18.100A Fall 2012: Assignment 16

Directions: Same as before. This is longer (20 pts.) than usual, but the questions are not difficult, and a lot of them deal with familiar calculus techniques.

For the two Questions, try to do them without looking at the answers, if possible.

Reading: Mon. 19.3, 20.1-.4(p. 276)

1. (3: 1,2) Riemann Sums:
a) Work Question 19.3/1.
b) Work 19.3/3ab. The trapezoidal rule is a standard numerical method used for evaluating integrals, in all calculus textbooks; it's meant to provide motivation for the problem, but all you need to do the problem is the formula given here.

2. (2: .5,1.5) The First Fundamental Theorem:
a) Read the proof, then do Question 20.1/1.
b) Work 20.1/1; include the question about the hypotheses needed.

3. (3: 1.5, 1.5) The Second Fundamental Theorem:
a) Work 20.2/3, for the case $\Delta x > 0$ (as in the book's proof).
b) Work Problem 20-2ab, for the function $f(x) = \text{sgn} x, \ x \neq 0; \ f(0) = 1$ (p. 128).

4. (2) Two easy applications of the theorems:
a) Work 20.2/4
b) Work 20.3/2. (You can assume the result in 20.3/1b.)

5. (2) Work Problem 20-3b, but skip the last questioning sentence; cite references throughout. Start with the inequalities
$$f(\underline{x}) \leq f(x) \leq f(\bar{x}) \ \text{ for all } x \ \epsilon \ [a,b], \quad (\underline{x} = \text{min. pt.}, \ \bar{x} = \text{max. pt.})$$

6. (2) Usually integration by parts is used to make an integral look better. In this somewhat bizarre application, it is used to make it look worse.

The idea is to prove Taylor's theorem (17.2), but just for the case $n = 1$: Theorem 16.1B, the Linearization Error Theorem. (The general case n arbitrary is proved the same way.)

a) Follow the directions in P20-4a for $n = 1$, getting the remainder term $R_1(x)$ as a definite integral ("the integral remainder").

(The integration by parts is used in a strange way. As advised, watch the signs carefully, and remember that x is just an unspecifed constant, as far as the integral is concerned.)

b) Convert the remainder term to the form it has in Theorem 16.1B ("the Lagrange remainder" or "the derivative remainder") following the method described in P20-4b, using the result in Problem 5 above.

Reading: (Wed.) 19.1-2, 19.4; 19.5: skip proofs. The Riemann integral: def'n, ppties.

7. (2) Work 19.2/1, using the lower sums rather than the upper sums.

8. (2: 1.5, .5) Work 19.4/2ab. (This is an important theorem.)

You have to find and interpose a "buffer" $g(x)$ – not a number this time, but rather a function such that $\int_a^b g(x) > 0$ and $f(x) \geq g(x) \geq 0$.

Use $>$ and $\geq$ carefully; and no circular reasoning – justifying a step by using as the reason for it the very theorem you're trying to prove!

9. (2) Work Problem 19-1(a). (Note that $f(x)$ is not assumed to be continuous.)

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