18.03SC Practice Problems 21

Fourier Series: Introduction

This problem session is intended as preparation for working with Fourier series.

1. What is the general solution to $\ddot{x} + \omega_n^2 x = 0$? Try to remember it rather than deriving it again.

2. Verify that (as long as $\omega \neq \pm\omega_n$)

$$x_p = a\,\frac{\cos(\omega t)}{\omega_n^2 - \omega^2} \quad \text{is a solution to} \quad \ddot{x} + \omega_n^2 x = a\cos(\omega t),$$

and that

$$y_p = b\,\frac{\sin(\omega t)}{\omega_n^2 - \omega^2} \quad \text{is a solution to} \quad \ddot{y} + \omega_n^2 y = b\sin(\omega t).$$

3. What about $\ddot{x} + \omega_n^2 x = \cos(\omega_n t)$? What is a particular solution? What is the general solution? Are there any solutions $x(t)$ such that $|x(t)| < 10^6$ for all t? Are there any periodic solutions?

A function is *periodic* if there is a number $P > 0$ such that $f(t + P) = f(t)$ for all t. Such a number P is then a "period" of $f(t)$. If $f(t)$ is a periodic function which is continuous and not constant, then there is a smallest period, often called *the* period.

4. On the same set of axes, sketch the graphs of $\sin(t)$ and $\sin(2t)$. Then sketch the graph of $f(t) = \sin(t) + \sin(2t)$.

Some pointers: $f(t)$ is easy to evaluate when one of the terms is zero. What is the derivative at points where both terms are zero? This information should be enough to let you make a rough sketch.

What are the periods of these three functions?

5. For what values of ω_n is there a periodic solution to the equation

$$\ddot{x} + \omega_n^2 x = b_1 \sin(t) + b_2 \sin(2t)$$

(where b_1 and b_2 are nonzero)? Name one if it exists.