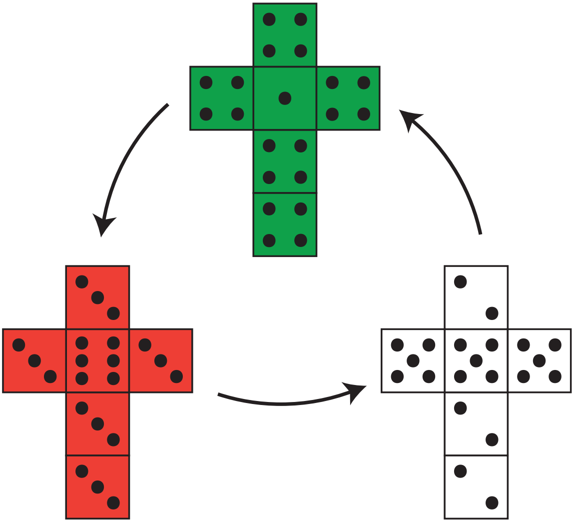
Probability: Terminology and Examples
18.05 Spring 2014



Board Question

Deck of 52 cards

- 13 *ranks*: 2, 3, ..., 9, 10, J, Q, K, A
- 4 *suits*: $\heartsuit, \spadesuit, \diamondsuit, \clubsuit$,

Poker hands

- Consists of 5 cards
- A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other ranks
- Example: $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 8\clubsuit, K\diamondsuit\}$

Question

(a) How many different 5 card hands have exactly one pair?

Hint: practice with how many 2 card hands have exactly one pair.
Hint for hint: use the rule of product.

(b) What is the probability of getting a one pair poker hand?

Clicker Test

Set your clicker channel to 41.

Do you have your clicker with you?

$$\text{No} = 0$$
$$\text{Yes} = 1$$

Probability Cast

Introduced so far

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes S (or Ω).
- Event: a subset of the sample space.
- Probability function, $P(\omega)$: gives the probability for each outcome $\omega \in S$
    1. Probability is between 0 and 1
    2. Total probability of all possible outcomes is 1.

Example (from the reading)

Experiment: toss a fair coin, report heads or tails.

Sample space: $\Omega = \{H, \ T\}$.

Probability function: $P(H) = .5, \quad P(T) = .5.$

Use tables:

Outcomes	H	T
Probability	$1/2$	$1/2$

(Tables can really help in complicated examples)

Discrete sample space

Discrete = listable

Examples:

$\{a, b, c, d\}$ (finite)

$\{0, 1, 2, \ldots\}$ (infinite)

Events

Events are sets:

- Can describe in words
- Can describe in notation
- Can describe with Venn diagrams

Experiment: toss a coin 3 times.

Event:

You get 2 or more heads $=$ { HHH, HHT, HTH, THH}

CQ: Events, sets and words

Experiment: toss a coin 3 times.

Which of following equals the event "exactly two heads"?

$A = \{THH, HTH, HHT, HHH\}$
$B = \{THH, HTH, HHT\}$
$C = \{HTH, THH\}$

(1) A (2) B (3) C (4) A or B

CQ: Events, sets and words

Experiment: toss a coin 3 times.

Which of the following describes the event $\{THH, HTH, HHT\}$?

(1) "exactly one head"
(2) "exactly one tail"
(3) "at most one tail"
(4) none of the above

CQ: Events, sets and words

Experiment: toss a coin 3 times.

The events "exactly 2 heads" and "exactly 2 tails" are disjoint.

(1) True (2) False

CQ: Events, sets and words

Experiment: toss a coin 3 times.

The event “at least 2 heads” implies the event “exactly two heads”.

(1) True (2) False

Probability rules in mathematical notation

Sample space: $S = \{\omega_1, \omega_2, \ldots, \omega_n\}$

Outcome: $\omega \in S$

Probability between 0 and 1:

Total probability is 1:

Event A: $P(A)$

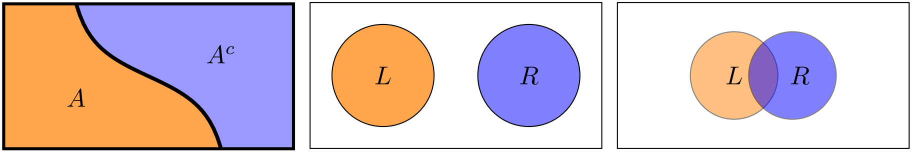
Probability and set operations on events

Events A, L, R

Rule 1. Complements: $P(A^c) = 1 - P(A)$.

