Areas Between Curves

Suppose you have two curves, $y = f(x)$ above and $y = g(x)$ below. You want to find the area between the two curves bounded on the left by $x = a$ and on the right by $x = b$.
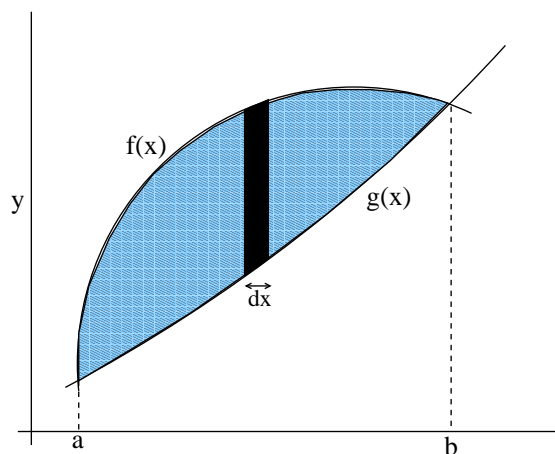

Figure 1: Finding the area between two intersecting functions.

As we did with Riemann sums, we can (approximately) chop this area up into thin rectangles. Each rectangle will have width dx and height $f(x) - g(x)$, so will have area

$$\underbrace{(f(x) - g(x))}_{\text{height}} \underbrace{dx}_{\text{base}}.$$

In order to get the whole area, sum the areas of all these rectangles:

$$A = \int_a^b (f(x) - g(x)) \, dx.$$

There are two key steps to solving problems with integrals. The first is figuring out what to integrate. The function being integrated is called the *integrand*. The second is finding the *limits of integration* — in this case a and b. Once we have these we can compute the integral, either numerically or by finding an antiderivative. Without the integrand and limits of integration we can't find the value of the integral.

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