Differentials

Today we move on from differentiation to integration. For this we'll need a new notation for quantities called differentials.

Given a function $y = f(x)$, the *differential* of y is

$$\boxed{dy = f'(x)dx}$$

Because $y = f(x)$ we sometimes call this the differential of f. Both dy and $f'(x)dx$ are called *differentials*. You can think of

$$\frac{dy}{dx} = f'(x)$$

as a quotient of differentials. Get used to this idea; it comes up in many contexts, including this class and multivariable calculus.

This arises from the Leibniz interpretation of a derivative as a ratio of "infinitesimal" quantities; differentials are sort of like infinitely small quantities.

Working with differentials is much more effective than using the notation coined by Newton; good notation can help you think much faster. Leibniz's notation was adopted on the Continent and Newton dominated in Britain; as a result the British fell behind by one or two hundred years in the development of calculus.