18.306 Advanced Partial Differential Equations with Applications
Fall 2009

Lecture 21 2009 11 23 MON

TOPICS. Gas dynamics in 1-D. Characteristics, simple waves, Riemann Invariants, rarefaction waves, shocks and shock conditions. Riemann problem. Generalizations to N by N systems.

Example: Gas Dynamics in 1-D. Isentropic % ----------

Formulation in terms of mass Lagrangian coordinates.
Riemann Invariants and simple waves. Wave breaking.
Shock conditions (Rankine-Hugoniot ) for systems.
Lax entropy: explain how it works for causality.

Shocks in the p-v plane. Right and left shocks.
Lax entropy equivalent to compressive shocks.
Shock curve: for a fixed ``right'' state on a ``right'' shock, states in phase space (u, v) that can be reached by a shock. Similar curve exists for left shocks, starting from left state.
Rarefaction curve: Same idea s for the shock curve. Write rarefactions using characteristic form, in particular: Riemann Invariants.

RIEMANN PROBLEM:
Show how to solve using the shock/rarefaction curves as a sort of coordinate system in phase space. Describe how solution looks in space-time.

General systems: there are N shock curves and N rarefaction curves. At least locally they can be used to solve the Riemann problem. In general not always clear as the states on the right and left in a Riemann problem get further appart.