

**18.03 Hour Exam I**  
**February 24, 2010**

Your Name
Your Recitation Leader's Name
Your Recitation Time

Problem	Points
1	
2	
3	
4	
5	
Total	

Do not open this booklet till told to do so. There are five problems. Use your test-taking skills—be sure you get to all the problems. Do all your work on these pages. No calculators or notes may be used. The point value (out of 100) of each problem is marked in the margin. Solutions will be available on the web after 4:00 today, and at recitation.

[8] **1. (a)** In a perfect environment, the population of Norway rat that breeds on the MIT campus increases by a factor of  $e \simeq 2.718281828459045 \dots$  each year. Model this natural growth by a differential equation.

What is the growth rate  $k$ ?

[4] **(b)** MIT is a limited environment, with a maximal sustainable Norway rat population of  $R = 1000$  rats. Write down the logistic equation modeling this. (You may use “ $k$ ” for the natural growth rate here if you failed to find it in **(a)**.)

[8] **(c)** The MIT pest control service intends to control these rats by killing them at a constant rate of  $a$  rats per year. If it wants to limit the rat population to 75% of the maximal sustainable population, what rate  $a$  it should aim for (in rats per year)?

2. For the autonomous equation  $\dot{x} = x(x - 1)(x + 2)$ , please sketch:

- [4] (a) the phase line, identifying the critical points and whether they are stable, unstable, or neither.
- [4] (b) at least one solution of each basic type (so that every solution is a time-translate of one you have drawn)



Below is a diagram of a direction field of the differential equation  $y' = (1/4)(x - y^2)$ . On it please plot and label:

- [3] (c) the nullcline
- [3] (d) at least two quite different solutions
- [3] (e) the separatrix (if there is one)
- [3] (f) True or false: If  $y(x)$  is a solution with a minimum, then for all large enough  $x$ ,  $y(x) < \sqrt{x}$ . (No explanation needed: just circle one.)

[10] **3. (a)** Use Euler's method with stepsize  $h = 1/2$  to estimate the value at  $x = 3/2$  of the solution to  $y' = x + y$  such  $y(0) = 1$ .

[10] **(b)** Find the solution of  $t\dot{x} + x = \cos t$  such that  $x(\pi) = 1$ .

[3] 4. (a) Find real  $a, b$  such that  $\frac{1}{3+2i} = a + bi$ .

[3] (b) Find real  $r, \theta$  such that  $1 - i = re^{i\theta}$ .

[3] (c) Find real  $a, b$  such that  $(1 - i)^8 = a + bi$ .

[3] (d) Find real  $a, b$  such that  $b > 0$  and  $a + bi$  is a cube root of  $-1$ .

[3] (e) Find real  $a, b$  such that  $e^{\ln 2 + i\pi} = a + bi$ .

[5] (f) Write  $f(t) = 2 \cos(4t) - 2 \sin(4t)$  in the form  $A \cos(\omega t - \phi)$ .

[5] **5. (a)** Find a particular solution to the equation  $\dot{x} + 3x = e^{2t}$ .

[5] **(b)** Find the solution to the same equation such that  $x(0) = 1$ .

[5] **(c)** Write down a linear equation with exponential right hand side of which  $\dot{x} + 3x = \cos(2t)$  is the real part.

[5] **(d)** Find a particular solution to the equation  $\dot{x} + 3x = \cos(2t)$ .

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18.03 Differential Equations  
Spring 2010

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