PROFESSOR: All right, now for something completely different. Before beginning though, I would like to raise a procedural question. It was my intent that quiz number two, which is like a little more than a week away, was going to be part symmetry and part tensors. I don't want to have your fortunes based by weight of 2:1 on symmetry theory as opposed to properties.

What I would suggest, and you can express your opinion either way, is that we postpone quiz number two by maybe as much as 10 days, so that it can be half symmetry and half the introductory discussion of tensors. And if that does not create a conflict with some of your other classes, I would propose that as a suggestion. If not, we can just have on the quiz what we're going to cover of tensors in the next couple days and have the rest symmetry.

So what I am suggesting is the quiz was scheduled for a week from November 8, which is a Tuesday. What I would suggest is putting that off to the 18th, and that will give us four extra lectures-- six extra lectures-- on tensors and we can make the quiz 50-50.

AUDIENCE: What does that do to quiz number three?

PROFESSOR: Quiz number three is going to stay in place because it's right at the end of the term.

AUDIENCE: What about the subject in three?

PROFESSOR: It will be all tensors. It will be all tensors and properties. And the third quiz is set up for longer than you think. It's December 8, so it'll be about three weeks.

AUDIENCE: So everything from here on?

PROFESSOR: Yeah. Does that make sense? Or is somebody going to have a big, traumatic responsibility at that point?

AUDIENCE: I don't know, but I think it would be probably be easiest for us not to have it three days before close of exams.

PROFESSOR: OK, OK, that's what I wanted to hear. How about if we went just a little bit further then and did it not on the-- when was the suggestion? The 18th. Let's say we did it on Tuesday the 15th? Does that conflict with anything?

AUDIENCE: That's a little bit.

AUDIENCE: We were doing it on the 18th.

AUDIENCE: Is that Friday?

[INTERPOSING VOICES]

PROFESSOR: I'm sorry, 18th. I said it would be 17th. Excuse me.

AUDIENCE: That definitely would be a little better.

PROFESSOR: OK, let me allow you to think about that, and that would be pushing it back one week and maybe it'd be-- we'll see how much material on tensors we cover. So let's table it for now but let's say, tentatively, let's consider seriously moving it to the 15th. OK? But you guys have final say if there's a preference that you have.

OK let's talk about properties in first a general and rather philosophic way. When we discuss properties we very commonly lump together a set of behaviors which have some common phenomenology. So for example, we will talk about mechanical properties.

And that's a basket in which we place many different sorts of behavior. We place things like fracture toughness, yield strength, elastic constants, a whole variety of things involving strength deformation and so on. Or another thing we very often lump together in a basket is something called electrical properties. And we talk about electrical conductivity, ionic conductivity, electronic conductivity. We talk about dielectric constants, we talk about permittivity and permeability, magnetic properties-- a whole bunch of other classes of behavior, which have some sort of root or description in terms of electromagnetism.

There are though a lot of strange aspects to properties. There are some properties that are determined for material which no longer exists when you're through determining the property. You have a sample you want to know what the yield strength is. So you put the gradually increasing load on it until finally, POP, it breaks. And now you know what the yield strength is for a material that no longer exists.

Another example is an old one. Back before the days of X-ray diffraction when people would try to characterize material, anything that is very intensely colored usually looks black, and that's not a very definitive property. The color of something is a very prominent characteristic.

So for mineralogists, in particular, anybody going out into the hills and wanting to be prepared to determine what a particular rock that they tripped over was had something on a rope around their neck called a streak plate. And what the streak plate was was a little rectangle of porcelain. And they would take this black mineral and rub it on this piece of porcelain and it would leave a mark-- that was called the streak.

And what this was was actually little bunch of fragments that rubbed off on to the porcelain because the porcelain was white and the fragments were very small. A black piece of rock could give you a streak that was brown or green or deep orange. And so you really were determining the color of the material when it was in a form finely divided enough that light could pass through it. So the streak test was a very important diagnostic for determining minerals.

There are some properties that we refer to as structure sensitive. In the sense that their value-- conductivity is a good example-- the value of ionic conductivity depends on the impurities and point defects that are present in the material. So to say that the ionic conductivity of a certain material is such and such, you have to specify the purity and perfection of the material or the property's meaningless.

