PROFESSOR: Good afternoon and welcome back. I'm glad you all came back even though there was a problem in finding a seat for every one last time. I would like you to turn in the problem set if you've been able to do it. If you haven't been able to do it, that's no great problem, but I hope you find it mildly amusing.

I have given that problem set out a couple of times over the years, and I can recall one very pale student who appeared at my door the next morning saying it's the first problem set, you've only talked an hour and I can't do it. And then there was another group of three who formed a consortium to attempt to solve the code using a computer. They didn't get very far either.

This is an interesting sort of problem because it requires you to think in a slightly different direction than you're accustomed to thinking, and therefore it's a little bit amusing. This is not my creation, it came from a book by a man named Polya, the title of the *book* is *Mathematics and Plausible Reasoning.* And he gives this as an example of a problem that can be solved only if you think in a slightly different direction.

I'll give you another example of a problem from his book. Suppose, not that the problem arises in this era when you do all of your graphics on a computer console, but suppose you had to, in solving a problem in short notice, draw a circle that had a diameter of 4 inches. And you went looking for your pair of compasses which you never use very often, and when you found them, they were rusted solid, and they were open to a distance of 5 inches.

OK, so there's the problem. You have a pair of compasses that can only draw a circle that's 5 inches in diameter, you must draw a circle that's 4 inches in diameter. What sort of construction, what sort of mapping out of arcs that you connected together could you do to create the circle of smaller diameter? Anybody have an idea?

It seems impossible doesn't it? Well suppose you got yourself a little block of wood that had a height that was equal to the square root of 5 squared minus 4 squared. And so you have one end of the compass is up on top of the block of wood the other end of the compass traces out a circle that has a smaller diameter. It's fairly obvious.

But what you have to do is to think of a problem that has poked your nose into two-dimensions and think of it in terms of a three-dimensional problem, and then the answer is easy. So that was the sort of thought provoking thing that Polya presented in one portion of this book.

OK, if you enjoyed that problem, I have another one for you in a similar vein. And perhaps you'll enjoy this one as well. So while I'm talking, let me pass this around, I think there's enough for everyone.

Last time I got so caught up with the displaying the heft of the *International Tables for X-Ray Crystallography* that I forgot to mention entirely that there is another text that we will use in the class. And this we will not need until halfway through the semester, and therefore I did not feel terribly remiss in not mentioning it.

It's a book by somebody named Nye, and it's called the *Physical Properties of Crystals.* And this is published by Oxford University Press and the publication date of the original addition was in 1967.

This is a book that is really, I don't think I'm being extravagant in calling it a classic. It is a beautifully written book. The first 2/3 of it deal systematically with tensor properties, the particular sort of mathematics that is used to set them up, transformation of axes, and then looks at specific physical properties and numbers that have to be described in terms of tensors.

This is actually the third book in a sequence. It was a book by Wooster that covered things very similarly, but the notation that was used for the tensors was not the modern current notation.

And then the subject started in the form of a third book, Woldemar Voigt. And the title of this book is *Lehrbuch der Kristallphysik.* And this was published back in 1910.

This is the first time anybody had anything to say on the matter. It in fact is a big fat book that contains some topics that are not covered in Wooster and Nye. Nye's book is beautifully written. The first 2/3 of it concerns tensor formalism and the last 1/3, which we will not touch at all, deals with thermodynamic relations between different properties that are represented by tensors.

So I don't recommend you go out and buy this book until you determine whether or not you need it because I'll try to make the lectures self-contained and I'll have lots of notes and handouts. The problem with Nye's book, and any book that is intensely mathematical, is that you can't jump in on page 73 to find the answer to a specific question.

Because when you go there, it will say as we showed back in Chapter 4, now what is he talking about? So you go back to Chapter 4, and Chapter 4 says, starting with our definition of Chapter 2, and you have to go back and read Chapter 2. So it's awfully hard to pick something out to answer a specific question. You have to really go all the way through it.

The good news is that Nye's book has been published in paperback. And it is available at the COOP and paperback means cheap, cheap, or relatively inexpensive. No books are really cheap these days. So we will cover material that's in there, we'll have a slightly different emphasis, but the notation and the general mathematics that's involved is in Nye's book.

The second thing that I mentioned last time is that there is a very, very nice and thorough and geometric treatment of crystal symmetry. And I said that's the good news. The bad news is that it's out of print, so I promised, what a guy, that I give you a Xerox copy of the first half of the book. So here is the text that we'll use in the first part of the term.

