Solutions

9.1 a) For each surface of the film, $H_{magstat}$ = - (1/2)M = -$M_S \cos\theta$, so $H_{magstat}$ = -M.

$f_{magstat} = -\mu_o M_S \cdot H_{magstat}$

$f = -\mu_o M_S H \cos\theta + (\mu_o/2) M_S^2 \cos^2\theta$

$\partial f/\partial\theta = 0$ gives: $H = M_S \cos\theta = M_\perp$ (after division by $\sin\theta$, which is zero only at and above saturation). Thus:

$H/M_S = M_\perp/M_S = m$

The system saturates when $H = M_S$ =1.27 MA/m or when $B = B_S$ = 1.6T

b) [111] is the easy axis, so the only anisotropy is shape. Answer is same as a) but $H_a = H_{magstat} = -NM$ with N = 1/3 instead of 1. $m = 3H/M_S$

Saturation is achieved at (1/3) $\mu_o M_S \approx$ 0.2T.

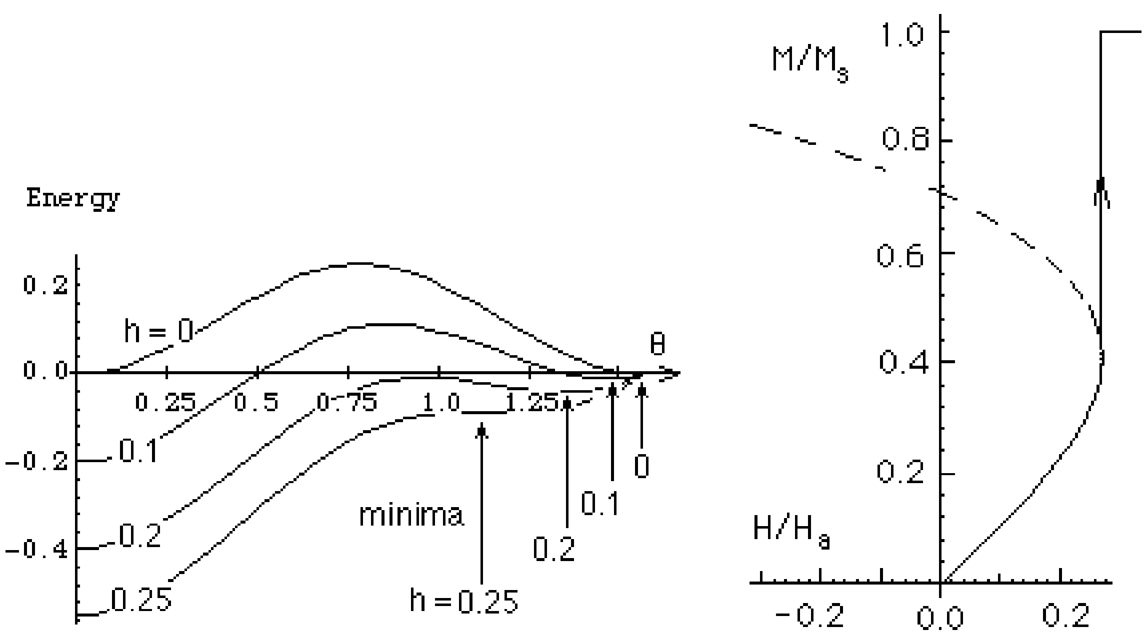
9.2 Putting m = 1 in Eq. 9.13 gives sin $2\theta_o = 0$ which can only be satisfied for $\theta_o = 0$ or $\pi/2$. So the $m(H)$ curves in Fig. 9.3 never reach m = 1 except for the two limiting cases, for $\theta_o = 0$ or $\pi/2$.

9.3 The energy density, $f = K_1 \sin^2\theta \cos^2\theta - M_S H \cos\theta$, is plotted below. f is minimized for the equation of motion: $(m - 2m^3) - h = 0$, where $h = M_S H/2K_1$. This cubic equation may have up to three different solutions. The physically meaningful one(s) can be discerned by considering the energy as a function of θ.

The equilibrium orientation θ_o decreases toward zero, i.e. $m = \cos\theta$ increases as H increases. At a field $h \approx 0.25$, the energy minimum near θ = 1.2 vanishes and the magnetization may jump abruptly to $\theta = 0$, m = 1.

The calculated form of m versus h is shown. The discontinuous change in m is a first order transition; it corresponds to what is called a *switching field*. It can be determined from the derivatives of the equation of motion, either $\partial h/\partial m = 0$ or $\partial m/\partial h = \infty$. Thus, the critical magnetiztion at switching is given by: $1 - 6m_c^2 = 0$. Thus m_c = 0.408... (or $\theta_c = 66^o$) which occurs for $h_c = 2/(3\sqrt{6}) = 0.272...$

Fig. for Sol. 9.3. Left, normalized energy density as a function of θ (radians)for different values of reduced field, $h = H/H_a$. Shift in equilibrium orientation with h is indicated. Right, calculated *m-h* behavior: m increases with increasing h then at m_c = 0.408, jumps to m = 1.0. The dashed line in the *m-h* curve shows the continuation of the mathematical solution. Note that the initial slope $\partial m/\partial h)_o = 1$ or $\partial M/\partial H = M_s/H_a$ gives the value for the anisotropy field H_a. Thus, the value of K_1 can be determined by measuring *m(h)* and using either the initial slope or the critical field h_c.



Prob. 9.3. Left, normalized energy density as a function of θ (radians) for different values of reduced field, $h = H/H_1$. Shift in equilibrium orientation with h is indicated. Right, calculated *m-h* behavior: m increases with increasing h then jumps to m = 1.0 at m_c = 0.408.

