How to Check Your Answer

While it may be difficult to solve a differential equation, it is fairly easy to see if a proposed solution is correct. Check the following results by plugging the proposed answer into the original equation.

a) $y = \frac{1}{3}e^x$ is a solution to $4y'' - y = e^x$.

b) $y = \frac{1}{x}$ is a solution to $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0$.

Solution

a) $y = \frac{1}{3}e^x$ is a solution to $4y'' - y = e^x$.

If $y = \frac{1}{3}e^x$ then $y' = \frac{1}{3}e^x$ and $y'' = \frac{1}{3}e^x$. We now plug these expressions in to the original equation:

$$\begin{aligned} 4y'' - y &= 4\left(\frac{1}{3}e^x\right) - \frac{1}{3}e^x \\ &= \frac{3}{3}e^x \\ &= e^x. \end{aligned}$$

It is true that $4y'' - y = e^x$ when $y = \frac{1}{3}e^x$.

b) $y = \frac{1}{x}$ is a solution to $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0$.

Here $y = x^{-1}$, $\frac{dy}{dx} = -x^{-2}$ and $\frac{d^2y}{dx^2} = 2x^{-3}$. Plugging in to the original equation we get:

$$\begin{aligned} x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y &= x^2 \cdot 2x^{-3} + 3x \cdot (-x^{-2}) + x^{-1} \\ &= 2x^{-1} - 3x^{-1} + x^{-1} \\ &= 0. \end{aligned}$$

Therefore $y = \frac{1}{x}$ is a solution to the differential equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0$.

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