

3.60 Symmetry, Structure and Tensor Properties of Materials

CONVENTIONS FOR RELABELING STRESS, STRAIN, STIFFNESS + COMPLIANCE

IN MATRIX NOTATION

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \longrightarrow \begin{bmatrix} \sigma_1 & \sigma_6 & \sigma_5 \\ \sigma_6 & \sigma_2 & \sigma_4 \\ \sigma_5 & \sigma_4 & \sigma_3 \end{bmatrix} \quad \text{ie } \sigma_m \equiv \sigma_{ij}$$

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \longrightarrow \begin{bmatrix} \epsilon_1 & \frac{1}{2}\epsilon_6 & \frac{1}{2}\epsilon_5 \\ \frac{1}{2}\epsilon_6 & \epsilon_2 & \frac{1}{2}\epsilon_4 \\ \frac{1}{2}\epsilon_5 & \frac{1}{2}\epsilon_4 & \epsilon_3 \end{bmatrix} \quad \text{ie } \epsilon_m \equiv \begin{cases} \epsilon_{ij} & i=j \\ \frac{1}{2}\epsilon_{ij} & i \neq j \end{cases}$$

$$C_{ijkl} \equiv C_{ijrk} \equiv C_{mn} \quad \text{ie, NO FACTORS OF 2 OR 4 INVOLVED}$$

$$\left\{ \begin{array}{l} S_{ijkl} \equiv S_{ijrk} \equiv S_{mn} \quad \text{for } m \text{ AND } n \neq 4, 5, 6 \\ S_{ijkl} \equiv S_{ijrk} \equiv \frac{1}{2}S_{mn} \quad \text{for } m \text{ OR } n = 4, 5, 6 \\ S_{ijkl} \equiv S_{ijrk} \equiv \frac{1}{4}S_{mn} \quad \text{for } m \text{ AND } n = 4, 5, 6 \end{array} \right.$$

CONDENSATION FROM TENSOR TO MATRIX NOTATION & VICE VERSA

(MORAL: YOU CAN'T WIN, BUT IF YOU PLAY IT RIGHT YOU CAN COME OUT EVEN)

(A) STRAIN IN TERMS OF STRESS

WRITING ONE REPRESENTATIVE LINE IN TENSOR NOTATION:

TENSOR: $\epsilon_{23} = S_{2311} \sigma_{11} + S_{2322} \sigma_{22} + \dots + S_{2321} \sigma_{21} + S_{2312} \sigma_{12}$

$\downarrow \epsilon_{23} \equiv \frac{1}{2} \epsilon_4 \quad \sigma_{11} \equiv \sigma_1 \quad \sigma_{22} \equiv \sigma_2 \quad \sigma_{21} \equiv \sigma_6 \quad \sigma_{12} \equiv \sigma_6$

$\frac{1}{2} \epsilon_4 = S_{2311} \sigma_1 + S_{2322} \sigma_2 + \dots + S_{2321} \sigma_6 + S_{2312} \sigma_6$

$\downarrow S_{2311} \equiv \frac{1}{2} S_{41} \quad S_{2322} \equiv \frac{1}{2} S_{42} \quad S_{2321} \equiv \frac{1}{4} S_{46} \quad S_{2312} \equiv \frac{1}{4} S_{46}$

$\frac{1}{2} \epsilon_4 = \frac{1}{2} S_{41} \sigma_1 + \frac{1}{2} S_{42} \sigma_2 + \dots + \frac{1}{4} S_{46} \sigma_6 + \frac{1}{4} S_{46} \sigma_6$

$\frac{1}{2} \epsilon_4 = \frac{1}{2} S_{41} \sigma_1 + \frac{1}{2} S_{42} \sigma_2 + \dots + \frac{1}{2} S_{46} \sigma_6$

MATRIX: $\therefore \epsilon_4 = S_{41} \sigma_1 + S_{42} \sigma_2 + \dots + S_{46} \sigma_6$