There are some properties that are not even single-valued. And the best example there are dielectric or magnetic properties. If you plot the magnetization of a material, the magnetic moment per unit volume as a function of the applied magnetic field B, the material is initially non-magnetized. When you apply B, you get a magnetic moment per unit volume that eventually saturates.

And then if you remove the magnetic field, the material keeps some of its magnetization. It doesn't go to zero when you reduce the field to zero. You have to reverse the field to get the property to go to zero and then magnetize it in another direction. And if you continue to cycle the magnetic field, you get a behavior like this. This is hysteresis, and this type of behavior is called generally hysteretic.

Same thing would be a relation between the displacement vector and an applied electric field. But clearly you can have any value of the magnetization in this range. And if you relate the magnetization as a function of the applied field, you can get any value of the proportionality constant you like between a negative maximum and a positive maximum including zero. So there's a property that's not even single-valued. Depends on the past history of the sample.

And then there are even more peculiar properties. There are properties that are-- we call them composite properties and very often qualitative. What do they mean by a composite property. Let me give you one example-- The property fuzzy.

If I say fuzzy, you know exactly what I mean. It means something that has a diffuse reflectivity, something that has a surface texture that's yielding. It's not like rubbing a wire brush. It's soft and giving, a whole collection of different properties. But yet when I say fuzzy, you know exactly what I mean.

Let me not be so facetious as talking about a fuzzy property. People such as ceramists or powder metallurgists very often will take a powder of a material and consolidate it by compacting it and heating it, very often subject to a compacting stress. And when you do that little necks grow between the particles and they hold together and the material is centric.

So you refer to a property of a powder as being centerable or the centerability of a powder. You know exactly what somebody means by that. But what does it depend on? It depends on the surface energy, it depends on vapor pressure, depends on bulk diffusion coefficients. It depends on surface diffusion coefficients. And all of these things have to be just right to make the powder something that easily densifies upon heating in the application or not of pressure.

So you know exactly what I mean by centerability, but it is a very complex property. And was one that really was not understood until probably the late 1950s. Until then, it was an art that was entirely empirical. And it was somebody here at MIT, a fellow named Robert Coble, who developed a theory of centering that was the first really workable theory that described densification by heating and compaction. OK, composite property that depends on many different individual properties of the material.

OK, what we are going to examine here are equilibrium properties that can be rigorously defined and measurable. So we're going to leave out of the picture things like fuzzy and centerability. And let me give a very nice, obscure definition of what I mean by a property. And what I mean by a property, in terms of a formal statement, is the response of a material to a specific change in a given set of conditions.

So the response of the material to a specific change in a set of conditions that relates independent properties and dependent properties for a particular process. So I'll state that again because I think it's terribly elegant. So a property is the response of the material to a specific change in a given set of conditions that relates independent and dependent quantities in a particular process.

Now there are a lot of properties that have their roots solidly embedded in thermodynamics. So let me give you a few of those. What we'll talk about when we specify a property is something that we will refer to as a displacement. We'll talk about a generalized displacement in response to a generalized force. So some of the thermodynamic quantities that are related in this fashion-- if we list some forces and some displacements, the thing that happens as a result of that stimulus that's applied to the material.

Temperature can be regarded as a force that results in all sorts of processes as a result-- thermal energy flow, thermal expansion, all sorts of things. But one of the things that will happen in response to a temperature change thermodynamically is an entropy. So you can view entropy as a generalized displacement resulting from the application of temperature as a generalized force.

Another example is electric field and the thing that happens there, and electromagnetism, is the quantity, D, which is displacement. Still another example, stress, and the result of applying a stress, among other things, is a strain. What is special about these forces in this displacement is that their product is, in each of these cases, energy, the change in internal energy of the particular body. And when that is the case, these are said to be conjugate-- a conjugate force and displacement. And there are other examples that one can come up .with.

Now to talk about a specific set of properties. Very often, and this crept into the earlier discussion, very often the thing that we do to a material, as a generalized force, is a vector. So very often the thing that we do to the material, apply an electric field, apply a magnetic field, apply a temperature gradient, apply a tensile stress, it has the characteristics of a vector, magnitude and direction.