I included at the beginning, the table of contents, so you can see each other topics that are covered in the book. We will not go through all of the material that's covered. There are a lot of different symmetries to be derived, and it turns out that if you get the general idea and you can summarize the results, there's no need to derive every single one of them.

It's nice to know that there's a place where you can find out how it is done if you really have a particular question. Did everybody get a copy or are there a few who did not? I made extras, OK, nobody in need of one, great.

OK, let me now start with a general rhetorical question. Crystallography, as we mentioned last time, is the geometry of crystals. It's the geometry of patterns and the sorts of symmetries that are in those patterns.

Now you might ask yourself, why should I as a material scientist or a physical scientist of some sort, worry about this stuff? I'm not training to be a wallpaper designer, I'm going to do physical things. I'm going to heat things and measure properties, and that sort of thing. Well there are at least three answers to that question.

First of all, whether you like it or not, the arcane language of symmetry is the language that's used to describe crystals. It's the language that's used to describe structures. The normal thing that you do when you're trying to describe verbally a ball and pin model of the geometrical arrangement of atoms in a crystal is to say the red balls are at the corners of the cube, the green balls are in the middle of the edges, and the chartreuse balls are sort of tucked up inside one of the corners, but slightly closer to 1 face than to the other 2 faces.

The point I'm trying to make is that is a language that has limited utility. It deals, it's capable of dealing only with the simplest sort of atomic configurations. So there is a general language based on symmetry theory, based on group theory, that is universally used to describe atomic arrangements.

So instead of saying red balls at the corners of the cube and green balls in the middle of the faces, I can say space group 4 over m3 bar 2 over m, atom a in position for b, m3m, atom b in position for c, m3m. That's what rock salt it. And that is the way, not only it, but especially more complicated structural arrangements are described in the literature. So this is the language of describing such arrangements.

And finally, sooner or later, I bet you that every one of you will be involved with some crystalline material, and the first question you will answer is what is its structure, what is the atomic arrangement. That's where properties start. And you'll go a book or a set of volumes that describe structural data.

There's a big long compendium of books that fill about that much of a bookshelf which are called structure reports. They started a number of years ago to compile all of the structures that had been determined within a given calendar year. They did a pretty good job of staying caught up back in 1915 and 1920.

And then as it became easier to obtain such results, partly due to the advent of rapid large computers, they fell further and further behind and I think now they are about 5 or 10 miles-- 5 or 10 years, miles as well. But this is one of the places to go to look up, without going to the original literature, whether the material that you're interested in has had its atomic arrangement determined.

When you go there you're going to find the atomic arrangement, not in terms of red balls at one position on the cell, but you're going to find it in terms of the language of symmetry theory. So one of the things I hope you'll be able to do by the time we finish this time together, is to be able to go to such literature and know exactly what to do and where to go to reconstruct the geometrical arrangement of the atoms.

OK, so hopefully you're at least mildly convinced that this exercise is going to be worthwhile. Before we continue where we left off last time, I would like to say a little bit about the language in which these geometries are described.

And we mentioned last time without thoroughly demonstrating why that in a 3-dimensional space there are 4 basically different kinds of operations. And one of these is something that all crystals must by definition display, and this is the operation of translation. Analytically it can be described as a mapping in which every coordinate in a space xyz is mapped to a location x plus some constant, y plus some constant, z plus some constant.

And if you do the operation again, this would go to a location x plus 2a y plus 2b, z plus 2c. A feature of translation that is unique to this particular symmetry transformation is that it has no origin. If I have a pair of motifs that are related by translation, we could think of them as being related by a vector, magnitude and direction, that takes this motif and moves it to this location.

We said that more generally we should view these operations, not just acting on one little domain and space, but acting on everything. So this implies that there be a infinite chain of motifs if the operation of translation is to be present. Because only that infinite, doubly infinite string is consistent with all of space being mapped into itself.

Like any vector, there's no unique origin. You could say it extends from here to here, or from here to here, or any other choice of translation, provided the direction and the magnitude are the same in every choice. As a result, it's not really possible to specify the locus of this particular operation. It has magnitude and direction, but no unique origin.

What we can do through a very neat device is to nevertheless, take some reference point and have each of these reference points separated by T, and have each motif lurking off in space in exactly the same location and distance from this point that we've constructed. And this array of abstractions, so these geometrical abstractions, these points, are what are called lattice points, and they are a very neat summary of the translational periodicity of the crystal.

