18.04 Recitation 12
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1. Use Rouche's theorem to show that all 5 zeros of $z^5 + 3z + 1$ are inside the curve $C_2 := \{z : |z| = 2\}$.

2. Use Rouche's theorem to show that $z + 3 + 2e^z$ has one zero in the left half plane.

3. Use Rouche's theorem to give another proof of the fundamental theorem of algebra i.e. to show that $z^n + a_{n-1}z^{n-1} + \cdots + a_0$ has exactly n roots in the complex plane.

4. Let $G(z)$ be a meromorphic function, and let $H(z) := \frac{G(z)}{1+G(z)}$. For a closed curve γ such that $G \circ \gamma$ does not go through -1 and G does not have any poles on γ, show that $P_{H,\gamma} = P_{G,\gamma} + \mathrm{Ind}(G \circ \gamma, -1)$.

5. Consider the method of images if there is a source at $(0, 0)$, and walls at $y = 1$ and $y = -1$. How many image sources do you need? What is the resulting complex potential?

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