And in many cases, the thing that happens as a generalized displacement, we may call this in general, q, is also a vector. An example is if we apply an electric field as a generalized force, one thing that might happen is a current flow, which is also a vector. And we are accustomed to writing, q, the generalized displacement, as a proportionality constant, sigma, which is in the case of electric field and current flow, the electrical conductivity. Let me call it, in general terms, proportionality constant, a, times the applied vector, p.

Is this something we would like to stick with as a general relation? Well let me submit that writing an expression of this form makes an inherent assumption. Namely that this will be true for small, and we'll have to define for each property what we mean by a small, applied vector. Let me give you an example.

If p were-- the applied vector was electric field, and the resulting vector was current flow, and the relation between those is the conductivity-- a relation of this sort says that if you double the field, you double the conductivity. You triple the field, you triple the conductivity. Obviously this can't go on indefinitely because all a sudden, POW, dielectric breakdown. The sample evaporates at the smoke.

And again, you have the property of a material which no longer exists. So a lot of properties, we inherently assume that the applied vector is small in order to write something in an expression of this form. So for conductivity, dielectric breakdown is going to destroy the nice linear relation between current flow and electric field.

That's not always the case. Let me give you an example of another property. And this is a property, magnetic susceptibility, which relates the magnetization, which is magnetic moment per unit volume, and relates that to an applied electric field-- an applied magnetic field, H. And the proportionality constant, represented by a Greek chi, is the magnetic susceptibility. So let me give you a specific example.

Suppose we have a chunk of glass and the glass contains a dilute concentration of iron. And iron carries a permanent magnetic moment. And the reason I want to make it a dilute concentration of iron is I don't want these magnetic moments close enough that they can interact with one another. I want this to, therefore, be a dilute system.

So we have different iron atoms in this. Each iron atom has a magnetic moment. And then we put this in a magnetic field, H. And the magnetic field acts on each of these little magnetic moments just as though it were a compass needle. And so it will try to take each of these moments and drag it into coincidence with the magnetic field.

But at a finite temperature, temperatures making these magnetic moments jiggle around, so at a finite temperature, the magnetic moments will just not simply zing into coincidence with the magnetic field. You'll have to increase the magnetic field to make it larger. When that happens, more and more of the magnetic moments will come into alignment. And if you put on a really strong magnetic field, then every single magnetic moment will be dragged into exact alignment with the magnetic field and the system has saturated. There's no way you could squeeze further magnetic moment per unit volume out of it.

So again, you would expect, for this particular property, magnetic susceptibility of a material, if you plot it as a function of the applied magnetic field-- linear maybe be at low applied fields-- but eventually if you make the field strong enough, the system is going to saturate. And then again, no longer will the direction of the magnetization, and that magnetic moment, be parallel to H. But the property becomes nonlinear if the applied vector is strong enough.

This is an example of a property where you would not go wrong at all by stating direct proportionality. For ordering to occur, temperature and applied field go hand in hand. Increased temperature tends to create more disorder. Increased magnetic field tends to align the moments.

In the days when MIT had a national magnet laboratory, the magnet laboratory held the record for the strongest magnetic field ever produced artificially by man. And if you took that magnetic field and applied it to this system of dilute iron in a glass, you would have to lower the temperature of the sample to about 3 Kelvin before you would begin to see saturation.

So even the strongest field that you could produce in a laboratory environment would not succeed in producing non-linearity until you cooled the sample down almost to absolute zero. So here would be a case where under any practical consideration whatsoever, assumption of strict proportionality would be right on the money. You'd be absolutely correct.

But there's another assumption built into this statement. P, the generalized force, is a vector in the cases we're discussing now, and q is also a vector. So when we write an expression of this form you're making another assumption. And that is that the vector displacement that results is always exactly parallel to the vector that you apply.

Do I make a big deal out of this? Isn't that always going to be the case? I mean whoever heard of taking a piece of metal and putting an electric field on it in this direction and having the current run off in this direction. It's absurd. Or maybe it isn't so absurd. So let's think of some of the atomistics of this process.

Now, since I know I'm among friends, I will not hesitate to display my ignorance, total ignorance, of polymer chemistry. So suppose we had a piece of polymer. That's what a polymer molecule looks like. It's a more or less linear molecule, and so these might, in a very highly ordered polymer, be chains that lined up like this.