It is absolutely essential not to mix up these lattice points which are a construct that we have created, and the atoms themselves that are present in a crystal. The atoms are atoms and they're not necessarily the lattice points. Another way of saying that not all atoms of the same chemical species need be translation equivalent. We'll see some examples of this later on, so do not mix up the atoms and the lattice points.

When I talk about the sodium chloride lattice, I mean an array of points in space that are located at the corners of a cube and in the middle of the faces of the cube. If I talk about the arrangement of sodium ions and chlorine ions, that is the sodium chloride structure and not the sodium chloride lattice.

And then last time I apologized for usage so as not to appear hypocritical. Everybody talks about lattice vibration, lattice energy, lattice dynamics, and so on, but that's a misuse of the term. But nevertheless, it is much more musical than saying structure energy, structure vibration. So we'll go on misusing the term lattice I'm afraid.

OK, let's look at another operation. Here we change the sense of no coordinate. Let's next look at an operation that might take xyz, and map it into minus xyz.

This would be a situation where if I set up a coordinate system, here's x, here's y, here's z. What I've done is to take an object that sits off here, at a coordinate plus x, and I've changed the sign of x so that this object now sits off here. This is exactly what happens when I take something and reflect it in a mirror.

And if that's not immediately obvious, it just so happens, not at all by accident, I brought along with me a mirror. OK, here is one hand, and if you look in the mirror, there is the other hand. It's the same distance behind the plane of the mirror, two coordinates have been left unchanged. The two coordinates within the plane of the mirror, if that is my choice of the reference system, and one of them has been reversed.

Now I'll take the second motif out of the geometric construct, and I'd like to point out one very curious feature of the pair of motifs that's generated by this transformation. They're both the same thing, clearly. But no matter how I try, I cannot move one so that it coincides with the other. And we intuitively appreciate this difference by saying we actually use our hands by analogy.

We say one is left-handed and one is right-handed, and they are not congruent. The fancy name that's used to describe this relation is to say that they are enantiomorph. Another term that's used, particularly in chemistry, is to say that they are chiral. Which one is the left-handed one, which is a right-handed one?

This is what I call my left hand, this is what I call my right hand. But can we distinguish them physically, any other way? No, these are terms that have come in to regular use in both our everyday language and also in science because we use our hands instinctively as readily available examples of enantiomorphs, readily at hand, I might say to almost make a pun.

One of the really brilliant figures in physics was a man named Richard Feynman. Recently deceased, Feynman gave a very famous series of lectures on science at Cornell University. And he has one entire vector that was devoted to the difference between right and left.

And he comes up with a funny story, he pretends that the hero of the story is someone who's trying to communicate with beings in outer space and suddenly he gets lucky and he gets a response to his message. And they work out a way to communicate and eventually they try to describe each other to the other individual.

Well what they look like? Well we're bipedal, and we have two organs related by reflection that let us sense light and form images. And we have an aperture through which we ingest things that can be metabolized. And our circulation and body works because we have a pump on the left hand side that circulates fluids through our--wait, I don't understand, what's left?

So then there follows along this course on how to define left in an absolute sense. How do you describe to someone what makes your left hand left, and your right hand right without being anthropomorphic about it. So it goes on and on and on and he gets into physical phenomena which are objective and independent of a human being.

And finally he comes to the anisotropic emission beta particles in radioactive decay. And that depends on direction relative to the magnetic moment and that defines an absolute sense of right and left. So that's something physical, it doesn't depend on the nature of the human being.

And then Feynman wraps up his story by saying, if finally our extraterrestrial being travels to space, gets out of his spaceship and he walks forward to greet you and you put out your right hand, and he puts out his left hand, get out of their fast because it means he is made out of anti-matter.

And nuclei of matter in this anisotropic emission of data rays shoot out the beta particle in one sense, anti-matter shoots out the beta particle in the chiral sense. So it's a cute little story that emphasizes the problem in defining absolutely left from right. But they're of opposite handedness that we can say.

Mirrors are interesting things and I brought along a couple of mirrors and they have very, very peculiar characteristics. And I would invite you to come up and look at these in private because if I hold them up in front of you, you're not going to be able to see what I'm doing at all, although I kid myself that you can, and I walk around and show you what I'm doing.

Here is something scientific, it's a chemical compound carbon dioxide, and-- OK, and if I hand this down you can see carbon dioxide in the mirror. Can you see that? Uh oh, it's not reflecting-- sorry, I didn't turn it on. Now I think we can get it. And is it working now?

