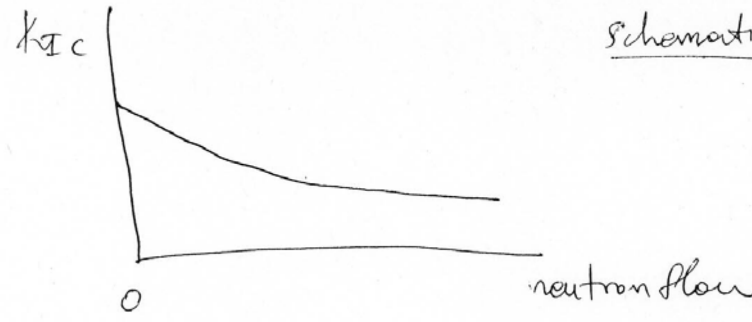
$g(n)$ weak function of n

$\Rightarrow g(n)^{-\frac{n+1}{2}}$ increases slowly with n

$\left(\sigma_y^*\right)^{\frac{n+1}{2}}$ decreases very fast with n

$\sigma_y^{\frac{n-1}{2}}$ decreases slowly with n, if σ_y is increasing

$\Rightarrow$ $\boxed{K_{IC} \text{ is decreasing with } \sigma_y \text{ and } n}$



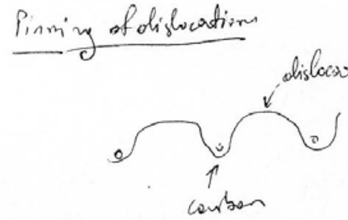
Schematically

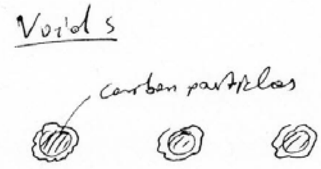
The influence of temperature could be important close to the transition temperature: the decrease could be smaller.

e) Austenitizing temperature controls the carbon content in steels. (increase it)

Small amounts of carbon increase the yield strength of steel, because carbon atoms can pin dislocations. Therefore, $\sigma_y \uparrow$. As a result, increasing the austenitizing temperature (AT), decreases the fracture toughness in the lower-shelf regime and increases it in the upper-shelf regime. So at low operating temperatures, the austenitizing temperature is detrimental and at high operating temperatures, the austenitizing temperature is beneficial.

High amount of carbon segregates into microparticles and may promote void growth due to the brittleness of the carbon microparticles (creates voids)





Finally, the austenite temperature make material transformation during the heat treatment more difficult, reducing the mechanical properties.

In addition, austenization increases the grain size of austenite phase (FCC). According to Hall-Petch law the yield strength decreases with increasing grain size ($\sigma_y \sim d_g^{-1/2}$). Therefore, the toughness should increase (because also $l_0^* \sim d_g$).

Typical austenization conditions: 900°C for several hours.