18.306 Advanced Partial Differential Equations with Applications
Fall 2009

Lecture 01 2009 09 09 WED

TOPICS: Mechanics of the course.
        Example pde. Initial and boundary value problems.
        Well and ill-posed problems.

Introduction: Syllabus issues; exams; lecturer; etc.
ODE's and PDE's
ODE solution: determined by a set of constants. Simple Examples.
PDE solution: determined by functions.   Simple Examples.
Example pde: heat, laplace, wave, ... more later.

Initial value and boundary value problems.
Quote existence-uniqueness theorem for ODE IV problem.
No analogous theorem for PDE's. Closest is C-K theorem, and need very
strong restrictions (e.g.: analytic functions.)

Well and ill-posed problems. Why is this important.
Examples (show these are badly ill-posed: growth rate of perturbations
          goes to infinity as the frequency grows. No control over
errors.
--- Can you recover the temperature in the past from today's data?
--- Can you recover the steady state temperature inside a body from
    knowledge of the temperature and heat flux along some part of the
    boundary? [Do example of square, with temperature and flux given on
    a side, zero temperature on the two adjoining sides, and nothing
    known about the opposite side].
--- Real life: issues like this (open problems) appear in multi-phase
    flows modeling, phase transition modeling, detonation waves (square
    wave model), image reconstruction.
Possible Fix: filtering. Works if filtering makes sense within context
    of problem (e.g. CAT scans or image reconstruction). It can also
    lead to nonsense if applied mechanically.