OK, now it's working. You can see why this is a very special kind of mirror because it reflects only red letters and it leaves the black letters unchanged. You're going to have to come up, I see some of you straining your necks, you'll have to come up and look at that in person. But the black letters are completely unchanged, the red letters are reflected into letters of an opposite chirality. It's a very special kind of mirror.

I've got another kind of mirror that works in a different way. Where Is my other piece of paper? OK. This is an interesting mirror because it reflects only male names are not female names. This was a very topical sort of mirror a few years ago when there was a lawsuit.

There was a college down South called the Citadel which would only admit male applicants and not female applicants. So I claim that this was a mirror that I got from the Citadel because it doesn't change the male names, but does change, does reject or reflect, female names. So you can play with this during our break.

But if I look at myself in a mirror, I take a look at myself. If I wink my left eye, the in there winks his right eye back at me. So I'm not really seeing myself, what I'm seeing is my enantiomorph. Doesn't that shake you up? You have never ever seen yourself in exactly the same way as other people see you. You are only familiar with your enantiomorph.

Does that make a difference? Well I'll bring in something that I put together and I couldn't put my hands on. We are very, very sensitive to the symmetry in our faces. And if they are reflected left to right, you surely are going to look different to the other person.

And the way to see that is to take a photograph of somebody and cut it down the middle, and put the two different sides reflected left to right. And the expression on the person's face, and the general spirit that that image conveys is entirely different if you use the one half of the face reflected left to right, and the other half reflected left to right.

So think of this, when you look in the mirror, you see your enantiomorph and other people see you differently. Let me ask you to scratch your head now. Is there any time when, in point of fact, you may have seen yourself without being reflected into the enantiomorph? Yeah.

AUDIENCE: Picture?

PROFESSOR: Absolutely. A Photograph or a TV monitor. When you look at a picture on television, you can read all the signs, and they didn't make up special signs in reflection so that they'd look right when they photographed you. So photography or a video camera does not change the chirality.

But I've got another way in which I can see myself in the exact same chirality. And this I can't really convince you of, you'll have to try it. If I put two mirrors together at 90 degrees, and then adjust them so that I am looking right the point of intersection so that my two images coincide, then I see myself in the normal way.

If I now blink my left eye, this guy blinks his left eye at me too. This is really astounding, two mirrors at 90 degrees, if you look at yourself right at their point of intersection, give you a non-chiral image of yourself. So I invite you to come up and try that, that's truly astounding.

So why should somebody in material science or chemistry care about chirality? Does it really make any difference? Let me give you a little experiment that you can try.

Suppose you have a little cell on a couple pieces of Polaroid, and the cell has a glass front and a glass back and you fill it with sugar solution. And then you pass a beam of polarized light through the sugar solution, and what happens is that the sugar solution rotates the direction of polarization in proportion to the thickness of solution at the light is passed through, in proportion to the concentration of sugar. The point of polarization gets rotated.

Now that's pretty curious, so you scratch your head about that. Why does that happen? Well, maybe I'd better go back and try it again. And a day or two later, you go back and you repeat the experiment. And once again, the point of polarization rotates, but it rotates in the opposite direction.

The reason for this is that if I was not careful in cleanliness and there were some little bugs lurking in the corners of that cell and when they sense the sugar solution, they said wow, free lunch, and they crawled out And gobbled it up. It turns out, those guys can gobble up just the sugar of one chirality. Sugar is a chiral molecule.

And in fact there is a product that's called invert sugar and this is sugar that is all of one handedness. But everything in the world around us, everything from sugar beats to sugar cane to other things that make sucrose, manufacture sugar of one chirality, not mixed.

All chiral molecules that are produced by living organisms are all of the same kind chirality. If we make them synthetically, there's no reason to favor synthesis of one molecule or the opposite handedness, so synthesized molecules are of equal proportion in the left-handed chirality and the right-hand chirality.

This means that in the case of pharmaceuticals at the very best, you are going to use only half of the product that you've made. There is a pharmaceutical product that is prescribed for attention deficit disorder, this is called Ritalin, and only one chirality of the Ritalin molecule does anything for you. The other part is just metabolized and doesn't do anything.

But there are other much more sinister cases. There was a serious problem about 20 years ago, primarily in Europe, where a particular pharmaceutical thalidomide was prescribed for pregnant women, it was to act as a sedative. Only one chirality of the molecule did this, the other chirality tragically caused birth defects. So you have to be very careful about the chirality of the pharmaceutical molecule that you synthesize.