And suppose we now put an electric field on this polymer and asked how the current will flow. Again, it would not be absurd to say that an electron sitting on this polymer chain in response to this field would be constrained only to flow in the direction of the chain and would find it rather difficult to hop from one chain to another. So maybe, just maybe, we could apply an electric field to a polymer and it would have a flow in this direction, that would be pointing in this direction, and the current flow, J, would be in that direction.

So should we maybe rethink this idea of the direction of the generalized displacement being not parallel to the direction of the generalized force. OK, these are hypothetical examples. Let me now give you an example of a real property for a real material, for which the thing that happens, the vector that happens, is decidedly not parallel to the direction of the applied vector.

OK, thermal conductivity is something that relates a heat flux, usually represented by the symbol K, and relates that to a temperature gradient, dt dx, which is a vector. The thing that gives rise to thermal conductivity can be either propagation of radiation as in propagation of light traveling through a transparent material.

But the other mechanism for thermal conductivity is modes of lattice vibration. When a material is hot, the atoms are jiggling around and you can make the displacement of an individual atom be represented by a sum of waves moving in all different directions with a variety of wavelengths in a variety of amplitudes. Take all those waves, add them together at a particular time, and you get the displacement of the atom.

Propagation of heat by this mechanism, by modes of thermal vibration, can be very,

very anisotropic and probably the best example of this is-- get a single crystal of graphite. And have the layers, the graphite sheets, which are hexagonal rings in which each carbon has three neighbors, and have the sheets be parallel to the surface of an extended two-dimensional slab.

Now we can't do that. Single crystals of graphite don't occur in sizes like that. But what you can do is make a material called pyrolytic graphite that you make by having a reaction in the vapor phase and having the soot that's formed settle down onto a substrate.

And what happens is you nucleate a graphite crystal in one orientation, and that grows preferentially with its layers parallel to the surface on which it's nucleated. And these nuclei occur at random. So you get a bunch of single crystals of graphite, which all have their layers, their three coordinated layers at parallel, but they are oriented at random about their c-axis. And that's a material that's very easy to prepare. And it's called pyrolytic graphite, and it has interesting applications and properties.

OK, now I'm going to suggest an experiment to you and I'll caution you, please do not try this at home. Get yourself a piece of pyrolytic graphite. Put a Bunsen burner underneath it. Play the Bunsen burner on the bottom of the sheet of pyrolytic graphite. And to determine temperature, we'll take a ball of cotton and put it directly above the flame. And then take another ball of cotton and put it down at the end of the sheet.

If you do that and bring up the Bunsen burner, this piece of cotton will sit there and sit there and sit there and sit there. And this piece of cotton will instantly burst into flame. So here's a case where dt dx, the thermal gradient, points in this direction. And the heat flow goes off in a direction at right angles to the temperature gradient, almost exactly at right angles.

It turns out that the thermal conductivity in this direction, the value of K in the-- I want to put crystallographic directions on this. But K parallel to the layers is equal to 10 to the third times K, perpendicular to the layers.

So there's an anisotropy of the property that amounts to a factor of 1,000. Very dramatic. So what I'm leading up is that in a general relation between a generalized displacement, p, and a generalized force-- I'm sorry, I'm using q-- p and q-- we just can't simply put some constant in front. Because that assumes inherently that the direction of what happens, the vector that happens, is exactly parallel to the direction of the applied vector and that generally is not true. The difference may be small but it has to be taken into consideration.

All right, let me now suggest a way of patching this up. What I'm going to do is assume-- and it is an assumption-- we're going to assume that each component of the vector that happens is given by-- in fancy terms, the vector that results as the generalized displacement is given by a linear combination of every component of the vector that's applied. And that's what we've defined as the generalized force.

So how would we express this analytically? I am going to use as my coordinate system, not x, y, and z as we usually do, I'm going to call it x1, x2, and x3. The reason is we'll see some unique properties of these indices that are very useful algebraically. So I'm going to now take my applied vector, p, and I'm going to assume that it has three components, p1, p2, and p3 along x1, x2, x3, respectively. My coordinate system, although I did not state it explicitly, is going to be a Cartesian coordinate system, not a crystallographic coordinate system.

