Approximating $\ln(1+x)$ and $(1+x)^r$

- $\sin x \approx x \quad (\text{if } x \approx 0)$
- $\cos x \approx 1 - \frac{x^2}{2} \quad (\text{if } x \approx 0)$
- $e^x \approx 1 + x + \frac{1}{2}x^2 \quad (\text{if } x \approx 0)$
- $\ln(1+x) \approx x - \frac{1}{2}x^2 \quad (\text{if } x \approx 0)$
- $(1+x)^r \approx 1 + rx + \frac{r(r-1)}{2}x^2 \quad (\text{if } x \approx 0)$

Now that we've seen a couple of examples of quadratic approximation, we'll derive the last two formulas in our library, shown above. The general formula for a quadratic approximation is:

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 \quad (x \approx 0)$$

As usual, we chose the base point $x_0 = 0$. Shown below are the first and second derivatives of the functions we're interested in and their values at $x_0 = 0$. Combining this with the general formula yields the quadratic approximations listed above.

$f(x)$	$f'(x)$	$f''(x)$	$f(0)$	$f'(0)$	$f''(0)$
$\sin x$	$\cos x$	$-\sin x$	0	1	0
$\cos x$	$-\sin x$	$-\cos x$	1	0	-1
e^x	e^x	3^x	1	1	1
$\ln(1+x)$	$\frac{1}{1+x}$	$\frac{-1}{(1+x)^2}$	0	1	-1
$(1+x)^r$	$r(1+x)^{r-1}$	$r(r-1)(1+x)^{r-2}$	1	r	$r(r-1)$

We can approximate most common functions using algebraic combinations of the functions in this library.