9.4 The energy density is $f = -M_s B_o \cos\theta + K_1 \cos^2 2\theta + B_1 e_{xx} (\cos^2\theta - \upsilon \sin^2\theta)$

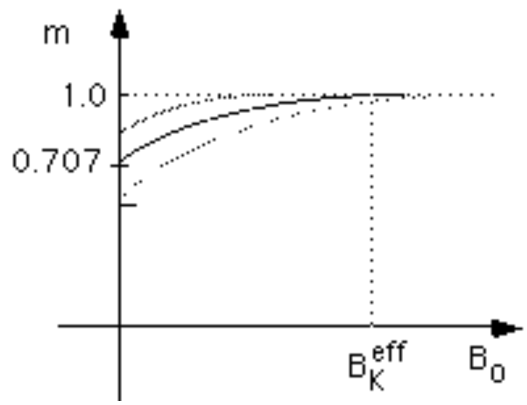
$\partial f/\partial\theta = 0$ gives $m[8K_1 (2m^2 - 1) + 2B_1 e_{xx} (1 + \upsilon)] = M_s B_o$ where $m = \cos\theta$ and $1-m^2 = \sin^2\theta$. For $e_{xx} = 0$ this gives the result sketched as the solid line: $m_r = 1/\sqrt{2}$, saturation ($m = 1$) occurs at $H = (8K_1/\mu_o M_s) = H_a$.

For $B_1 e_{xx} > 0$, $m_r < 1/\sqrt{2}$

$B_1 e_{xx} < 0$, $m_r > 1/\sqrt{2}$,

and $H_a = [8K_1 + 2 B_1 e_{xx} (1 + \upsilon)]/(\mu_o M_s)$.

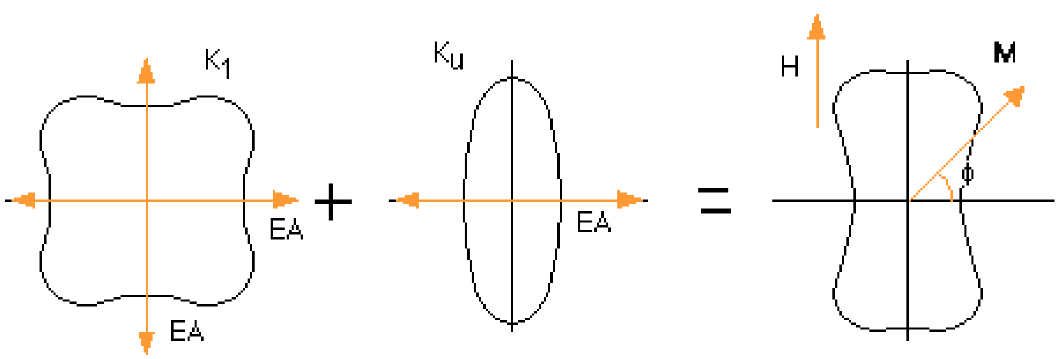
Thus, saturation occurs at higher fields for $B_1 e_{xx} > 0$ and lower fields for $B_1 e_{xx} < 0$.



9.5. **a)** From Eq. 6.6, setting $\theta = 90^o$, we have

$$f = K_u \sin^2\phi + (K_1/4)\sin^2 2\phi. \; -\mu_o M_s H \sin\phi.$$

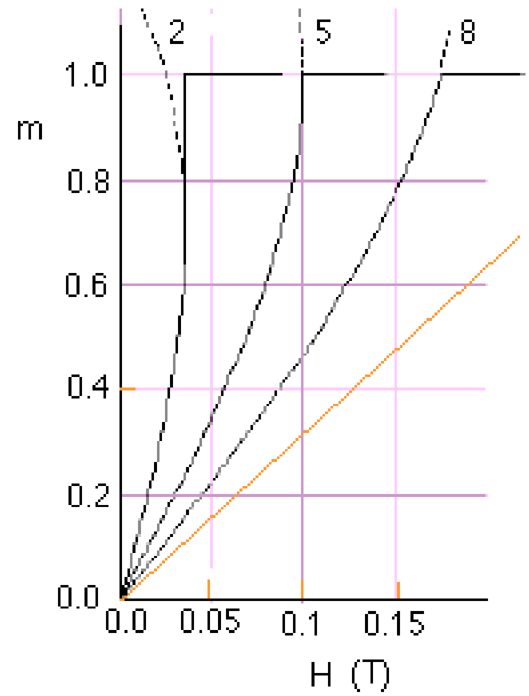
b) Energy surfaces:



c) Zero torque gives $2K_u \sin\phi \cos\phi + K_1 \sin 2\phi \cos 2\phi = \mu_o M_S H \cos\phi$. Divide by cos ϕ which is zero only at and above saturation. The parameter of interest is the component of magnetization along the field direction, $\sin\phi$, which we define as m. The equation of motion is then expressed:

$$2K_u m + 2K_1 m(1 - 2m^2) = \mu_o M_s H$$

d) Numerical solutions are shown at the right for three values of the ratio of uniaxial to cubic anisotropy constants, K_u/K_1 = 2, 5 and 8. $K_1 = 10^4$ J/m^3 and the field scale is $\mu_o H$ (T). Note that for K_u/K_1 = 5, the infinite slope point occurs at m = 1. For smaller K_u, the magnetization shows a discontinuity as was found in Prob. 9.3. For larger K_u, the m-h curve approaches a linear form typical of pure uniaxial, hard-axis magnetization.



9.7 Coercivity goes inversely as permeability. More exactly H_C is proportional to $(K_u + (3/2)\lambda_S\, \sigma)/\mu_o M_S$. In amorphous materials there is no magnetocrystalline anisotropy so K is very small. The coercivity then vanishes or goes through a minimum when magnetostriction vanishes.

9.9. g = 2.11.