So what I'm proposing here is that we take each component of the resulting vector q-- let's say q1-- and we'll assume that that's given by a linear combination of all three components of the vector p. So it'll be some number times p1 plus some number times p2 plus some number times p3. And those numbers in general will be different.

So let's say I call the coefficient a, and now I'm going to define a convention that will stay with us. I'm going to define each of these coefficients, a, in terms of the index of the component of the generalized displacement which is being computed, and the coefficient modifies the component of the generalized force for that particular term.

So I'm going to call this a11, where the 1 goes with this and the 1 goes with this. I'm going to call this a12, so that the 1 again says it's a contribution to q1. The 2 says that this term modifies p2. . And this similarly would be a13. For the term component of the generalized displacement, q2, I'll use the coefficients a21 times p1 plus a22 times p2, plus 23 times p3. And q3 similarly will be a31 times p1, plus a32 times p2 plus a33 times p3.

So I've got three simultaneous equations then, one for each component of the vector that results, the generalized displacement. So I can sum up this set of three equations by saying that the i-th component of q, where this is some particular component, q1, q2, or q3, is given by the sum over j from 1 to 3 of aij times p sub j. So I've got ai1 times p1, ai2 times p2 plus ai3 times p3. So this is in a nice, compact little nugget the expression that we are assuming will apply for all sorts of physical properties in which this is a vector and this is a vector-- electrical conductivity, magnetic susceptibility, thermal conductivity, and so on.

OK, this is a bad point to introduce something as hairy as what comes next. But I've got one minute left, and I think I can do it. I'm going to introduce something called the Einstein convention after old Albert Einstein himself. I'm sorry, but nobody said everything this term had to be easy. So let me introduce, if you're ready for it, the Einstein convention.

Old Albert, I think, was just as lazy as anybody else. And he said it is going to be a bloody pain in the butt-- I don't know if he put it exactly this way. It's going to be a pain in the butt to write this summation every time we want to combine three terms in a linear combination. So the Einstein convention is let us throw out the sigma and write this expression just as qi times aij p sub j. And whenever we see a subscript repeated, summation over repeated subscript is implied.

OK so that's the Einstein convention. It will save us the trouble of writing in a lot of summation signs. But I would caution you that this convention, compact and convenient as it is, can define some polynomials that are nightmares. So let me give you one example of this, and this is actually a physical property.

Let me say that Cijkl is given by ai capital I aj capital J ak capital K, al capital L times let's say D, capital I, capital J, capital K, capital L. That actually, believe it or not, means something physically and we're going to get to that in due course.

But what this is, taking the Einstein convention into account, this is a quadruple summation over capital I, capital J, capital K, and capital L. These are very often referred to in this business as dummy indices, meaning that they don't really mean anything physically. They're just indices of summation.

I would caution you that this is a term that does not permute. If I say those are dummy indices, it means one thing. If I say those are indices, dummy, it means something completely different, and you're apt to get a poke in the nose.

OK so these are dummy indices. Now, what does this represent? I won't say what it represents physically, but this consists of terms in four variables times a coefficient. So there are five quantities in each term.

If I sum over capital I, J, and K from 1 to 3, there are going to be 81 such terms, each with five elements in each term. So it's going to be on the order of 405 terms in this summation, and we've collapsed the whole thing down, 405 terms in this nice summation. So it is a great facility for writing down expressions explicitly, but these terms can hide a terribly, terribly complex polynomial.

All right, I think that's a good place to quit. It continues to amaze me, though, how some absolutely trivial convention, when first proposed by a great man, will carry that man's name no matter how stupid it is. So this is called the Einstein convention by everybody who works in this field.

Another one, just briefly to finish up, when you discuss dislocations, you talk about a circuit of steps around the dislocation. That's called a Burgers circuit. And if you do the same circuit in a corresponding piece of material that doesn't have a dislocation, if this circuit closes, this circuit fails to close by something that's called the Burgers vector.

Every book on dislocation theory says, Shockley called this good material. I'm sorry, Shockley called this good material, and Shockley called this bad material. Big deal-- good material, bad material because it's got a dislocation in it. But that is identified with Shockley's name in every discussion of dislocation. I'm just jealous because nobody has ever said according to me, such and such is-- all right, see you later in the week for more great things.