18.01 Exam 5

Name: _____

Problem 1: _____	/25
Problem 2: _____	/20
Problem 3: _____	/20
Problem 4: _____	/25
Problem 5: _____	/10

Total: _____ /100

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Problem 1(25 points) Use integration by parts to compute the antiderivative,

$$\int x^3 e^{-x^2} dx.$$

Problem 2(20 points) Use polynomial division and factoring to compute the antiderivative,

$$\int \frac{x^3 - 1}{x^2 - 3x + 2} dx.$$

You will not need to use partial fractions (though you are free to do so).

Problem 3(20 points)

(a)(15 points) Find the partial fraction decomposition of,

$$\frac{x+3}{x^2 - 2x + 1}.$$

(b)(5 points) Use your answer from (a) to compute the antiderivative of

$$\int \frac{x+3}{x^2 - 2x + 1} dx.$$

Problem 4(25 points) Let a be a positive real number.

(a)(3 points) Determine the range of x on which $2ax - x^2$ is nonnegative.

(b)(22 points) For that range, compute the antiderivative,

$$\int \sqrt{2ax - x^2}\,dx.$$

Problem 5(10 points) Compute the following derivatives. Please show all work to receive full credit.

(a)(5 points)

$$y = \sinh(t) = \frac{e^t - e^{-t}}{2}, \quad \frac{dy}{dx} = ?$$

(b)(5 points)

$$y = \sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1}), \quad \frac{dy}{dx} = ?$$

There are at least 2 methods. You may use the one you prefer.

Extra credit(5 points)

$$y = \sin(\tan^{-1}(2x)), \quad \frac{dy}{dx} = ?$$

Only solutions in simplest terms will be accepted.