Rule 2. Disjoint events: If L and R are disjoint then $P(L \cup R) = P(L) + P(R)$.

Rule 3. Inclusion-exclusion principle: For any L and R: $P(L \cup R) = P(L) + P(R) - P(L \cap R)$.



$\Omega = A \cup A^c$, no overlap — $L \cup R$, no overlap — $L \cup R$, overlap $= L \cap R$

Table question

- Class has 50 students
- 20 male (M),   25 brown-eyed (B)

For a randomly chosen student what is the range of possible values for $p = P(M \cup B)$?

(a) $p \leq .4$
(b) $.4 \leq p \leq .5$
(c) $.4 \leq p \leq .9$
(d) $.5 \leq p \leq .9$
(e) $.5 \leq p$

Table Question

Experiment:

1. Your table should make 9 rolls of a 20-sided die (one each if the table is full).
2. Check if all rolls at your table are distinct.

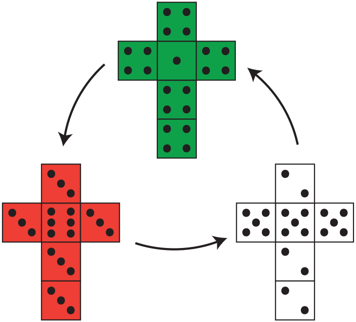
Repeat the experiment five times and record the results.

For this experiment, how would you define the sample space, probability function, and event?

Compute the true probability that all rolls (in one trial) are distinct and compare with your experimental result.

Jon's dice

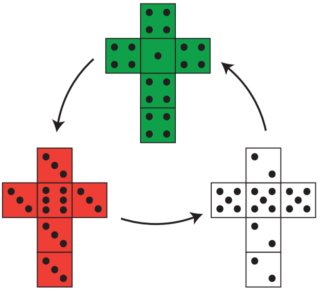
Jon has three six-sided dice with unusual numbering.



A game consists of two players each choosing a die. They roll once and the highest number wins.

Which die would you choose?

Board Question



1. Make probability tables for the red and which dice.
2. Make a probability table for the product sample space of red and white.
3. Compute the probability that red beats white.

4. Pair up with another group. Have one group compare red vs. green and the other compare green vs. red. Based on the three comparisons rank the dice from best to worst.

Computations for solution

	Red die		White die		Green die	
Outcomes	3	6	2	5	1	4
Probability	5/6	1/6	3/6	3/6	1/6	5/6

- The 2×2 tables show pairs of dice.
- Each entry is the probability of seeing the pair of numbers corresponding to that entry.
- The color gives the winning die for that pair of numbers. (We use black instead of white when the white die wins.)

		White		Green	
		2	5	1	4
Red	3	15/36	15/36	5/36	25/36
	6	3/36	3/36	1/36	5/36
Green	1	3/36	3/36		
	4	15/36	15/36		

Answer to board question continued

		White		Green	
		2	5	1	4
Red	3	15/36	15/36	5/36	25/36
	6	3/36	3/36	1/36	5/36
Green	1	3/36	3/36		
	4	15/36	15/36		

The three comparisons are:

$$\begin{aligned} P(\text{red beats white}) &= 21/36 = 7/12 \\ P(\text{white beats green}) &= 21/36 = 7/12 \\ P(\text{green beats red}) &= 25/36 \end{aligned}$$

Thus: red is better than white is better than green is better than red.

There is no best die: the property of being 'better than' is non-transitive.

Concept Question

Lucky Larry has a coin that you're quite sure is not fair.

- He will flip the coin twice
- It's your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?

1. Same
2. Different
3. It doesn't matter, same and different are equally likely

Board Question

Lucky Larry has a coin that you're quite sure is not fair.

- He will flip the coin twice
- It's your job to bet whether the outcomes will be the same (HH, TT) or different (HT, TH).

Which should you choose?

1. Same 2. Different 3. Doesn't matter

Question: Let p be the probability of heads and use probability to answer the question.

(If you don't see the symbolic algebra try p = .2, p=.5)

MIT OpenCourseWare
https://ocw.mit.edu

18.05 Introduction to Probability and Statistics
Spring 2014

For information about citing these materials or our Terms of Use, visit: https://ocw.mit.edu/terms.