Another example, there is-- I don't remember the name of it. This is something that is taken to-- this is something called Ethambutal which is used to treat tuberculosis. Only the molecule of one handedness does this, the other one causes blindness. That's really a sinister and antiomorph.

Then there's some even crazier examples. Ibuprofen is a chiral molecule, and this in a most remarkable situation is a molecule which you're body converts to the molecule of the chirality that has the intended purpose. So here your body is clever enough to change ibuprofen into the molecule which is the one that you need for its pharmaceutical effect.

OK, so mirrors are interesting things. I would invite you to come up and play with the special mirrors that do strange things and see yourself as others see you. And now I would like to continue on in this discussion to mention the ways in which we can represent a mirror plane in a graphic language.

This is what a mirror plane does, it changes the sense of one coordinate. If there is a locus across which that transformation is performed, we would like first of all, an analytic symbol. Some way of indicating the presence of that particular operation in the pattern, and a mirror is very descriptive so the symbol m is used to represent the presence of a mirror plane in a particular symbol.

We might want to indicate a specific operation. There are only two operations in the case of a mirror plane reflecting left to right and reflecting right to left. But there are other operations such as rotation. If we have a 16-fold rotation axis, there is one operation that consists of rotating 1/16 of 2 pi, another operation that will also leave the space invariant that's rotating 2/16 of 2 pi, and so on.

So an individual operation is something that we will want to designate. And for a mirror plane, something that is used commonly in physics is to use an operation sigma for a particular reflection. This is not done in Buerger. If you get into reading it, he uses m for both.

And then finally, it's going to be convenient when we have a pattern before us to use a geometric symbol to indicate in the pattern the locus of this particular operation. And what we use in the case of a mirror plane is a bold line. And that if this were the pattern and we wanted to indicate where the mirror plane was, or the mirror line in 2-dimensions that relates those two motifs, we draw it in thusly.

We began last time to examine the properties of rotation, but that's another sort of symmetry and that is a rotation which takes place about a rotation axis. The symbol that is used to represent the collection of operations, the analytic symbol, is based on the fact that the angular rotation, alpha, has to be equal to some sub-multiple of 2 pi. 2 pi over n, where n is some integer.

And the reason for that I think is quite clear, if I take a particular motif and rotate through an angle alpha, if I am not rotating by some sub-multiple of 2 pi, I'll just go round and round and round and I will never get a finite set of objects that is separated from its neighbor by the same angular interval alpha.

This will only happen if alpha is an integral sub-multiple of 2 pi. And the symbol that is used for the collection of operations that is usually embodied in a rotation axis is n, the same n that is in the denominator. The symbol for individual rotation, so we mentioned last time we have to specify the location of the point about which we rotate, and we have to indicate the angle alpha through which we've rotated.

So A alpha will be an individual operation, and the geometric symbol will be an n-Gon which has the symmetry of the rotation axis. So for a sixfold axis we would use a hexagon. For a fivefold axis we will use a pentagon, for a fourfold, a square, for a threefold, a triangle. Now an n-Gon with 180 degree rotation is a line segment.

And that would be easily overlooked and it's not very aesthetic, so here we indulge in a little bit of artistic license and fatten out the middle of the line segment to get an oval with pointed ends. And that's the symbol for a twofold axis. What about a one-fold axis? One-fold axes exist anywhere, so you can sprinkle them around with reckless abandon.

A one-fold axis has no symmetry at all, but that is a very nice symbol to use for no symmetry at all. So symmetry 1 is the absence of symmetry. So it does come up occasionally in notation.

Now if you look at what we've done so far, we have a transformation that changes the sense of no coordinate. We have a transformation that changes the sense of two coordinates, one coordinate, no coordinate, one coordinate.

Rotation is interchanging the sense of two coordinates in a plane, and in a 2-dimensional pattern, that's all there is. But for a 3-dimensional space, we have the option of changing the sense of no coordinate, the sense 1, the sense of 2, or change the sense of all 3 coordinates.

So if this is x and this is y and this is z, and up here in space lurks my initial motif, if I change the sense of x, the sense of y, and the sense of z, namely take xyz and map it to minus x, minus y, minus z, what I'm going to do is to essentially turn the object inside out. And if my initial one was right-handed, I will produce a chiral object, a left-handed object.