$\downarrow S_{41} \equiv 2S_{2311} \quad S_{42} \equiv 2S_{2322} \quad S_{46} \left(\frac{1}{2} \sigma_6 \right) + S_{46} \left(\frac{1}{2} \sigma_6 \right) \downarrow S_{46} \equiv 4S_{2321} \equiv 4S_{2312}$

$\epsilon_4 = 2S_{2311} \sigma_1 + 2S_{2322} \sigma_2 + \dots + 4S_{2321} \frac{1}{2} \sigma_6 + 4S_{2312} \frac{1}{2} \sigma_6$

$\downarrow \epsilon_4 \equiv 2\epsilon_{23} \equiv 2\epsilon_{32} \quad \sigma_1 \equiv \sigma_{11} \quad \sigma_2 \equiv \sigma_{22} \quad \sigma_6 \equiv \sigma_{21} \equiv \sigma_{12}$

$2\epsilon_{23} = 2S_{2311} \sigma_{11} + 2S_{2322} \sigma_{22} + \dots + 4S_{2321} \frac{1}{2} \sigma_{21} + 4S_{2312} \frac{1}{2} \sigma_{12}$

$2\epsilon_{23} = 2S_{2311} \sigma_{11} + 2S_{2322} \sigma_{22} + \dots + 2S_{2321} \sigma_{21} + 2S_{2312} \sigma_{12}$

TENSOR: $\therefore \epsilon_{23} = S_{2311} \sigma_{11} + S_{2322} \sigma_{22} + \dots + S_{2321} \sigma_{21} + S_{2312} \sigma_{12}$

B STRESS IN TERMS OF STRAIN

WRITING ONE REPRESENTATIVE LINE IN TENSOR NOTATION:

TENSOR: $\sigma_{21} = C_{2111} \epsilon_{11} + C_{2122} \epsilon_{22} + \dots + C_{2123} \epsilon_{23} + C_{2132} \epsilon_{32} + \dots$

\downarrow $\epsilon_{11} \equiv \epsilon_1$ \downarrow $\epsilon_{22} \equiv \epsilon_2$ \downarrow $\epsilon_{23} \equiv \frac{1}{2} \epsilon_4$ \downarrow $\epsilon_{32} \equiv \frac{1}{2} \epsilon_4$
 $\sigma_{21} \equiv \sigma_6$

$\sigma_6 = C_{2111} \epsilon_1 + C_{2122} \epsilon_2 + \dots + C_{2123} \frac{1}{2} \epsilon_4 + C_{2132} \frac{1}{2} \epsilon_4 + \dots$

\downarrow $C_{ijkl} \equiv C_{jikl} \equiv C_{ijlk} \equiv C_{klij}$
 $\sigma_6 = C_{61} \epsilon_1 + C_{62} \epsilon_2 + \dots + C_{64} \frac{1}{2} \epsilon_4 + C_{64} \frac{1}{2} \epsilon_4 + \dots$

MATRIX: $\sigma_6 = C_{61} \epsilon_1 + C_{62} \epsilon_2 + \dots + C_{64} \epsilon_4 + \dots$

\downarrow $\epsilon_{11} \equiv \epsilon_1$ \downarrow $\epsilon_{22} \equiv \epsilon_2$ \downarrow $\epsilon_{23} \equiv \epsilon_{32} \equiv \frac{1}{2} \epsilon_4$
 $\sigma_6 = C_{61} \epsilon_1 + C_{62} \epsilon_2 + \dots + C_{64} (\frac{1}{2} \epsilon_4) + C_{64} (\frac{1}{2} \epsilon_4)$

\downarrow $C_{ijkl} \equiv C_{jikl} \equiv C_{ijlk} \equiv C_{klij}$
 $\sigma_6 = C_{61} \epsilon_{11} + C_{62} \epsilon_{22} + \dots + C_{64} \epsilon_{23} + C_{64} \epsilon_{32} + \dots$

TENSOR: $\sigma_{21} = C_{2111} \epsilon_{11} + C_{2122} \epsilon_{22} + \dots + C_{2123} \epsilon_{23} + C_{2132} \epsilon_{32} + \dots$