

## Introduction to Related Rates

Next we'll look at the subject of related rates, which will give us another opportunity to practice working with several different variables and equations.

**Example:** Police are 30 feet from the side of the road. Their radar sees your car approaching at 80 feet per second when your car is 50 feet away from the radar gun. The speed limit is 65 miles per hour (which translates to 95 feet per second). Are you speeding?

How do you set up a problem like this? First, draw a diagram of the setup (as in Fig. 1):

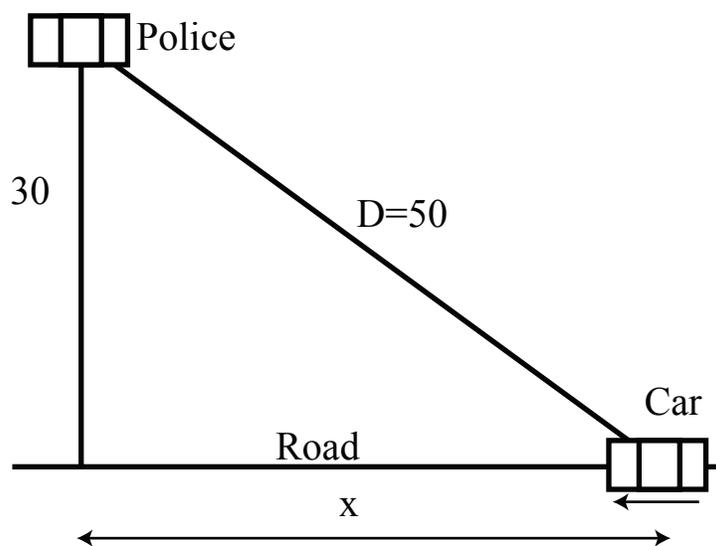


Figure 1: Illustration of example 1: triangle with the police, the car, the road, and labeled variables.

We see that this is a question about a right triangle — the line from the police car to your car is the hypotenuse, the road is one leg of the triangle, and the other leg is a segment from the police car to the road. The Pythagorean Theorem tells us that you're 40 feet away from the right angled vertex of the triangle.

How do we name the variables needed to turn this into a calculus problem? The important thing to figure out is which variables are changing.

We'll choose variable  $t$  to stand for time in seconds. The distance along the road from your car to the police car is an important value which we will call  $x$ . (We know that  $x = 40$  to begin with, but that value will change as the car moves.) Once we have those two variables, we can rewrite the question as “Is  $\frac{dx}{dt}$  bigger than 95 feet per second?”

There's another value that is changing, which is the length of the hypotenuse,

or the distance between the police car and your car. We'll call this  $D$ . (Because we know something about speed limit enforcement, we can correctly assume that the police car is not moving and so the length of the third side of the triangle is not variable.) The rate at which  $D$  is changing is exactly what the police are measuring with their radar gun:

$$\frac{dD}{dt} = D' = -80.$$

The derivative must be negative because the value of  $D$  is decreasing.

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