This is an operation which is called inversion. In this operation of turning the object inside out if you will, is inverting it to a new location. And this analytically is the exchange in coordinates provided the point of inversion is at the center. The analytic symbol for inversion is 1 with a bar over the top, pronounced 1 bar. And I'll have to leave to later indication of exactly where that notation comes from.

The individual operation is also called 1 bar, and the geometric symbol that is used to indicate the location of an inversion center is a tiny little open circle large enough so that you don't miss it, but not so large that it might be confused with an atom in a drawing of an atomic arrangement. So in this case, we would adorn our sketch was a little circle at the origin if that was the point through which the space was being inverted.

So that, ladies and gentlemen, is our basic bag of tricks in 3-dimensions. Let me point out that inversion can exist in 3-dimensions only because I have to have 3 coordinates to play with or else I cannot define the operation.

Suppose I have a mapping operation xyz that goes to minus x, minus y, minus z, and I get rid of z to make it 2-dimensional. Then my transformation is xy going to minus x, minus y and that's exactly what a twofold axis does. So inversion, when you throw out the third coordinate, looks like a 180 degree rotation. So you need 3 dimensions in order to define that transformation.

If we really wanted to go crazy, we could go on to say what happens in 4-dimensions? There should in principle be five different operations and yes, mathematically you can define them. They're very difficult to draw because we have to have some sort of operation that take something and pulls it out of our 3-dimensional world. We have no idea where it went and then all of sudden, [POP], it pops back into our space.

But mathematically there are cases when you need a fourth variable to describe the symmetry of an arrangement. And this generally occurs in something called a modulated structure where there's a periodic change in some variable other than the atomic positions. And let me give you two quick examples without going into it exhaustively.

One characteristic of an atom besides its location and its atomic mass and things like that, is perhaps a magnetic atom that has a magnetic moment attached to it. There are magnetically ordered structures. One of them looks exactly like rock salt.

And I'll draw just the magnetic cations which sit in locations like this. And the magnetic moment here is up, the magnetic moment here is up, the magnetic moment here is down, the magnetic moment here is down. So what I've drawn here is no longer the lattice and in fact, the lattice constant of this material looks like a rock salt as far as the atomic positions are concerned, but the magnetic moments have to be continued on in another direction and some extra distance.

Actually some examples of this sort of behavior is FeO, cobalt oxide, nickel oxide. All of these cations are magnetic, they have magnetic moments which are ordered and the unit cell turns out to be when you take magnetic moment into account, a larger cell, a super cell.

There's a more interesting type of magnetic structure though in which the magnetic moment is inclined relative to some translation in the structure. And the magnetic moments all lie on the generators of cones, but as you walk along the chain of atoms, the orientation of the moment rotates to different orientations.

There is a family of materials that are said to have cubicle spin structures in which the periodicity of the march of the magnetic moment around the surface of the cone occurs with a period that is in commensurate with the spacing of the chain of atoms. So strictly speaking, this material does not have a lattice in this direction, so it's not a crystal unless you use a fourth variable to describe the periodicity of the orientation of the moment.

And one final one at the risk of carrying this too far, here's a pattern that is based on a square lattice. It has a fourfold axis in it unless I make the pattern out of squares that are black and white, make a checkerboard. This is now no longer a fourfold axis because I can't rotate 90 degrees and leave the pattern invariant.

So this is an example of something called a black-white symmetry, or a color symmetry. And it requires more than just 4 operations to describe the relation between one motif and another. We need a fourth operation, switching of a color from black to white, or switching it from white to black, that's a forth operation. Again, within the confines of a pattern that exists in our space. Yes?

AUDIENCE: Does that actually have-- so you're saying it doesn't add value to the [INAUDIBLE]?

PROFESSOR: I did say no rotational symmetry if it has a fourfold axis here but this used to be a fourfold axis, and that now changes into a twofold axis. And then I have the problem of describing how this square is a square exactly like this square except for its color. So I need then an operation which rotates 90 degrees and switches from white to black. And then rotates 90 degrees again and switches from black to white. So there's a fifth operation, a color change that is necessary in a 3-dimensional space or a forth operation, a color change in a 2-dimensional checkerboard for example.

So there are lots of nuances to symmetry theory, it's mathematics and the nice thing about mathematics is it's your ballgame, you could make up the rules and as long as you play according to those rules consistently, then you've got something that people can't quarrel with.

OK, I think my internal clock has just told me that it's five minutes of the hour and it's time to take our break. Come up by all means and play with the mirrors if you'd like and we'll resume the lecture part of our discussion in 10 minutes.