PROFESSOR: Welcome back to recitation. In this video I'd like us to work on the following problem. We're going to let capital F of x equal the integral from 0 to x of little f of t dt. And I want us to find the quadratic approximation for big F near x equals 0 in terms of the little f. And then I want us to answer this question. What assumptions do we have to make about little f? You'll see we're making some assumptions about little f. I want to know what those assumptions are. So I'm going to give you a little bit of time to work on this problem. And then I'll be back and I will work on it with you.

Welcome back. Hopefully you were able to make some good headway with this problem. So again, what we're trying to do, is we've defined capital F. And we want to find the quadratic approximation for it near x equals 0 in terms of little f. Knowing that capital F of x is equal to the integral from 0 to x of little f of t dt. And then at the end, we'll talk about what are the assumptions we've made.

So to start off, what I actually want to do is remind us, what is the quadratic approximation for capital F, just in terms of itself? Of capital F? So let me come over to the right and we'll write down what the quadratic approximation is, near x equals 0.

So, this gives us that F of x is approximately F of 0 plus F prime of 0 times x plus F double prime at 0 over 2 times x squared. This is just the quadratic approximation of capital F at x equals 0. That's what it is. So now all we need to do, I've asked you to write it in terms of little f.

So what we need to do is figure out what each of these things actually is. What capital F of 0 is, what capital F prime of 0 is, and what capital F double prime of 0 is, in terms of the function little f. So let's see. Well first, let's figure out what capital F of 0 is. Let's write down what it is. The integral from 0 to 0 of f of t dt. Well we know, when you take the integral from a to a of a function, you should get 0. So capital F at 0 is just 0. So we have one piece.

Now, let's figure out what capital F prime is. Well, capital F prime of x, we know using fundamental theorem of calculus-- let's just write it down so we can see it easily-- we know this is just little f of x. So if I wanted capital F prime at 0, I just come over here. That's just going to be little f at 0.

So now, if we look back up at what we have at the top, we filled in this part-- or we know that

part, that's 0-- we know this part. So now all we need is capital F double prime evaluated at x equals 0. And then we'll be done. Now, how are we going to do this? Well we know what capital F prime is. So let's come in, sort of up here, and look again. We know what capital F prime is. We know it's little f. So if I want to take a second derivative, of capital F-- let me write that down here-- that's going to be taking a derivative of little f. Right? That'll be, that's d/dx of little f of x. So I can just write this as little f prime of x.

So again, let me show you where that came from. We knew big F prime was equal to little f. So when I take another derivative of this, I have to take a derivative of this. So that's why capital F double prime is equal to little f prime. So we're almost there. I think I'm about to run out of room down here, so let me come over a little bit further.

So now, we just need to evaluate this at 0. Well, that's just going to be evaluating little f prime at 0. So now I have all the pieces. Let's come back in and fill it in one more time. So what do I get? I get-- let me, now I have all the pieces. Now to answer the question. Little f-- or sorry-- big F of x, its quadratic approximation is going to be big F of 0, which we said was 0. So let's come over and look. Here's big F of 0. I know that's 0. And then I'm going to add to that big F prime at 0 times x. That's little f of 0 times x. Let's come back over and do that. So we add to that little f at 0 times x.

And let's come back. One more thing we're going to need. We need one more term. That's big F double prime at 0 over 2 times x squared. That I've left big F double prime at 0 is little f prime at 0. So that's f prime at 0 over 2 times x squared. So we had all the pieces. We just had to put them in. This does not look like an approximately by the way. Sorry. There we go.

So what do we get? This expression above is equal to-- well why did I write the 0 again? Sorry I was trying to avoid writing the 0. It's equal to f of 0 times x plus f prime of 0 over 2 times x squared.

So the quadratic approximation of capital F in terms of little f is exactly this. Now what assumptions are we making about little f? Well to apply the fundamental theorem of calculus, you need little f is continuous. But we actually need more than that. Because what have we done? We've actually taken a derivative of little f. So we need that this function, little f, is actually differentiable.

So that is, that actually includes, of course, that little f is continuous. So we don't even have to say continuous in the assumptions. We just have to say, we're assuming little f is differentiable,

at least near 0.

So let's go back and just remind ourselves what we did. I'm going to go all the way over, remind ourselves of the problem and kind of bring us back through. So we started off with this definition of big F. And I wanted us to find a quadratic approximation for it near x equals 0. And then we just mentioned, what are the assumptions? So I reminded you, first, of what is the quadratic approximation of big F in terms of itself, in terms of big F. And we just write it out.

And then we had defined what is this in terms of little f, what is this in terms of little f, and what is this in terms of little f? And so what do we look at? Well, we evaluate f of 0. That's easy enough. We found big F prime in terms of little f. That's fundamental theorem of calculus. And then we evaluate that at x equals 0. And then we know that if we want to take a second derivative of this function, we need to take a derivative of the little f. And so we get big F double prime is actually little f prime. So we evaluate that at 0 to get big F double prime at 0.

And then we come in and we just fill in everything we needed. And so ultimately, this is the final answer of the quadratic approximation of big F in terms of little f near x equals 0. So I think that's where we'll stop.